DOI https://doi.org/10.24297/jap.v21i.9545

Spin effects from Four Bosons EM

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Abstract

Electromagnetism is the theory of electric charge and spin. Our study is on a spin-valued four bosons electromagnetism. An EM under the charge exchange $\{+, 0, -\}$ intermediated by four bosons $\{A_{\mu I}\} \equiv \{A_\mu, U_\mu, V_\mu^\pm\}$ where A_μ means the usual photon, U_μ a massive photon, $V_\mu^\pm$ charged photons.

EM should express electric charge and spin together. Understand from first principles on the spin role in the electric and magnetic properties of particles. Theoretically, the spin is a space-time physical entity derived from Lorentz group. Phenomenologically, it appears as a vectorial entity inserted in the magnetic moment and electric dipole. A theoretical closure between them is expected.

A spin-valued four bosons EM is constituted by introducing Lorentz group Lie Algebra valued fields. Consider the quadruplet fields as $A_\mu^I = A_{\mu,\kappa\lambda}^I (\Sigma^{\kappa\lambda})_{\alpha\beta}$ where $(\Sigma^{\kappa\lambda})_{\alpha\beta}$ is the Lorentz generator. It provides spin as an intrinsec entity compatible with relativity and group theory. Similarly to the non-abelian gauge theory, where $A_{\mu a} = A_{\mu a} t_a$, one incorporates the spin valued field through a Lie algebra.

From first principles. Electric charge and spin are unified under a constructivist Lagrangian. Spin effects are studied through equations of motion and Bianchi identities. Enlarging the EM for interactions beyond electric charge. Four types are derived. Usual electric charge interaction, neutral interaction, electric charge and spin, neutral and spin.

A formalism is expressed. The spin valued performance is related through a Lagrangian. Spin interactions are derived. The magnetic moment and electric dipole are expressed by vectors $\vec{S}$ and $\vec{s}$ respectivity. They are able to couple spin with granular and collective fields strengths. Developing interacting terms constitutive as $\vec{B} \cdot \vec{S}$, $\vec{E} \cdot \vec{s}$, $\vec{e} \cdot \vec{S}$ and so on. Faraday interaction between magnetic field and photon is reproduced from first principles.

Keywords: nonlinear electrodynamics; new Faraday lines; granular and collective fields strengths; potential fields interacting with EM fields; self interacting photons.

1. Introduction

Electrodynamics incorporates electric charge and spin, but as two distinct physical entities. The first one imposed from a Lagrangian; other written heuristically. Physics has to connect them through first principles. Express their EM features from a common field theory.

Maxwell considerations were on an electromagnetism just based on electric charge [1]. Nevertheless the development of microscopic electromagentism through elementary particles developed three new EM aspects. Electric charge ported by particles with different flavours and spins, charge exchange $\Delta Q = 0, \pm 1$ and spin working as a magnet. Respectively, adding to Maxwell an electrodynamics with spin 1/2, spin-0, spin-1 [2]; a four bosons electromagnetism with charge

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transfer $\{+, 0, -\}$ under four intermediated bosons $\{A_{\mu I}\} = \{A_\mu, U_\mu, V_\mu^I\}$ where A_μ is the usual photon, U_μ massive photon, $V_\mu^{\pm}$ charged photons [3]; and a spin-valued four bosons electromagnetism [4].

Electric charge has a long history in physics [5]. The spin history is recent. It may be divide in three parts. First one, understoond by Tomonaga's book 'The story of spin' [6]. He narrated the period between 1913-1925 when the Bohr model was trying to describe the Balmer spectral emission rays. Relating that, there was a missing concept, the spin. The second, 1921-1925, since Compton with the electron spinning, Stern-Gerlach experience, Goudsmit-Unlenbeck introducing the elecron intrinsec angular momentum and magnetic moment to explain the Zeeman effect [7]. And by third, between 1925-1928, with the spin incorporation into the physical equations. In 1926, Heisenberg and Jordan introduced spin at Quantum Mechanics [8]; in 1927, Pauli the so-called non-minimal coupling by coupling the spin of the electron to an external magnetic field [9]; and in 1928, Dirac describing the relativistic dynamics of the spin of the electron [10].

Dirac equation introduced a four component function known as spinor which describes the behaviour of relativistic particles. This wave function did not just describe the position of the particle, Dirac equation showed that besides mass and charge the electron has intrinsec angular momentum and magnetic moment.

The magnetic moment relating quantities as mass, charge and spin become the next object of phenomenological study. It was heuristically defined as

$$\vec{\mu} = g_L \frac{eh}{2m} \vec{S} \tag{1}$$

Pauli equation found the interaction of spin with the magnetic field, $\vec{\mu} \cdot \vec{B}$, where $\vec{B}$ represents of the external photon EM field, $\vec{S}$ the spin of matter and g_L the gyromagnetic factor.

Dirac equation introduced a gyromagnetic value g=2. In 1947, it was observed anomalies in the factor (g-2) for electron by Kush and Foley [11]. Also shifts in the hyperfine structure hydrogen and deuterium fundamental states. In response Schwinger proposer the 1-loop radioactive [12]. Actually, one gets the following precision for the (g-2) measurements: 10^{-12} for electron, 10^{-10} for muon, 10^{-2} for tau, 10^{-18} for proton, 10^{-7} for neutron [13].

In 1992 Ferrara, Porrati, Telegdgi have shown that any elementary (point form) charge particle owes at tree level the gyromagnetic factor g = 2, independently on its spin. Consequently, for spin-1, a massive charge vectorial bosons $W^{\pm}$, in order to obtain g = 2 has to introduce a non-minimal coupling given $F_{\mu\nu} W^{\mu *} W^{\nu +}$.

Similarly, one obtains the electric dipole moment $\vec{d}$ interacting with the electric field

$$\vec{d} = \varsigma \frac{e}{2m_e} \vec{S} \tag{2}$$

where ς is a dimensionless constant analogous to g_L at magnetic case. The non-relativistic hamiltonian for EM interaction with spin-1/2 was found to be

$$H = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E} \tag{3}$$

The introduces os spin, spin magnetic moment and dipole electric dipole moment played a crucial role in electromagnetism. However, the quest is how to derive eqs. (1.1-1.3) from first principles. Our insight is to interpret spin through Lorentz Group [14]. Define the potential fields as $A^I_{\mu;\kappa\lambda}$ where he first space-time index represents the space-time symmetry $x'^{\mu} = \Lambda^{\mu}_{\nu}\, x^{\nu}$ and the others are field rotation symmetry $A'_{\mu I} = (e^{\frac{i}{2}\omega_{\alpha\beta}\Sigma^{\alpha\beta}})^{\nu}_{\mu} A_{\nu I}$. Introducing the spin-valued fields.

$$A_\mu^I \equiv A_{\mu;\kappa\lambda}^I(\Sigma^{\kappa\lambda}) \tag{4}$$

with

$$(\Sigma_{\mu\nu})_{\alpha\beta} = -i(g_{\mu\nu}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}) \tag{5}$$

obeying the Lie Algebra

$$[\Sigma_{\mu\nu}, \Sigma_{\rho\sigma}] = g_{\mu\rho}\Sigma_{\nu\rho} + g_{\nu\rho}\Sigma_{\mu\rho} - g_{\mu\rho}\Sigma_{\nu\rho} - g_{\nu\sigma}\Sigma_{\mu\rho} \tag{6}$$

Eq. (1.4) introduces spin at ab initio. It is showing that, similarly to the Yang-Mills Lie algebra valued, $A_\mu \equiv A_{\mu a}t_a$, there is a spin physics associated to generator $(\Sigma_{\mu\nu})_\beta^\alpha$. It rewrites the four bosons EM in spin terms. A vector spintronics is generated.

A new origin appears to express an electromagnetism with electric charge and spin together. The required minimal coupling between fields written as

$$L^{spin} \approx g_{IJ}F_{\alpha\beta}A_I^\mu(\Sigma^{\alpha\beta})_{\mu\nu}A_J^\nu \tag{7}$$

Thus, in order to identify the spin interaction, take the correspondent equation of motion

$$\partial_\nu F_I^{\mu\nu} = gF_{\alpha\beta}(\Sigma^{\alpha\beta})_\mu^\nu A_{\mu I} + \text{other terms} \tag{8}$$

Expanding

$$F_{\alpha\beta}(\Sigma^{\alpha\beta})_\nu^\mu A_I^\nu = F_{0i}(\Sigma^{0i})_\nu^\mu A_I^\nu + F_{ij}(\Sigma^{ij})_\nu^\mu A_I^\nu \tag{9}$$

Defining the vectorial entity relate to electric field.

$$\Sigma^{0i} = s^i \tag{10}$$

one gets, for the first term

$$F_{0i}(\Sigma^{0i})_\nu^\mu A_I^\nu = -(E_i s_i)_\nu^\mu A_I^\nu = -(\vec{E}\cdot\vec{s})_\nu^\mu A_I^\nu \tag{11}$$

The vectorial entity corresponding to magnetic field is

$$F_{ij}(\Sigma^{ij})_\nu^\mu A_I^\nu = -(\epsilon_{ijk}B_k)(\Sigma^{ij})_\nu^\mu A_I^\nu \tag{12}$$

Defining other vectorial entity

$$S_k \equiv \epsilon_{ijk}\Sigma_{jk} \tag{13}$$

one gets, for the second term

$$gF_{ij}(\Sigma^{ij})^{\alpha}_{\nu}A^{\nu}_{I} = -g(B_k S_k)^{\mu}_{\nu}A^{\nu}_{I} = -g(\vec{B}\cdot\vec{S})^{\mu}_{\nu}A^{\nu}_{I} \tag{14}$$

Thus, the spin interaction is expressed in vectorial form as

$$gF_{\alpha\beta}(\Sigma^{\alpha\beta})^{\mu}_{\nu}A^{\nu}_{I} = -g[(\vec{E}\cdot\vec{s})^{\mu}_{\nu} + (\vec{B}\cdot\vec{S})^{\mu}_{\nu}]A^{\nu}_{I} \tag{15}$$

Eq. (1.15) defines the presence of a spin charge. It provides a coupling constant g which can be the electric charge or not. It may express the electric charge, modulated electric charge or a neutral charge. Something showing that EM contains a spin interaction beyond electric charge.

The quadri-electric dipole and quadri-magnetic moment of a given field A^{μ}_{I} are expressed as

$$\vec{d}^{\mu}_{I} = \vec{s}^{\mu}_{\nu}A^{\nu}_{I}\ ;\ \vec{\mu}^{\mu}_{I} = \vec{S}^{\mu}_{\nu}A^{\nu}_{I} \tag{16}$$

which yields

$$\vec{d}^{\mu}_{I} = \vec{s}^{\mu}_{0}A^{0}_{I} + \vec{s}^{\mu}_{i}A^{i}_{I};\ \vec{\mu}^{\mu}_{I} = \vec{S}^{\mu}_{0}A^{0}_{I} + \vec{S}^{\mu}_{i}A^{i}_{I} \tag{17}$$

Eqs. (1.16-1.17) are defining the electric dipole and magnetic moment from first principles. They are extending the usual definition given by eqs. (1.1) and (1.2). New terms appear to be measured.

2. Spin valued Four Bosons Electromagnetism

The introduction of the concept of spin, starting with electron, become a beginning of a wide application of this physics. Indead, after the electron the spin turned out an attribute to all elentarity particles. Our objective here is to study for spin-1.

A spintronic with spin-1 will be studied based on an electromagnetism based on three charges transmissions $\{+, 0, -\}$ and four intermediated by four bosons $\{A_{\mu}, U_{\mu}, V^{+}_{\mu}, V^{-}_{\mu}\}$. A fields set associated through the following spin-valued gauge transformation.

$$\begin{aligned}
A'_{\mu;\kappa\lambda}(\Sigma^{\kappa\lambda}) &= A_{\mu;\kappa\lambda}(\Sigma^{\kappa\lambda}) + k_1\partial_{\mu}\alpha \\
U'_{\mu;\kappa\lambda}(\Sigma^{\kappa\lambda}) &= U_{\mu;\kappa\lambda}(\Sigma^{\kappa\lambda}) + k_1\partial_{\mu}\alpha \\
V'^{+}_{\mu;\kappa\lambda}(\Sigma^{\kappa\lambda}) &= e^{iq\alpha}V'^{+}_{\mu;\kappa\lambda}(\Sigma^{\kappa\lambda}) \\
V'^{-}_{\mu;\kappa\lambda}(\Sigma^{\kappa\lambda}) &= e^{-iq\alpha}V'^{-}_{\mu;\kappa\lambda}(\Sigma^{\kappa\lambda})
\end{aligned} \tag{18}$$

2. Spin-valued fields strengths

Recapitulating, the antisymmetric sector:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad F'_{\mu\nu} = F_{\mu\nu} \tag{19}$$

$$U_{\mu\nu} = \partial_\mu U_\nu - \partial_\nu U_\mu, \quad U'_{\mu\nu} = U_{\mu\nu} \tag{20}$$

$$V^{\pm}_{\mu\nu} = D_\mu V^{\pm}_\nu - D_\nu V^{\pm}_\mu, \quad V^{\pm'}_{\mu\nu} = e^{\pm iq\alpha} V^{\pm}_{\mu\nu} \tag{21}$$

where the covariant derivative is given by $D_\mu = \partial_\mu + i(q_1 A_\mu + q_2 U_\mu)$. The couplings q_1 and q_2 are modulated electric charge given by $q_1 = a \cdot q$; $q_2 = b \cdot q$, where $ak_1 + bk_2 = -1$.

The corresponding antisymmetric vectorial fields strengths are written as

$$F^I_{\mu\nu} = \partial_\mu A^I_\nu - \partial_\nu A^I_\mu; \qquad \vec{E}^I_i = F^I_{0i}; \qquad \vec{B}^I_i = \frac{1}{2}\epsilon_{ijk} F^I_{jk} \tag{22}$$

where $A^\mu_I \equiv \left(\phi_I, \vec{A}_I\right)$ and I is a flavour indice $I = 1, ...4$ corresponding to $A_\mu, U_\mu, V^{\pm}_\mu$

For the symmetric sector:

$$S_{\mu\nu 1} = \partial_\mu A_\nu + \partial_\nu A_\mu; \; S_{\mu\nu 2} = \partial_\mu U_\nu + \partial_\nu U_\mu. \tag{23}$$

the invariance is under the condition

$$S^{1'}_{\mu\nu} + S^{2'}_{\mu\nu} = S^1_{\mu\nu} + S^2_{\mu\nu} \tag{24}$$

Charged fields are transforming as

$$S^{\pm}_{\mu\nu} = D_\mu V^{\pm}_\nu + D_\nu V^{\pm}_\mu, \quad S^{\pm'}_{\mu\nu} = e^{\pm iq\alpha} S^{\pm}_{\mu\nu} \tag{25}$$

Longitudinal terms are written as

$$S^\alpha_{\alpha 1} = 2\partial_\alpha A^\alpha, S^\alpha_{\alpha 2} = 2\partial_\alpha U^\alpha. \tag{26}$$

with

$$S^{\alpha 1'}_\alpha + S^{\alpha 2'}_\alpha = S^{\alpha 1}_\alpha + S^{\alpha 2}_\alpha \tag{27}$$

and

$$S^{\alpha\pm}_\alpha = D_\alpha V^{\alpha\pm}, \quad S^{\alpha\pm'}_\alpha = e^{\pm iq\alpha} S^{\alpha\pm}_\alpha \tag{28}$$

For the granular symmetric fields strengths, one gets

$$S^I_{\mu\nu} = \partial_\mu A^I_\nu + \partial_\nu A^I_\mu; \qquad S^{\alpha I}_\alpha = 2\partial_\alpha A^{\alpha I} \tag{29}$$

$$S^I_{0i} = \partial_0 A^I_i + \partial_i A^I_0, \qquad S^I_{ij} = \partial_i A^I_j + \partial_j A^I_i \tag{30}$$

Collective fields strengths are expressed as

The antisymmetric collective fields strengths are written as

$$e_{[\mu\nu]} = \mathbf{e}_{[IJ]} A^I_\mu A^J_\nu, \qquad \vec{\mathbf{e}}_i = \mathbf{e}_{[0i]}, \qquad \vec{\mathbf{b}}_i = \frac{1}{2}\epsilon_{ijk}\mathbf{e}_{[jk]} \tag{31}$$

It gives,

$$\mathbf{e}_{[\mu\nu]}^{[12]'} = \frac{1}{2}\mathbf{e}_{[12]}\left(A_\mu U_\mu - U_\mu A_\mu\right) = \mathbf{e}_{[\mu\nu]}^{[12]} \tag{32}$$

$$\mathbf{e}_{[\mu\nu]}^{[+-]'} = i\mathbf{e}_{[34]}\left(V_\mu^+ V_\nu^- - V_\mu^- V_\nu^+\right) = \mathbf{e}_{[\mu\nu]}^{[12]} \tag{33}$$

and

$$\mathbf{e}_{[\mu\nu]}^{[12+]} = \frac{1}{2}\mathbf{e}_{[12]}\left[\left(A_\mu - U_\mu\right)V_\nu^+ + \left(A_\nu - U_\nu\right)V_\mu^+\right] \tag{34}$$

$$\mathbf{e}_{[\mu\nu]}^{[12-]} = \frac{1}{2}\mathbf{e}_{[12]}\left[\left(A_\mu - U_\mu\right)V_\nu^- + \left(A_\nu - U_\nu\right)V_\mu^-\right] \tag{35}$$

transforming as

$$\mathbf{e}_{[\mu\nu]}^{[12+]'} = e^{iq\alpha(x)}\mathbf{e}_{[\mu\nu]}^{[12+]}, \tag{36}$$

$$\mathbf{e}^{[12-]'} = e^{-iq\alpha(x)}\mathbf{e}_{[\mu\nu]}^{[12-]} \tag{37}$$

The collective symmetric fields strengths are

$$\mathbf{e}_{(\mu\nu)} = \mathbf{e}_{(11)}A_\mu A^\nu + \mathbf{e}_{(12)}\left(A_\mu U_\nu + U_\mu A_\nu\right) + \mathbf{e}_{(22)}U_\mu U_\nu = \mathbf{e}'_{(\mu\nu)}$$

$$\mathbf{e}_{(\mu\nu)}^{(+-)} = \mathbf{e}_{(34)}V_\mu^+ V_\nu^- = \mathbf{e}_{(\mu\nu)}^{(+-)'} \tag{38}$$

and

$$\mathbf{e}_\alpha^\alpha = \mathbf{e}_{(11)}A_\alpha A^\alpha + 2\mathbf{e}_{(12)}A_\alpha U^\alpha + \mathbf{e}_{(22)}U_\alpha U^\alpha = \mathbf{e}_\alpha^{\alpha'} \tag{39}$$

$$\mathbf{e}_\alpha^{(+-)\alpha} = \mathbf{e}_{(34)}V_\alpha^+ V^{-\alpha} = \mathbf{e}_\alpha^{(+-)\alpha'} \tag{40}$$

Other collective fields strengths are included

$$\mathbf{e}_{(\mu\nu)}^{(12+)} = \left[\mathbf{e}_{(11)}A_\mu + \mathbf{e}_{(12)}\left(A_\mu + U_\mu\right) + \mathbf{e}_{(22)}U_\mu\right]V_\nu^+ \tag{41}$$

$$\mathbf{e}_{(\mu\nu)}^{(12-)} = \left[\mathbf{e}_{(11)}A_\mu + \mathbf{e}_{(12)}\left(A_\mu + U_\mu\right) + \mathbf{e}_{(22)}U_\mu\right]V_\nu^- \tag{42}$$

$$\mathbf{e}_{(\mu\nu)}^{(++)} = \frac{1}{2}\left(\mathbf{e}_{(33)} - \mathbf{e}_{(44)}\right)V_\mu^+ V_\nu^+ \tag{43}$$

$$\mathbf{e}_{(\mu\nu)}^{(--)} = \frac{1}{2}\left(\mathbf{e}_{(33)} - \mathbf{e}_{(44)}\right)V_\mu^- V_\nu^- \tag{44}$$

transforming as

$$\mathbf{e}_{(\mu\nu)}^{(12+)'} = e^{iq\alpha(x)}\mathbf{e}_{(\mu\nu)}^{(12+)}, \quad \mathbf{e}_{(\mu\nu)}^{(12-)'} = e^{-iq\alpha}\mathbf{e}_{(\mu\nu)}^{(12-)} \tag{45}$$

$$\mathbf{e}_{(\mu\nu)}^{(++)'} = e^{2iq\alpha(x)}\mathbf{e}_{(\mu\nu)}^{(++)'}, \quad \mathbf{e}_{(\mu\nu)}^{(--)'} = e^{-2iq\alpha(x)}\mathbf{e}_{(\mu\nu)}^{(--)'} \tag{46}$$

Similarly,

$$\mathbf{e}_\alpha^{(12+)\alpha} = \left[\mathbf{e}_{(11)}A_\alpha + \mathbf{e}_{(12)}\left(A_\alpha + U_\alpha\right) + \mathbf{e}_{(22)}U_\alpha\right]V^{+\alpha}, \tag{47}$$

$$\mathbf{e}_\alpha^{(12-)\alpha} = \left[\mathbf{e}_{(11)}A_\alpha + \mathbf{e}_{(12)}\left(A_\alpha + U_\alpha\right) + \mathbf{e}_{(22)}U_\alpha\right]V^{-\alpha} \tag{48}$$

$$\mathbf{e}_\alpha^{(++)\alpha} = \frac{1}{2}\mathbf{e}_{(34)}V_\alpha^+ V^{\alpha+}, \mathbf{e}_\alpha^{(--)\alpha} = \mathbf{e}_{(34)}V_\alpha^- V^{\alpha-} \tag{49}$$

$$\mathbf{e}_{\alpha}^{(12+)\alpha}=e^{iq_2\alpha(x)}\mathbf{e}_{\alpha}^{(12+)\alpha},\mathbf{e}_{\alpha}^{(12-)\alpha}=e^{-iq_2\alpha(x)}\mathbf{e}_{\alpha}^{(12-)\alpha} \tag{50}$$

$$\mathbf{e}_{\alpha}^{(++)\alpha}=e^{2iq_2\alpha(x)}\mathbf{e}_{\alpha}^{(++)\alpha},\mathbf{e}_{\alpha}^{(--)\alpha}=e^{-2iq_2\alpha}\mathbf{e}_{\alpha}^{(--)\alpha} \tag{51}$$

The simetric sector for collective

$$s_{\alpha IJ}^{\alpha}=\mathbf{e}_{(IJ)}A_{\alpha}^{I}A^{\alpha J} \tag{52}$$

$$s_{ij}^{IJ}=\mathbf{e}_{(IJ)}\left(A_i^I A_j^J+U_i^J A_j^J\right) \tag{53}$$

Thus the corresponding spin-valued granular and collective fields strengths will transform as

$$\begin{aligned}&F_{\mu\nu}\rightarrow(\Sigma_{\mu\nu})_{\rho\sigma}F^{\rho\sigma}\\&e_{[\mu\nu]}\rightarrow(\Sigma_{\mu\nu})_{\rho\sigma}e^{[\rho\sigma]}\\&\beta_I S_{\mu\nu}\rightarrow(\Sigma_{\mu\nu})_{\rho\sigma}\beta_I S^{\rho\sigma}\end{aligned} \tag{54}$$

with

$$F_{\mu\nu}F^{\mu\nu}=\frac{1}{8}(\Sigma^{\mu\nu})_{\sigma}^{\rho}(\Sigma^{\kappa\lambda})_{\rho}^{\sigma}F_{\mu\nu}F_{\kappa\lambda}+\frac{1}{4}(\Sigma^{\mu\kappa})_{\sigma}^{\rho}(\Sigma^{\nu\lambda})_{\rho}^{\sigma}F_{\mu\nu}F_{\kappa\lambda} \tag{55}$$

and soon [4].

2. Lagrangian

The abelian Lagrangian is subdivided in three parts

$$L=L_K+L_I^3+L_I^4 \tag{56}$$

where each one contains antisymmetric and symmetric pieces.

2.2.1 Antisymmetric sector

Considering eq. (2.40),

$$L_K=6a_1F_{\mu\nu}F^{\mu\nu}+6a_2U_{\mu\nu}U^{\mu\nu}+6a_3V_{\mu\nu}^{+}V^{\mu\nu-}, \tag{57}$$

$$\begin{aligned}&L_I^3=-2i(\Sigma_{\mu\nu})_{\kappa\lambda}[(b_1F^{\mu\nu}+b_2U^{\mu\nu})(e^{[12][\kappa\lambda]}+e^{[+-][\kappa\lambda]})+\\&+b_3(V^{\mu\nu+}e^{[12-][\kappa\lambda]}+V^{\mu\nu-}e^{[12+][\kappa\lambda]})]-2i(\Sigma_{\mu\kappa})_{\nu\lambda}[(b_1F^{\mu\nu}+\\&b_2U^{\mu\nu})(e^{[12][\kappa\lambda]}+e^{[+-][\kappa\lambda]})+b_3(V^{\mu\nu+}e^{[12-][\kappa\lambda]}+V^{\mu\nu-}e^{[12+][\kappa\lambda]})]\end{aligned} \tag{58}$$

and

$$L_I^4 = -4i(\Sigma_{\mu\nu})_{\kappa\lambda}[(e^{[12][\mu\nu]} + e^{[+-][\mu\nu]})(e^{[12][\kappa\lambda]} + e^{[+-][\kappa\lambda]}) + e^{[12+][\mu\nu]}e^{[12+][\kappa\lambda]}] \tag{59}$$

Notice that L_I^3 contains the non-minimal coupling, eq. 1.7, which is the necessary fields theory condition for $g = 2$. It also shows the photon spin coupling with an EM external field.

2.2.2 Symmetric sector

$$L_S = L_{SK} + L_{SI}^3 + L_{SI}^4 \tag{60}$$

where

$$\begin{aligned} &L_{SK} = -2i(\Sigma_{\mu\kappa})_{\nu\lambda}(\beta_1 S_1^{\mu\nu} + \beta_2 S_2^{\mu\nu})(\beta_1 S_1^{\kappa\lambda} + \beta_2 S_2^{\kappa\lambda}) - 2i(\Sigma_{\mu\kappa})_{\nu\lambda}S^{\mu\nu+}S^{\kappa\lambda-} \\ &+24(\rho_1 S_{\alpha 1}^{\alpha} + \rho_2 S_{\alpha 2}^{\alpha})(\rho_1 S_{\beta 1}^{\beta} + \rho_2 S_{\beta 2}^{\beta}) + 24\rho_3 S_\alpha^{\alpha+} S_\beta^{\beta-}, \end{aligned} \tag{61}$$

$$\begin{aligned} &L_{SI}^3 = -2i(\Sigma_{\mu\kappa})_{\nu\lambda}(\beta_1 S_1^{\mu\nu} + \beta_2 S_2^{\mu\nu})(\mathbf{e}^{(11)(\kappa\lambda)} + \mathbf{e}^{(22)(\kappa\lambda)} + \mathbf{e}^{(12)(\kappa\lambda)} + \mathbf{e}^{(+-)(\kappa\lambda)}) \\ &-2i\beta_3(\Sigma_{\mu\kappa})_{\nu\lambda}(S^{\mu\nu+}\mathbf{e}^{(12-)(\kappa\lambda)} + S^{\mu\nu-}\mathbf{e}^{(12+)(\kappa\lambda)}) + 24(\rho_1 S_{\alpha 1}^{\alpha} + \rho_2 S_{\alpha 2}^{\alpha})(\mathbf{e}_\alpha^{(11)\alpha} + \\ &+\mathbf{e}_\alpha^{(12)\alpha} + \mathbf{e}_\alpha^{(22)\alpha} + \mathbf{e}_\alpha^{(+-)\alpha}) + 24\rho_3 S_\alpha^{\alpha+} + 24\rho_3 S_\alpha^{\alpha-}\mathbf{e}_\beta^{(12+)\beta} \end{aligned} \tag{62}$$

and

$$\begin{aligned} &L_{SI}^4 = -2i(\Sigma_{\mu\kappa})_{\nu\lambda}(\mathbf{e}^{(11)(\mu\nu)} + \mathbf{e}^{(22)(\mu\nu)} + \mathbf{e}^{(12)(\mu\nu)} + \mathbf{e}^{(+-)(\mu\nu)})(\mathbf{e}^{(11)(\kappa\lambda)} + \mathbf{e}^{(22)(\kappa\lambda)} \\ &+\mathbf{e}^{(12)(\kappa\lambda)} + \mathbf{e}^{(+-)(\kappa\lambda)}) - 2i(\Sigma_{\mu\kappa})_{\nu\lambda}\mathbf{e}^{(12+)(\mu\nu)}\mathbf{e}^{(12-)(\kappa\lambda)} + 24(\mathbf{e}_\alpha^{(11)\alpha} + \mathbf{e}_\alpha^{(12)\alpha} + \mathbf{e}_\alpha^{(22)\alpha} \\ &+\mathbf{e}_\alpha^{(+-)\alpha})^2 + \mathbf{e}_\alpha^{(12+)\alpha}\mathbf{e}_\alpha^{(12-)\beta} \end{aligned} \tag{63}$$

3. Fields equations

We should study now how fields equations are narrating the spin-valued physics. Four types of equations are derived.

3. Euler-Lagrange equations

A differential hyperbolic equations system is derived for four fields. It gives,

$$\begin{aligned} &\partial_\nu[24\sigma_I Z^{[\nu\mu]} + 18\eta\tilde{Z}^{[\nu\mu]} - 8\beta_I Z^{(\nu\mu)} + 4(2\beta_I - 3\rho_I)g^{\mu\nu}Z_\alpha^\alpha] = \\ &= 12e_{[IJ]}(\Sigma_\nu^\mu)_{\alpha\beta}Z^{[\alpha\beta]}G^{\nu J} + 6i\eta e_{[IJ]}(\Sigma_{\mu\nu})_{\alpha\beta}Z^{[\alpha\beta]}G^{\nu J} + \\ &+8ie_{(IJ)}(\Sigma_\alpha^\mu)_{\nu\beta}Z^{(\alpha\beta)}G^{\nu J} - 3i\eta e_{[IJ]}(\Sigma_{\alpha\beta})_{\kappa\lambda}\epsilon^{\nu\mu\kappa\lambda}Z^{[\alpha\beta]}G_\nu^J + \\ &-4\tau_{(IJ)}Z_\alpha^\alpha G^{\mu J} \end{aligned} \tag{64}$$

where

$$Z^{[\mu\nu]} = \sigma_I G^{\mu\nu I} + b_I e^{[\mu\nu]}, \tag{65}$$

$$Z^{(\mu\nu)} = \beta_I S^{\mu\nu I} + \rho_I g^{\mu\nu} S_\alpha^{\alpha I} + e^{(\mu\nu)} + g^{\mu\nu}\omega_\alpha^\alpha \tag{66}$$

and

$$Z_\alpha^\alpha = (\beta_I + \rho_I) S_\alpha^{\alpha I} + e_\alpha^\alpha + 4\omega_\alpha^\alpha \tag{67}$$

The above equations are showing that every field the quadruplet contains its own charge. It is expressed through the correspondent continuity equations. Indicating that electric charge is no more the only one source for electromagnetism.

Considering, as example, just the photon field, one gets for the sin-1 sector.

$$\partial_\nu \{12a_1 F^{\nu\mu} + 6b_1(e^{[12][\nu\mu]} + e^{[+-][\nu\mu]})\} = j_{AT}^\mu \tag{68}$$

one gets,

$$\begin{aligned} j_{AT}^\mu &= i(\Sigma_\nu^\mu)_{\alpha\beta}[\mathbf{e}_{[12]}(b_1 F^{\alpha\beta} + b_2 U^{\alpha\beta})U^\nu + b_3\mathbf{e}_{[34]}(V^{\alpha\beta+}V^{\nu-} + V^{\alpha\beta-}V^{\nu+})] \\ &-i(\Sigma_\alpha^\mu)_{\beta\nu}[\mathbf{e}_{[12]}(b_1 F^{\alpha\beta} + b_2 U^{\alpha\beta})U^\nu + b_3\mathbf{e}_{[34]}(V^{\alpha\beta+}V^{\nu-} + V^{\alpha\beta-}V^{\nu+})] \\ &+2i\mathbf{e}_{[12]}(\Sigma_\nu^\mu)_{\alpha\beta}(e^{[12][\mu\nu]} + e^{[+-][\alpha\beta]})U^\nu + i(\Sigma_\nu^\mu)_{\alpha\beta}(e^{[12-][\alpha\beta]}V^{\nu+} + e^{[12+][\alpha\beta]}V^{\nu-}) + \\ &-2i(\Sigma_\alpha^\mu)_{\beta\nu}(e^{[12][\mu\nu]} + e^{[+-][\alpha\beta]})U^\nu - i(\Sigma_\alpha^\mu)_{\beta\nu}(e^{[12-][\alpha\beta]}V^{\nu+} + e^{[12+][\alpha\beta]}V^{\nu-}) \end{aligned} \tag{69}$$

Eq. (3.6) shows spin-valued charged fields interacting with granular and collective fields strengths where modulated electric charges and chargeless field are included.

3. Noether equations

A second type of equation from eq. (2.1) is the Noether theorem. It yields the following three equations.

Charge conservation:

$$\partial_\mu J_N^\mu = 0 \tag{70}$$

Symmetry equation:

$$\partial_\mu K^{\mu\nu} + J_N^\nu = 0 \tag{71}$$

Constraint:

$$\partial_\mu \partial_\nu K^{\mu\nu} = 0 \tag{72}$$

where

$$\begin{aligned} J_N^\mu &= iq\{[6a_3 V^{\mu\nu-} + 3a_3 e^{[12-][\mu\nu]} - 4\beta_3 S^{\mu\nu-} + (48\rho_3 + \beta_3)g^{\mu\nu}S_\alpha^{\alpha-} + \\ &-4\beta_3 e^{(12-)(\mu\nu)} + (4\beta_3 + 48\rho_3)g^{\mu\nu}e_\alpha^{(12-)\alpha}](\Sigma^{\kappa\lambda})V_{\nu;\kappa\lambda}^+ - [6a_3 V^{\mu\nu+} + \\ &3a_3 e^{[12+][\mu\nu]} - 4\beta_3 S^{\mu\nu+} + (48\rho_3 + \beta_3)g^{\mu\nu}S_\alpha^{\alpha+} - 4\beta_3 e^{(12+)(\mu\nu)} + \\ &(4\beta_3 + 48\rho_3)g^{\mu\nu}e_\alpha^{(12+)\alpha}](\Sigma^{\kappa\lambda})V_{\nu;\kappa\lambda}^-\} \end{aligned} \tag{73}$$

Considering the symmetry equation one gets the following electric charge conservation expression.

$$\begin{aligned}
&\partial_\mu\{k_1[24a_1F^{\mu\nu}+12b_1(e^{[12][\mu\nu]}+e^{[+-][\mu\nu]})-8(\beta_1S_1^{\mu\nu}+\beta_1\beta_2S_2^{\mu\nu})\\
&g^{\mu\nu}(8\beta_1S_{\alpha1}^{\alpha}+96\rho_1S_{\alpha1}^{\alpha}+8\beta_1\beta_2S_{\alpha}^{\alpha}+96\rho_1\rho_2S_{\alpha2}^{\alpha}-4\beta_1e^{(\mu\nu)})+\\
&+g^{\mu\nu}(4\beta_1+48\rho_1)e_{\alpha}^{\alpha}]+k_2[24a_2U^{\mu\nu}+12b_2(e^{[12][\mu\nu]}+e^{[+-][\mu\nu]})\\
&-8(\beta_2S_2^{\mu\nu}+\beta_1\beta_2S_1^{\mu\nu})+g^{\mu\nu}(8\beta_2S_{\alpha2}^{\alpha}+96\rho_2S_{\alpha2}^{\alpha}+8\beta_1\beta_2S_{\alpha1}^{\alpha},\\
&+96\rho_1\rho_2S_{\alpha1}^{\alpha}-4\beta_2e^{(\mu\nu)})+g^{\mu\nu}(4\beta_2+48\rho_2)e_{\alpha}^{\alpha}]\}=-J_N^\mu
\end{aligned}\tag{74}$$

where J_N^μ means the electric charge working as fields sources.

Separating eq. (3.11) in transverse and longitudinal sectors, one gets

$$\begin{aligned}
&\partial_\mu\{(24a_1k_1-8\beta_1k_1-8\beta_1\beta_2k_2)F^{\nu\mu}+(24a_2k_2+k_1\beta_1\beta_2-8\beta_2k_2U^{\nu\mu})\\
&+12(b_1k_1+b_2k_2)(e^{[12][\nu\mu]}+e^{[+-][\nu\mu]})\}=J_{NT}^\nu-j_N^\nu
\end{aligned}\tag{75}$$

where $J_{NT}^\mu=\theta^{\mu\nu}J_{\nu N}$

with

$$J_{NT}^\mu=iq\{\big[6a_3V^{\mu\nu-}+3a_3e^{[12-][\mu\nu]}\big]V_\nu^+-\big[6a_3V^{\mu\nu+}+3a_3e^{[12+][\mu\nu]}\big]V_\nu^+\tag{76}$$

and

$$\begin{aligned}
&j_N^\mu=-\frac{1}{2}S_{\alpha2}^{\alpha}A^\mu+\mathbf{e}_{(11)}S^{\mu\nu1}A_\mu+\frac{1}{2}\mathbf{e}_{(22)}S_{\alpha2}^{\alpha}U^\mu+\mathbf{e}_{(22)}S^{\mu\nu2}U_\nu\\
&+\frac{1}{2}\mathbf{e}_{(12)}(S_{\alpha2}^{\alpha}A^\nu+S_{\alpha}^{\alpha1}U^\nu)+\mathbf{e}_{(12)}(S_1^{\mu\nu}U_\nu+S_2^{\mu\nu}A_\nu)+\\
&\frac{1}{2}\mathbf{e}_{(34)}(S_\alpha^{\alpha+}V^{\mu-}+S_\alpha^{\alpha-}V^{\mu+})+(S^{\mu\nu+}V_\nu^-+S^{\mu\nu-}V_\nu^+)
\end{aligned}\tag{77}$$

For spin-0:

$$\begin{aligned}
&\partial^\nu\{(96\rho_1k_1+8\beta_2k_2)S_{\alpha1}^{\alpha}+(96\rho_2k_2+8\beta_1k_1)S_{\alpha2}^{\alpha}\\
&+[k_1(8\beta_1+48\rho_1)+k_2(8\beta_2+48\rho_2)]e_\alpha^\alpha\}=J_{NL}^\nu-j_N^\nu
\end{aligned}\tag{78}$$

where $J_{NL}^\mu=\omega^{\mu\nu}J_{\nu N}$.

and

$$\begin{aligned}
&J_{NL}^\mu=iq\{\big[-4\beta_3S^{\mu\nu-}+(48\rho_3+\beta_3)g^{\mu\nu}S_\alpha^{\alpha-}-4\beta_3e^{(12-)(\mu\nu)}+\\
&+(4\beta_3+48\rho_3)g^{\mu\nu}e_\alpha^{(12-)\alpha}\big]V_\nu^+-\big[-4\beta_3S^{\mu\nu+}+(48\rho_3+\beta_3)g^{\mu\nu}S_\alpha^{\alpha+}\\
&-4\beta_3e^{(12+)(\mu\nu)}+(4\beta_3+48\rho_3)g^{\mu\nu}e_\alpha^{(12+)\alpha}\big]V_\nu^-\}
\end{aligned}$$

Considering on charge conservation law, one gets

For spin-1 sector:

$$
\begin{aligned}
&\partial_\mu J^\mu_{NT} = iq\{-2qe_{[34]}(\Sigma^\mu_\nu)_{\alpha\beta}[(A_\alpha + U_\alpha)V^+_\beta(A_\nu - U_\nu) - (A_\beta + U_\beta)V^+_\alpha(A_\nu - U_\nu)]V^{\mu-} \\
&+ie_{[34]}(b_1F^{\alpha\beta} + b_2U^{\alpha\beta})(\Sigma^\mu_\nu)_{\alpha\beta}V^{\nu+}V^-_\mu - ie_{[34]}(b_1F^{\alpha\beta} + b_2U^{\alpha\beta})(\Sigma^\mu_\alpha)_{\beta\nu}V^{\mu+}V^-_\nu \\
&-ib_3e_{[34]}V^{\alpha\beta+}[(\Sigma^\mu_\nu)_{\alpha\beta} + (\Sigma^\mu_\alpha)_{\beta\nu}](A_\nu + U_\nu)V^{\mu-} + \partial_\mu V^+_\nu[a_3V^{\mu\nu-} + b_3e^{[12-][\mu\nu]}] \\
&-2qe_{[34]}(\Sigma^\mu_\nu)_{\alpha\beta}[(A_\alpha + U_\alpha)V^-_\beta(A_\nu - U_\nu) - (A_\beta + U_\beta)V^-_\alpha(A_\nu - U_\nu)]V^{\mu+} \\
&+ie_{[34]}(b_1F^{\alpha\beta} + b_2U^{\alpha\beta})(\Sigma^\mu_\nu)V^{\nu-}V^+_\mu - ie_{[34]}(b_1F^{\alpha\beta} + b_2U^{\alpha\beta})(\Sigma^\mu_\alpha)_{\beta\nu}V^{\nu-}V^+_\mu \\
&-ib_3e_{[34]}V^{\alpha\beta-}[(\Sigma^\mu_\nu)_{\alpha\beta} + (\Sigma^\mu_\alpha)_{\beta\nu}](A_\nu + U_\nu)V^{\mu+} + \partial_\mu V^-_\nu[a_3V^{\mu\nu+} + b_3e^{[12+][\mu\nu]}]\}
\end{aligned}
\tag{79}
$$

For spin-0 sector:

$$
\begin{aligned}
&\partial_\mu J^\mu_{NL} = iq\big\{V^{\mu+}\big[-2qe_{[34]}(\Sigma^\mu_\nu)_{\alpha\beta}[(A_\alpha + U_\alpha)V^-_\beta(A_\nu - U_\nu) \\
&-(A_\beta + U_\beta)V^-_\alpha(A_\nu - U_\nu)] - 2i[\beta_1S^{\alpha\beta}_1(\Sigma^\mu_\alpha)_{\beta\lambda}V^{\lambda-} + \beta_2S^{\alpha\beta}_2(\Sigma^\mu_\alpha)_{\beta\lambda}V^{\lambda-}] + \\
&-i\mathbf{e}_{(12)}\beta_3S^{\alpha\beta-}(\Sigma^\mu_\alpha)_{\beta\lambda}(A^\lambda + U^\lambda) + 24\mathbf{e}_{(34)}(\rho_1S^\alpha_{\alpha1} + \rho_2S^\alpha_{\alpha2})V^{\mu-} + \\
&24\mathbf{e}_{(12)}\beta_3S^{\alpha-}_\alpha(A^\mu + U^\mu)\big] - V^{\mu-}\big[-(A_\beta + U_\beta)V^+_\alpha(A_\nu - U_\nu)] + \\
&-2i[\beta_1S^{\alpha\beta}_1(\Sigma^\mu_\alpha)_{\beta\lambda}V^{\lambda+} + \beta_2S^{\alpha\beta}_2(\Sigma^\mu_\alpha)_{\beta\lambda}V^{\lambda+}] - i\mathbf{e}_{(12)}\beta_3S^{\alpha\beta+}(\Sigma^\mu_\alpha)_{\beta\lambda}(A^\lambda + U^\lambda) + \\
&+24\mathbf{e}_{(34)}(\rho_1S^\alpha_{\alpha1} + \rho_2S^\alpha_{\alpha2})V^{\mu+} + 24\mathbf{e}_{(12)}\beta_3S^{\alpha+}_\alpha(A^\mu + U^\mu)\big]\big\}
\end{aligned}
\tag{80}
$$

Notice that Noether current and corresponding charge conservation one expliciting on fields spin valued interaction with fields strengths.

3. Constitutive equations.

A third type of equation are the so-called constitutive equation. They are consequence from the fields set $\{A_\mu, U_\mu, V^\pm_\mu\}$ interdepence under a common gauge symmetry, eq. (2.1). They join the Euler-Lagrange and Noether equation

A. Spin-1 sector:

- For the photon field A_μ:

$$
\begin{aligned}
&\partial_\nu\{\tilde{a}_1F^{\nu\mu} + \tilde{b}_1(e^{[12][\nu\mu]} + e^{[+-][\nu\mu]})\} + l^\mu_{AT} + c^\mu_{AT} = \\
&24\tilde{a}_2m^2_UU^\mu + j^\mu_{AT} + j^\mu_{NT} - J^\mu_{NT}
\end{aligned}
\tag{81}
$$

where $\tilde{a}_1$, $\tilde{b}_1$, are multiple of the constants a_1 and b_1.

Eq. (3.5) may be composed. The first term is

$$
l^\mu_{AT} = 6\mathbf{e}^2_{[12]}U_\nu U^\nu A^\mu - \tilde{a}_2U_\nu U^\nu U^\mu + 3\mathbf{e}^2_{[12]}V^-_\nu V^{\nu+}A^\mu - 3\mathbf{e}^2_{[12]}V^-_\nu V^{\nu+}U^\mu
\tag{82}
$$

which is called the London term. It provides a mass term generating dynamically. A scalar with mass dimension 2, as $U_\nu U^\nu$.

The second termc^{μ}_{AT} is identified as conglomerate term. It gives

$$
\begin{aligned}
c^{\mu}_{AT} &= -6\mathbf{e}^2_{[12]}A_\nu U^\nu U^\mu + 6a_2\mathbf{e}^2_{[12]}U_\nu A^\nu A^\mu - 3\mathbf{e}^2_{[12]}A_\nu V^{\nu+}V^{\mu-} + \\
&+\mathbf{e}^2_{[12]}U_\nu V^{\nu+}V^{\mu-} + 6\mathbf{e}_{[34]}\mathbf{e}_{[12]}V^+_\nu U^\nu V^{\mu-} - 6\mathbf{e}_{[12]}\mathbf{e}_{[34]}V^-_\nu U^\nu V^{\mu+} \\
&-3\tilde{a}_2\mathbf{e}^2_{[12]}U_\nu V^{\nu+}V^{\mu-} + 6\tilde{a}_2\mathbf{e}_{[34]}\mathbf{e}_{[12]}V^+_\nu A^\nu V^{\mu-} - 6\mathbf{e}_{[12]}\mathbf{e}_{[34]}V^-_\nu A^\nu V^{\mu+}
\end{aligned}
\tag{83}
$$

It provides another type mass, a scalar as $A_\nu U^\nu$. So eqs (3.18) and (3.19) are showing mass terms to be analysed through the dispersion relation [15].

The spin-valued term appears through the coupled current term.

$$
\begin{aligned}
j^{\mu}_{AT} &= -2qe_{[12]}(\Sigma^\mu_\nu)_{\alpha\beta}[(A_\alpha + U_\alpha)V^+_\beta V^-_\nu - (A_\beta + U_\beta)V^+_\alpha V^-_\nu] \\
&-2qe_{[12]}(\Sigma^\mu_\nu)_{\alpha\beta}[(A_\alpha + U_\alpha)V^-_\beta V^+_\nu - (A_\beta + U_\beta)V^-_\alpha V^+_\nu] \\
&+ib_3(1+\tilde{a}_2)\mathbf{e}_{[34]}[V^{\alpha\beta+}(\Sigma^\mu_\nu)_{\alpha\beta}V^{\nu-}) + V^{\alpha\beta-}(\Sigma^\mu_\nu)_{\alpha\beta}V^{\nu+}] \\
&-i\mathbf{e}_{[12]}(b_1F^{\alpha\beta} + b_2U^{\alpha\beta})(\Sigma^\mu_\alpha)_{\beta\nu}(U^\nu + \tilde{a}_2A^\nu) + \\
&-ib_3\mathbf{e}_{[34]}[V^{\alpha\beta+}(\Sigma^\mu_\alpha)_{\beta\nu}V^{\nu-} + V^{\alpha\beta-}(\Sigma^\mu_\alpha)_{\beta\nu}V^{\nu+}] \\
&-(2iq_1 + a_3q_2)(\partial^\mu V^+_\nu V^{\nu-} - \partial_\nu V_{\mu+}V^{\nu-} + \partial^\mu V^-_\nu V^{\nu+} - \partial_\nu V^{\mu-}V^{\nu+}) \\
&i\mathbf{e}_{[12]}(b_1F^{\alpha\beta} + b_2U^{\alpha\beta})(\Sigma^\mu_\nu)_{\alpha\beta}(U^\nu + \tilde{a}_2A^\nu)
\end{aligned}
\tag{84}
$$

- For the massive photon field U^μ:

$$
\begin{aligned}
&\partial_\nu\{\tilde{a}_2U^{\nu\mu} + b_2(e^{[12][\nu\mu]} + e^{[+-][\nu\mu]})\} - 24m^2_U U^\mu + l^\mu_{UT} + c^\mu_{UT} \\
&+c^\mu_{UT} = j^\mu_{UT} + j^\mu_{NT} - J^\mu_{NT} + \tilde{a}_1 j^\mu_{AT}
\end{aligned}
\tag{85}
$$

with

$$
\begin{aligned}
l^\mu_{UT} &= 6\mathbf{e}^2_{[12]}A_\nu A^\nu U^{mu} + \tilde{a}_1\mathbf{e}^2U_\nu U^\nu A^\mu + 3\mathbf{e}^2_{[12]}V^-_\nu V^{\nu+}U^\mu \\
&-3\mathbf{e}^2_{[12]}V^-_\nu V^{\nu+}A^\mu,
\end{aligned}
\tag{86}
$$

$$
c^\mu_{UT} = 6\mathbf{e}^2_{[12]}A_\nu U^\nu A^\mu - 6\tilde{a}_1U_\nu A^\nu U^\mu - 3\mathbf{e}^2_{[12]}U_\nu V^{\nu+}V^{\mu-} +
$$

$$
+3\mathbf{e}^2_{[12]}U_\nu V^{\nu+}V^{\mu-} + 6\mathbf{e}_{[34]}\mathbf{e}_{[12]}V^+_\nu A^\nu V^{\mu-} - 6\mathbf{e}_{[12]}\mathbf{e}_{[34]}V^-_\nu A^\nu V^{\mu+} \tag{87}
$$

$$
\begin{aligned}
&-3\tilde{a}_1\mathbf{e}^2_{[12]}A_\nu V^{\nu-}V^{\mu+} + 3\mathbf{e}^2_{[12]}U_\nu V^{\nu-}V^{\mu+} + 6\bar{a}_1\mathbf{e}_{[34]}\mathbf{e}_{[12]}V^+_\nu A^\nu V^{\mu-} \\
&-6\mathbf{e}_{[12]}\mathbf{e}_{[34]}V^-_\nu U^\nu V^{\mu+},
\end{aligned}
\tag{88}
$$

and

$$\begin{aligned}
&j^{\mu}_{UT} = -2qe_{[12]}(\Sigma^{\mu}_{\nu})_{\alpha\beta}[(A_{\alpha}+U_{\alpha})V^{+}_{\beta}V^{-}_{\nu} - (A_{\beta}+U_{\beta})V^{+}_{\alpha}V^{-}_{\nu}] \\
&-2qe_{[12]}(\Sigma^{\mu}_{\nu})_{\alpha\beta}[(A_{\alpha}+U_{\alpha})V^{-}_{\beta}V^{+}_{\nu} - (A_{\beta}+U_{\beta})V^{-}_{\alpha}V^{+}_{\nu}] \\
&i\mathbf{e}_{[12]}(b_1F^{\alpha\beta}+b_2U^{\alpha\beta})(\Sigma^{\mu}_{\nu})_{\alpha\beta}(A^{\nu}+\tilde{a}_1U^{\nu}) + \\
&ib_3(1+\tilde{a}_1)\mathbf{e}_{[34]}[V^{\alpha\beta+}(\Sigma^{\mu}_{\nu})_{\alpha\beta}V^{\nu-} + V^{\alpha\beta-}(\Sigma^{\mu}_{\nu})_{\alpha\beta}V^{\nu+}] + \\
&i\mathbf{e}_{[12]}(b_1F^{\alpha\beta}+b_2U^{\alpha\beta})(\Sigma^{\mu}_{\alpha})_{\beta\nu}(A^{\nu}+\tilde{a}_1U^{\nu}) \\
&-ib_3\mathbf{e}_{[34]}[V^{\alpha\beta+}(\Sigma^{\mu}_{\alpha})_{\beta\nu}V^{\nu-} + V^{\alpha\beta-}(\Sigma^{\mu}_{\alpha})_{\beta\nu}V^{\nu+}] \\
&-(2iq_1+u_3q_2)(\partial^{\mu}V^{+}_{\nu}V^{\nu-} - \partial_{\nu}V_{\mu+}V^{\nu-} + \partial^{\mu}V^{-}_{\nu}V^{\nu+} - \partial_{\nu}V^{\mu-}V^{\nu+}) \\
&-2iq_1(\partial_{\nu}V^{\mu\pm}A^{\nu} - \partial^{\mu}V^{\nu\pm}A_{\nu}) - 2iq_2(\partial_{\nu}V^{\mu\pm}U^{\nu} - \partial^{\mu}V^{\nu\pm}A_{\nu}).
\end{aligned} \tag{89}$$

- For charged photons $V^{\mu\pm}$:

$$\partial_{\nu}\{(6a_3-4\beta_3)V^{\nu\mu\pm} + 3a_3e^{[12\pm][\nu\mu]}\} - 12m^2_VV^{\mu\pm} = l^{\mu}_{VT} + c^{\mu\pm} = j^{\mu\pm}_{VT} \tag{90}$$

with

$$l^{\mu\pm}_{VT} = 6\mathbf{e}^2_{[34]}(V^{+}_{\nu}V^{\nu\pm}V^{\mu-} - V^{-}_{\nu}V^{\nu\pm}V^{\mu+}), \tag{91}$$

$$\begin{aligned}
&c^{\mu\pm}_{VT} = 6\mathbf{e}^2_{[34]}(V^{+}_{\nu}U^{\nu}A^{\mu} - V^{+}_{\nu}A^{\nu}U^{\mu}) \\
&3\mathbf{e}_{[34]}\mathbf{e}_{[12]}(V^{+}_{\nu}U_{\nu}A^{\mu} - V^{+}_{\nu}A^{\nu}U^{\mu}) \\
&+3\mathbf{e}_{[12]}\mathbf{e}_{[34]}[A_{\nu}V^{\nu\pm}(A^{\mu}-U^{\mu}) + \\
&+U_{\nu}V^{\nu\pm}(A^{\mu}-U^{\mu})],
\end{aligned} \tag{92}$$

and

$$j^{\mu\pm}_{VT} = -2qe_{[34]}(\Sigma^{\mu}_{\mu})_{\alpha\beta}[(A_{\alpha}+U_{\alpha})V^{\pm}_{\beta}(A_{\nu}-U_{\nu}) - \tag{93}$$

$$\begin{aligned}
&-(A_{\beta}+U_{\beta})V^{\pm}_{\alpha}(A_{\nu}-U_{\nu})] \\
&i\mathbf{e}_{[34]}(b_1F^{\alpha\beta}+b_2U^{\alpha})(\Sigma^{\mu}_{\nu})_{\alpha\beta}V^{\nu\pm} + \\
&-i\mathbf{e}_{[34]}(b_1F^{\alpha\beta}+b_2U^{\alpha\beta})(\Sigma^{\mu}_{\alpha})_{\beta\nu}V^{\nu\pm} + \\
&-ib_3\mathbf{e}_{[34]}(V^{\alpha\beta\pm})[(\sigma^{\mu}_{\nu})_{\alpha\beta} + (\Sigma^{\mu}_{\alpha})_{\beta\nu}](A^{\nu}+U^{\nu})
\end{aligned} \tag{94}$$

Spin-0 sector:

- For the scalar photon:

$$\begin{aligned}
&\partial^{\mu}\{(s^*_1+\tilde{s}_1)S^{\alpha}_{\alpha 1} + (c^*_1+\tilde{c}_1)e^{\alpha}_{\alpha}\} = 48(k^*_1+\tilde{M}_1)m^2_UU^{\mu} + \\
&+(t^*_1+\tilde{t}_1)j^{\mu}_{AL} + (l^*_1+\tilde{\beta}_1)j^{\mu}_{UL} + j^{\mu}_{NL} - J^{\mu}_{NL}
\end{aligned} \tag{95}$$

with

$$
\begin{aligned}
&l^{\mu}_{AL} = -4\mathbf{e}_{(11)}\mathbf{e}_{(12)}A_{\nu}A^{\nu}A^{\mu} - 2\mathbf{e}_{(11)}\mathbf{e}_{(22)}A_{\nu}A^{\nu}U^{\mu} - 4\mathbf{e}_{(11)}\mathbf{e}_{(12)}U_{\nu}U^{\nu}U^{\mu} \\
&-8\mathbf{e}^{2}_{(11)}{}^{2}A_{\nu}A^{\nu}A^{\mu} + 8\mathbf{e}_{(22)}\mathbf{e}_{(11)}U_{\nu}U^{\nu}A^{\mu} - 8\mathbf{e}^{2}_{(11)}A_{\nu}A^{\nu}U^{\mu} - 8\mathbf{e}_{(11)}\mathbf{e}_{(22)}U_{\nu}U^{\nu}U^{\mu} + \\
&+96\mathbf{e}_{(12)}\mathbf{e}_{(11)}A_{\nu}A^{\nu}U^{\mu} + 96\mathbf{e}_{(12)}\mathbf{e}_{(22)}U_{\nu}U^{\nu}U^{\mu} + 106\mathbf{e}_{(12)}\mathbf{e}_{(11)}A_{\nu}A^{\nu}A^{\mu} + \\
&+106\mathbf{e}_{(12)}\mathbf{e}_{(22)}U_{\nu}U^{\nu}A^{\mu} - 2\tilde{l}_{1}A_{\nu}A^{\nu}U^{\mu} - 4\tilde{l}_{1}\mathbf{e}_{(11)}\mathbf{e}_{(12)}A_{\nu}A_{\nu}U^{\mu} + \\
&-4\tilde{l}_{1}\mathbf{e}_{(11)}\mathbf{e}_{(12)}U_{\nu}U^{\nu}A^{\mu} - 8\tilde{1}\mathbf{e}^{2}_{(11)}A_{\nu}A^{\nu}U^{\mu} - 8\tilde{l}_{1}U_{\nu}U^{\nu}U^{\mu} + \\
&-8\tilde{l}_{1}\mathbf{e}^{2}_{(11)}A_{\nu}A^{\nu}U^{\mu} - 8\mathbf{e}^{2}_{(22)}\tilde{l}_{1}U_{\nu}U^{\nu}A^{\mu} + 96\tilde{l}_{1}\mathbf{e}^{2}_{(12)}A_{\nu}A^{\nu}A^{\mu} + \\
&96\tilde{l}_{1}\mathbf{e}_{(12)}\mathbf{e}_{(22)}U_{\nu}U^{\nu}A^{\mu} + 2\mathbf{e}_{(34)}\mathbf{e}_{(11)}V^{+}_{\nu}V^{\nu-}A^{\mu} + 2\mathbf{e}_{(11)}\mathbf{e}_{(34)}V^{+}_{\nu}V^{\nu-}U^{\mu} + \\
&+2\tilde{l}_{1}\mathbf{e}_{(12)}\mathbf{e}_{(34)}V^{+}_{\nu}V^{\nu-}A^{\mu} + 2\tilde{l}_{1}\mathbf{e}_{(22)}\mathbf{e}_{(34)}V^{+}_{\nu}V^{\nu-}U^{\mu} \\
&-4\mathbf{e}^{2}_{(12)}V^{+}_{\nu}V^{\nu-}(A^{\mu} + U^{\mu})] - 4\tilde{l}_{1}\mathbf{e}^{2}V^{+}_{\nu}V^{\nu-}(A^{\mu} + U^{\mu}),
\end{aligned} \tag{96}
$$

$$
\begin{aligned}
&c^{\mu}_{AL} = -\mathbf{e}^{2}_{(11)}U_{\nu}A^{\nu}U^{\nu} - 2\mathbf{e}_{(11)}\mathbf{e}_{(12)}A_{\nu}U^{\nu}A^{\mu} - 2\mathbf{e}_{(11)}\mathbf{e}_{(34)}(V^{+}_{\nu}A^{\nu}V^{\mu-} - V^{-}_{\nu}A^{\nu}V^{\mu+}) \\
&-4\mathbf{e}^{2}_{(11)}U_{\nu}A^{\nu}A^{\mu} - 2\mathbf{e}_{(11)}\mathbf{e}_{(12)}(A_{\nu}U^{\nu}U^{\mu} + U_{\nu}U^{\nu}A^{\mu}) - 2\mathbf{e}_{(11)}\mathbf{e}_{(34)}(V^{+}_{\nu}U^{\nu}V^{\nu-} + V^{-}_{\nu}U^{\nu}V^{\mu+} \\
&-\mathbf{e}_{(11)}\mathbf{e}_{(12)}A_{\nu}U^{\nu}A^{\mu} - 8\mathbf{e}_{(11)}\mathbf{e}_{(12)}A_{\nu}U^{\nu}U^{\mu} - 2\mathbf{e}^{2}_{(12)}(A_{\nu} + U_{\nu})V^{\nu-}V^{\mu+} \\
&-2\mathbf{e}^{2}_{(12)}V^{+}_{\nu}(A^{\nu} + U^{\nu})V^{\mu-} + 4\mathbf{e}_{(11)}\mathbf{e}_{(12)}(A_{\nu} + U_{\nu})V^{\nu+}V^{\mu-} + \\
&+4\mathbf{e}_{(11)}\mathbf{e}_{(12)}(A_{\nu} + U_{\nu})V^{\nu-}V^{\mu+} - 2\mathbf{e}^{2}_{(22)}\tilde{l}_{1}U_{\nu}A^{\nu}A^{\mu} - 2\tilde{l}_{1}\mathbf{e}_{(22)}\mathbf{e}_{(12)}A_{\nu}U^{\nu}U^{\mu} \\
&-\mathbf{e}_{(22)}\mathbf{e}_{(12)}\tilde{l}_{1}(A_{\nu}U^{\nu}A^{\mu} + A_{\nu}U^{\nu}U^{\mu}) - 2\tilde{l}_{1}\mathbf{e}_{(22)}\mathbf{e}_{(34)}(V^{+}_{\nu}A^{\nu}V^{\mu-} + V^{-}_{\nu}U^{\mu}V^{\mu+}) \\
&-8\tilde{l}_{1}\mathbf{e}_{(22)}\mathbf{e}_{(12)}A_{\nu}U^{\nu}U^{\mu} - 8\tilde{l}_{1}\mathbf{e}_{(22)}\mathbf{e}_{(12)}\mathbf{e}_{(22)}A_{\nu}U^{\nu}A^{\mu} - 2\tilde{l}_{1}\mathbf{e}^{2}_{(12)}(A_{\nu} + U_{\nu})V^{\nu-}V^{\mu+} \\
&-2\mathbf{e}^{2}_{(12)}\tilde{l}_{1}V^{+}_{\nu}(A^{\nu} + U^{\nu})V^{\mu-} + 4\mathbf{e}_{(11)}\mathbf{e}_{(12)}\tilde{l}_{1}(A_{\nu} + U_{\nu})V^{\nu+}V^{\mu-} \\
&+4\mathbf{e}_{(22)}\mathbf{e}_{(12)}\tilde{l}_{1}(A_{\nu} + U_{\nu})V^{\nu-}V^{\mu+}
\end{aligned} \tag{97}
$$

and

$$
\begin{aligned}
&j^{\nu}_{AL} = -2i\mathbf{e}_{(12)}(1 + \tilde{l}_{1})(\Sigma^{\mu}_{\alpha})_{\beta\lambda}[(\beta_{1}S^{\alpha\beta}_{1} + \beta_{2}S^{\alpha\beta}_{2})(U^{\lambda} + A^{\lambda})] + \\
&-\beta_{3}\mathbf{e}_{(12)}(1 + \tilde{l}_{1})(\Sigma^{\mu}_{\alpha})_{\beta\lambda}[S^{\alpha\beta+}V^{\lambda-} + S^{\alpha\beta-}V^{\lambda+}] + \\
&48\mathbf{e}_{(12)}(1 + \tilde{l}_{1})(\rho_{1}S^{\alpha}_{\alpha 1} + \rho_{2}S^{\alpha}_{\alpha 2}S^{\alpha}_{\alpha 2})(U^{\mu} + A^{\mu}) + \\
&+24(1 + \tilde{1})\rho_{3}(S^{\alpha+}_{\alpha}V^{\mu-} + S^{\alpha-}_{\alpha}V^{\mu+})
\end{aligned} \tag{98}
$$

- For scalar U^{μ}:

$$
\begin{aligned}
&\partial^{\mu}\{(s^{*}_{2} + \tilde{s}_{2})S^{\alpha}_{\alpha 2} + (c^{*}_{2} + \tilde{c}_{2})e^{\alpha}_{\alpha}\} + l^{\mu}_{UL} + c^{\mu}_{UL} - 48m^{2}_{U}U^{\mu} + \\
&+(t^{*}_{2} + \tilde{t}_{2})j^{\mu}_{AL} + (l^{*}_{2} + \tilde{\beta}_{2})j^{\mu}_{UL} + j^{\mu}_{NL} - J^{\mu}_{NL}
\end{aligned} \tag{99}
$$

with

$$
\begin{aligned}
& l_{UL}^{\mu}=-4\mathbf{e}_{(22)}\mathbf{e}_{(12)}U_{\nu}U^{\nu}U^{\mu}-2\mathbf{e}_{(22)}\mathbf{e}_{(11)}U_{\nu}U^{\nu}A^{\mu}-4\mathbf{e}_{(22)}\mathbf{e}_{(12)}A_{\nu}A^{\nu}A^{\mu} \\
& -8\mathbf{e}_{(22)}^{2}{}^{2}U_{\nu}U^{\nu}U^{\mu}+8\mathbf{e}_{(22)}\mathbf{e}_{(11)}A_{\nu}A^{\nu}U^{\mu}-8\mathbf{e}_{(22)}^{2}U_{\nu}U^{\nu}A^{\mu}-8\mathbf{e}_{(11)}\mathbf{e}_{(22)}A_{\nu}A^{\nu}A^{\mu}+ \\
& +96\mathbf{e}_{(12)}\mathbf{e}_{(22)}U_{\nu}U^{\nu}A^{\mu}+96\mathbf{e}_{(12)}\mathbf{e}_{(11)}A_{\nu}A^{\nu}A^{\mu}+106\mathbf{e}_{(12)}\mathbf{e}_{(22)}U_{\nu}U^{\nu}U^{\mu}+ \\
& +106\mathbf{e}_{(12)}\mathbf{e}_{(11)}A_{\nu}A^{\nu}U^{\mu}-2\tilde{l}_{2}U_{\nu}U^{\nu}A^{\mu}-4\tilde{l}_{2}\mathbf{e}_{(22)}\mathbf{e}_{(12)}U_{\nu}U_{\nu}A^{\mu}+ \\
& -4\tilde{l}_{2}\mathbf{e}_{(22)}\mathbf{e}_{(12)}A_{\nu}A^{\nu}U^{\mu}-8\tilde{2}\mathbf{e}_{(22)}^{2}U_{\nu}U^{\nu}A^{\mu}-8\tilde{l}_{2}A_{\nu}A^{\nu}A^{\mu}+ \\
& -8\tilde{l}_{2}\mathbf{e}_{(22)}^{2}U_{\nu}U^{\nu}A^{\mu}-8\mathbf{e}_{(11)}^{2}\tilde{l}_{2}A_{\nu}A^{\nu}U^{\mu}+96\tilde{l}_{2}\mathbf{e}_{(12)}^{2}U_{\nu}U^{\nu}U^{\mu}+ \\
& 96\tilde{l}_{2}\mathbf{e}_{(12)}\mathbf{e}_{(11)}A_{\nu}A^{\nu}U^{\mu}+2\mathbf{e}_{(34)}\mathbf{e}_{(22)}V_{\nu}^{+}V^{\nu-}U^{\mu}+2\mathbf{e}_{(22)}\mathbf{e}_{(34)}V_{\nu}^{+}V^{\nu-}A^{\mu}+ \\
& +2\tilde{l}_{2}\mathbf{e}_{(12)}\mathbf{e}_{(34)}V_{\nu}^{+}V^{\nu-}U^{\mu}+2\tilde{l}_{2}\mathbf{e}_{(11)}\mathbf{e}_{(34)}V_{\nu}^{+}V^{\nu-}A^{\mu} \\
& -4\mathbf{e}_{(12)}^{2}V_{\nu}^{+}V^{\nu-}(A^{\mu}+U^{\mu})]-4\tilde{l}_{2}\mathbf{e}_{(11)}\mathbf{e}_{(12)}V_{\nu}^{+}V^{\nu-}(A^{\mu}+U^{\mu})
\end{aligned}
\tag{100}
$$

$$
\begin{aligned}
& c_{UL}^{\mu}=-\mathbf{e}_{(22)}^{2}A_{\nu}U^{\nu}A^{\nu}-2\mathbf{e}_{(22)}\mathbf{e}_{(12)}U_{\nu}A^{\nu}U^{\mu}-2\mathbf{e}_{(22)}\mathbf{e}_{(34)}(V_{\nu}^{+}U^{\nu}V^{\mu-}-V_{\nu}^{-}U^{\nu}V^{\mu+}) \\
& -4\mathbf{e}_{(22)}^{2}U_{\nu}A^{\nu}U^{\mu}-2\mathbf{e}_{(22)}\mathbf{e}_{(12)}(A_{\nu}U^{\nu}A^{\mu}+A_{\nu}U^{\nu}U^{\mu})-2\mathbf{e}_{(22)}\mathbf{e}_{(34)}(V_{\nu}^{+}A^{\nu}V^{\nu-}+V_{\nu}^{-}A^{\nu}V^{\mu+} \\
& -\mathbf{e}_{(22)}\mathbf{e}_{(12)}A_{\nu}U^{\nu}U^{\mu}-8\mathbf{e}_{(22)}\mathbf{e}_{(12)}A_{\nu}U^{\nu}A^{\mu}-2\mathbf{e}_{(12)}^{2}(A_{\nu}+U_{\nu})V^{\nu-}V^{\mu+} \\
& -2\mathbf{e}_{(12)}^{2}V_{\nu}^{+}(A^{\nu}+U^{\nu})V^{\mu-}+4\mathbf{e}_{(22)}\mathbf{e}_{(12)}(A_{\nu}+U_{\nu})V^{\nu+}V^{\mu-}+ \\
& +4\mathbf{e}_{(22)}\mathbf{e}_{(12)}(A_{\nu}+U_{\nu})V^{\nu-}V^{\mu+}-2\mathbf{e}_{(11)}^{2}\tilde{l}_{2}A_{\nu}U^{\nu}U^{\mu}-2\tilde{l}_{2}\mathbf{e}_{(11)}\mathbf{e}_{(12)}A_{\nu}U^{\nu}A^{\mu} \\
& -\mathbf{e}_{(11)}\mathbf{e}_{(12)}\tilde{l}_{2}(A_{\nu}U^{\nu}U^{\mu}+A_{\nu}U^{\nu}A^{\mu})-2\tilde{l}_{2}\mathbf{e}_{(11)}\mathbf{e}_{(34)}(V_{\nu}^{+}A^{\nu}V^{\mu-}+V_{\nu}^{-}U^{\mu}V^{\mu+}) \\
& -8\tilde{l}_{2}\mathbf{e}_{(11)}\mathbf{e}_{(12)}A_{\nu}U^{\nu}A^{\mu}-8\tilde{l}_{2}\mathbf{e}_{(11)}\mathbf{e}_{(12)}\mathbf{e}_{(22)}A_{\nu}U^{\nu}U^{\mu}-2\tilde{l}_{2}\mathbf{e}_{(12)}^{2}(A_{\nu}+U_{\nu})V^{\nu-}V^{\mu+} \\
& -2\mathbf{e}_{(12)}^{2}\tilde{l}_{2}V_{\nu}^{+}(A^{\nu}+U^{\nu})V^{\mu-}+4\mathbf{e}_{(22)}\mathbf{e}_{(12)}\tilde{l}_{2}(A_{\nu}+U_{\nu})V^{\nu+}V^{\mu-} \\
& +4\mathbf{e}_{(11)}\mathbf{e}_{(12)}\tilde{l}_{2}(A_{\nu}+U_{\nu})V^{\nu-}V^{\mu+}
\end{aligned}
\tag{101}
$$

and

$$
\begin{aligned}
& j_{UL}^{\nu}=-2i\mathbf{e}_{(12)}(1+\tilde{l}_{2})(\Sigma_{\alpha}^{\mu})_{\beta\lambda}[(\beta_{1}S_{1}^{\alpha\beta}+\beta_{2}S_{2}^{\alpha\beta})(U^{\lambda}+A^{\lambda})]+ \\
& -\beta_{3}\mathbf{e}_{(12)}(1+\tilde{l}_{2})(\Sigma_{\alpha}^{\mu})_{\beta\lambda}[S^{\alpha\beta+}V^{\lambda-}+S^{\alpha\beta-}V^{\lambda+}]+ \\
& 48\mathbf{e}_{(12)}(1+\tilde{l}_{2})(\rho_{1}S_{\alpha1}^{\alpha}+\rho_{2}S_{\alpha2}^{\alpha}S_{\alpha2}^{\alpha})(U^{\mu}+A^{\mu})+ \\
& +24(1+\tilde{2})\rho_{3}(S_{\alpha}^{\alpha+}V^{\mu-}+S_{\alpha}^{\alpha-}V^{\mu+})
\end{aligned}
\tag{102}
$$

- For scalar charge fields $V^{\mu\pm}$:

$$
\partial^{\mu}\{s_{1}^{*}S_{\alpha}^{\alpha\pm}+c_{3}^{*}e_{\alpha}^{(12\pm)\alpha}\}+l_{VT}^{\mu}+c_{VT}^{\mu}-24m_{V}^{2}V^{\mu\pm}=j_{VL}^{\mu}
\tag{103}
$$

with

$$
\begin{aligned}
& l_{VL}^{\mu\pm}=-4\mathbf{e}_{(34)}^{2}V_{\nu}^{+}V^{\nu\pm}V^{\mu-}-4\mathbf{e}_{(34)}^{2}V_{\nu}^{-}V^{\nu\pm}V^{\mu+}+4\mathbf{e}_{(34)}\mathbf{e}_{(11)}A_{\nu}A^{\nu}V^{\mu\pm} \\
& 4\mathbf{e}_{(34)}\mathbf{e}_{(22)}U_{\nu}U^{\nu}V^{\mu\pm}+4\mathbf{e}_{(34)}^{2}V_{\nu}^{+}V^{\nu-}V^{\mu\pm}-2\mathbf{e}_{(12)}^{2}A_{\nu}A^{\nu}V^{\mu\pm}- \\
& -\mathbf{e}_{(12)}^{2}U_{\nu}U^{\nu}V^{\mu\pm}+48\mathbf{e}_{(34)}\mathbf{e}_{(11)}A_{\nu}A^{\nu}V^{\mu\pm}+48\mathbf{e}_{(34)}\mathbf{e}_{(34)}U_{\nu}U^{\nu}V^{\mu\pm}+ \\
& +48\mathbf{e}_{(34)}^{2}V_{\nu}^{+}V^{\nu-}V^{\mu\pm},
\end{aligned}
\tag{104}
$$

$$c_{VL}^{\mu\pm} = -4\mathbf{e}_{(34)}(\mathbf{11})A_\nu V^{\nu\pm}A^\mu - 4\mathbf{e}_{(34)}(\mathbf{22})U_\nu V^{\nu\pm}U^\mu + 4\mathbf{e}_{(34)}(\mathbf{22})A_\nu U^\nu V^{\mu\pm}$$
$$-4\mathbf{e}_{(34)}(\mathbf{12})(U_\nu V^{\nu\pm}A^\mu + A_\nu V^{\nu\pm}U^\mu) - 2\mathbf{e}^2_{(12)}V_\nu^\pm A^\nu A^\mu - 2\mathbf{e}^2_{(12)}V_\nu^\pm A^\nu U^\mu$$
$$-2\mathbf{e}^2_{(12)}V_\nu^\pm U^\nu A^\mu - 2\mathbf{e}^2_{(12)}V_\nu^\pm U^\nu U^\mu - 2\mathbf{e}^2_{(12)}(A_\nu V^{\nu\pm}A^\mu + A_\nu V^\nu U^\mu +$$
$$+U_\nu V^{\nu\pm}A^\mu + U_\nu V^{\nu\pm}U^\mu) + 48\mathbf{e}_{(34)}\mathbf{e}_{(12)}A_\nu U^\nu V^{\mu\pm}, \tag{105}$$

and

$$j_{VL}^{\mu\pm} = -2i[\beta_1 S_1^{\alpha\beta}(\Sigma^\mu_\alpha)_{\beta\lambda}V^{\lambda\pm} + \beta_2 S_2^{\alpha\beta}(\Sigma^\mu_\alpha)_{\beta\lambda}V^{\lambda\pm}] +$$
$$-i\mathbf{e}_{(12)}\beta_3 S^{\alpha\beta\pm}(\Sigma^\mu_\alpha)_{\beta\lambda}(A^\lambda + U^\lambda) + 24\mathbf{e}_{(34)}(\rho_1 S^\alpha_{\alpha 1} + \rho_2 S^\alpha_{\alpha 2})V^{\mu\pm}$$
$$24\mathbf{e}_{(12)}\beta_3 S_\alpha^{\alpha\pm}(A^\mu + U^\mu) \tag{106}$$

A new electromagnetism emerges. Vectors and scalars fields equations are developed. Their left-hand side are constituted by their fields strengths dynamics and the right-hand working sources. A physicality based on potential fields is explicited. Potential fields are not more subsidiary. Spin-valued potential fields became physical ingredients.

3. Bianchi identities

A fourth type of equation are the Bianchi identities. It yields for antisymmetric sector:

3.4.1 Granular:

$$\partial_\mu F_{\nu\rho I} + \partial_\rho F_{\mu\nu I} + \partial_\nu F_{\rho\mu I} = 0 \tag{107}$$

3.4.2 Collective:

$$\partial_\mu e_{[\nu\rho]} + \partial_{rho} e_{[\mu\nu]} + \partial_\nu e_{[\rho\mu} = e_{[12]}(\Sigma_{\rho\nu})_{\kappa\lambda}A_\mu U^{\kappa\lambda}$$
$$+e_{[12]}(\Sigma_{\rho\mu})_{\kappa\lambda}A_\nu U^{\kappa\lambda} + e_{[12]}(\Sigma_{\nu\mu})_{\kappa\lambda}A_\rho U^{\kappa\lambda}$$
$$-e_{[12]}(\Sigma_{\rho\nu})_{\kappa\lambda}U_\mu F^{\kappa\lambda} - e_{[12]}(\Sigma_{\rho\mu})_{\kappa\lambda}U_\nu F^{\kappa\lambda}$$
$$-e_{[12]}(\Sigma_{\nu\mu})_{\kappa\lambda}A_\rho F^{\kappa\lambda} \tag{108}$$

and

$$\partial_\mu e^{[+-]}_{[\nu\rho]} + \partial_\nu e^{[+-]}_{[\rho\mu]} + \partial_\rho e^{[+-]}_{[\mu\nu]} = -ie_{[34]}\{V_\mu^+(\Sigma^{\nu\rho})_{\kappa\lambda}V^{\kappa\lambda-}$$
$$+V_\nu^+(\Sigma^{\rho\mu})_{\kappa\lambda}V^{\kappa\lambda-} + V_\rho^+(\Sigma^{\mu\nu})_{\kappa\lambda}V^{\kappa\lambda} - V_\mu^-(\Sigma^{\nu\rho})_{\kappa\lambda}V^{\kappa\lambda}$$
$$-V_\nu^-(\Sigma^{\rho\mu})_{\kappa\lambda}V^{\kappa\lambda+} - V_\rho(\Sigma^{\mu\nu})_{\kappa\lambda}V^{\kappa\lambda+}\} \tag{109}$$

3.4.3 For symmetric sector:

Collective neutral:

$$\begin{aligned}
&\partial_\mu e_{(\nu\rho)} + \partial_\nu e_{(\nu\mu)} + \partial_\rho e_{(\mu\nu)} = \\
&e_{(11)}\{A_\mu(\Sigma^{nu\kappa})_{\lambda\rho}S^1_{\kappa\lambda} + A_\nu(\Sigma^{\rho\kappa})_{\lambda\mu}S^1_{\kappa\lambda} \\
&+A_\rho(\Sigma^{\mu\kappa})_{\lambda\nu}S^1_{\kappa\lambda} + g^{\nu\kappa}S^{\alpha1}_\alpha A_\mu + g^{\rho\mu}S^\alpha_{\alpha1}A_\nu + g^{\mu\nu}S^\alpha_{\alpha1}A_\rho\} \\
&+ie_{(11)}\{g^{\nu\rho}A_\alpha\partial_\mu A^\alpha + g^{\rho\mu}A_\alpha\partial_\nu A^\alpha + g^{\mu\nu}A_\alpha\partial_\rho A^\alpha\} \\
&e_{(22)}\{U_\mu(\Sigma^{\nu\kappa})_{\lambda\rho}S^2_{\kappa\lambda} + U_\nu(\Sigma^{\rho\kappa})_{\lambda\mu}S^2_{\lambda\mu} + U_\rho(\Sigma^{\mu\kappa}_{\lambda\nu}S^2_{\kappa\lambda} + \\
&+g^{\nu\rho}S^{\alpha2}_\alpha U_\mu + g^{\rho\mu}S^{\alpha2}_\alpha U_\nu + g^{\mu\nu}S^{\alpha2}_\alpha U_\rho\} \\
&+ie_{(22)}\{g^{\nu\rho}U_\alpha\partial_\mu U^\alpha + g^{\rho\mu}U\alpha\partial_\nu U^\alpha + g^{\mu\nu}U_\alpha\partial_{rho}U^\alpha\} \\
&+e_{(12)}\{U_\mu(\Sigma^{\nu\kappa})_{\lambda\rho}S^1_{\kappa\lambda} + U_\nu(\Sigma^{\rho\kappa})_{\lambda\mu}S^1_{\kappa\lambda} + U_\rho(\Sigma^{\mu\kappa})_{\lambda\nu}S^1_{\kappa\lambda} + \\
&A_\mu(\Sigma^{\nu\kappa})_{\lambda\rho}S^2_{\kappa\lambda} + A_\nu(\Sigma^{\rho\kappa})_{\lambda\mu}S^2_{\kappa\lambda} + A_\rho(\Sigma^{\mu\kappa})_{\lambda\nu}S^2_{\kappa\lambda}\} \\
&+ie_{(12)}\{g^{\nu\rho}(S^{\alpha1}_\alpha U_\mu + S^{\alpha2}_\alpha A_\mu) + g^{\rho\mu}(S^{\alpha1}_\alpha U_\nu + S^{\alpha2}_\alpha A_\nu) + \\
&g^{\mu\nu}(S^{\alpha1}_\alpha U_\rho + S^{\alpha2}_\alpha A_\rho) + g^{\nu\rho}(U_\alpha\partial_\mu A^\alpha + A_\alpha\partial_\mu U^\alpha) \\
&+g^{\rho\mu}(U_\alpha\partial_\nu A^\alpha + A_\alpha\partial_\nu U^\alpha) + g^{\mu\nu}(U_\alpha\partial_\rho A^\alpha + A_\alpha\partial_\rho U^\alpha\}
\end{aligned} \tag{110}$$

Collective charged

$$\begin{aligned}
&\partial_\mu e^{(+-)}_{(\nu\rho)} + \partial_\nu e^{(+-)}_{(\rho\mu)} + \partial_\rho e^{(+-)}_{\mu\nu} = ie_{(34)}\{V^+_\mu(\Sigma^{\nu\kappa})_{\lambda\rho}S^-_{\kappa\lambda} + V^-_{nu}(\Sigma^{\rho\kappa})_{\kappa\lambda} \\
&V^+_\rho(\Sigma^{\mu\kappa})_{\lambda\nu}S^-_{\kappa\lambda} + g^{\nu\rho}S^{\alpha-}_\alpha V^+_\mu + g^{\rho\mu}S^{\alpha-}_\alpha V^+_\nu + g^{\mu\nu}S^{\alpha-}_\alpha V^+_\nu \\
&+V^-_\mu(\Sigma^{\nu\kappa})_{\lambda\rho}S^+_{\kappa\lambda} + V^-_\nu(\Sigma^{\rho\kappa})_{\lambda\mu}S^+_{\kappa\lambda} + V_\rho(\Sigma^{\mu\kappa})_{\lambda\nu}S^+_{\kappa\lambda} \\
&+g^{\nu\rho}S^{\alpha+}_\alpha V^-_\mu + g^{\rho\mu}S^{\alpha+}_\alpha V^-_\nu + g^{\mu\nu}S^{\alpha+}_\alpha V^-_\rho\} + \\
&+e_{(34)}\{g^{\nu\rho}(V^+_\alpha\partial_\mu V^{\alpha-} + V^-_\alpha\partial_\mu V^{\alpha+)+g^{\rho\mu}(V^+_\alpha\partial_\nu V^{\alpha-}+V_\alpha\partial_\nu V^{\alpha+})} \\
&+g^{\mu\nu}(V^+_\alpha\partial_\rho V^{\alpha-} + V^-_\alpha\partial_\rho V^{\alpha+}\}
\end{aligned} \tag{111}$$

Notice that Bianchi identities do not introduce electric charge as source.

4. Euler-Lagrange vectorial

Electromagnetism is a physical theory to be undersood in terms of electric and magnetic fields. It requires to rewrite the previous covariant form into a vectorial expression. For instance, taking photon case, one gets.

For photon Gauss law;

$$\vec{\nabla}\cdot\{6a_1\vec{E}_A + b_1(\vec{e}_{AU} + \vec{e}_{+-})\} = \rho_{AT} \tag{112}$$

where

$$\rho_{AT} = -i\mathbf{e}_{[12]}b_1[(\vec{E}_A\vec{s})_i^0\vec{U} + (\vec{B}_A\cdot\vec{S})_i^0\vec{U}] + ib_3\mathbf{e}_{[34]}[(\vec{E}^+\cdot\vec{s})_i^0\vec{V}^-$$
$$+(\vec{B}^+\cdot\vec{S})_i^0\vec{V}^- + (\vec{E}^-\cdot\vec{s})_i^0\vec{V}^+ + (\vec{B}^-\cdot\vec{S})_i^0\vec{V}^+] + 2i\mathbf{e}_{[12]}[(\vec{e}_{AU}\cdot\vec{s})_i^0\vec{U}$$
$$+(\vec{b}_{AU}\cdot\vec{S})_i^0\vec{U} + (\vec{e}_{+-}\cdot\vec{s})_i^0\vec{U} + (\vec{b}_{+-}\cdot\vec{S})_i^0\vec{U}] + i[(\vec{e}_{AU-}\cdot\vec{s})_i^0\vec{V}^+$$
$$+(\vec{b}_{AU-}\cdot\vec{S})_i^0\vec{V}^+ + (\vec{e}_{AU+}\cdot\vec{s})_i^0\vec{V}^- + \vec{b}_{AU+}\cdot\vec{S})_i^0\vec{V}^-] + \mathbf{e}_{[12]}(b_1\vec{E}_A +$$
$$+\vec{E}_U)\cdot\vec{U} + b_3\mathbf{e}_{[34]}(\vec{E}^+\cdot\vec{V}^- + \vec{E}^-\cdot\vec{V}^+) + 2(\vec{e}_{AU} + \vec{e}_{+-})\cdot\vec{U} +$$
$$+(\vec{e}_{AU-}\cdot\vec{V}^+ + \vec{e}_{AU+}\cdot\vec{V}^-) + 2iq_1(\frac{\partial\phi^+}{\partial t}\phi^- + \frac{\partial\vec{V}^-}{\partial t}\vec{V}^+ - \frac{\partial\phi^+}{\partial t}\phi^-$$
$$-\vec{\nabla}\phi^+\cdot\vec{V}^- + \frac{\partial\vec{V}^-}{\partial t}\cdot\vec{V}^+ - \vec{\nabla}\phi^-\cdot\vec{V}^+) - q_1^2(2\phi^+\phi^-\phi_A + 2\vec{V}^+\cdot\vec{V}^-\phi_A$$
$$-\vec{V}^+\cdot\vec{A}\phi^- - \phi^-\phi_A\phi^+ - \vec{V}^-\cdot\vec{A}\phi^+) \tag{113}$$

For photon Ampère law;

$$\vec{\nabla}\times\{6a_1\vec{B}_A + b_1(\vec{b}_{AU} + \vec{b}_{+-})\} - \partial_t\{a_1\vec{E}_1 + b_1(\vec{e}_{AU} + \vec{e}_{+-})\} = \vec{j}_{AU} \tag{114}$$

$$\vec{j}_{AU} = i\mathbf{e}_{[12]}[(\vec{E}_A\vec{s})_0^i\phi_U + (\vec{B}_A\cdot\vec{S})_0^i\phi_U + (\vec{E}_A\vec{s})_j^iU^j + (\vec{E}_A\vec{s})_j^iU^j]$$
$$+ib_3\mathbf{e}_{[34]}[(\vec{E}^+\vec{s})_0^i\phi^- + (\vec{B}_+\cdot\vec{S})_0^i\phi^- + (\vec{E}^-\vec{s})_0^i\phi^+ + (\vec{B}_-\cdot\vec{S})_i^0\phi^+ +$$
$$+(\vec{E}^+\vec{s})_j^iV^{j-} + (\vec{B}^+\cdot\vec{S})_j^iV^{j-} + (\vec{E}^-\vec{s})_j^iV^{j+} + (\vec{B}^-\cdot\vec{S})_j^iV^{j+}] +$$
$$+2\mathbf{e}_{[12]}[(\vec{e}_{AU}\cdot\vec{s})_0^i\phi_U + (\vec{b}_{AU}\cdot\vec{S})_0^i\phi_U + (\vec{e}_{AU}\cdot\vec{s})_j^iU^j + (\vec{b}_{AU}\cdot\vec{S})_j^iU^j] +$$
$$+i[(\vec{e}_{AU+}\cdot\vec{s})_0^i\phi_- + (\vec{b}_{AU+}\cdot\vec{S})_0^i\phi_- + (\vec{e}_{AU+}\cdot\vec{s})_j^iV^{j-} + (\vec{b}_{AU+}\cdot\vec{S})_j^iV^{j-} +$$
$$(\vec{e}_{AU-}\cdot\vec{s})_0^i\phi_+ + (\vec{b}_{AU-}\cdot\vec{S})_0^i\phi_+ + (\vec{e}_{AU-}\cdot\vec{s})_j^iV^{j+} + (\vec{b}_{AU-}\cdot\vec{S})_j^iV^{j+}]$$
$$+2\mathbf{e}_{[12]}[(\vec{b}_{AU} + \vec{b}_{+-})\times\vec{U} + (\vec{e}_{AU} + \vec{e}_{+-})\phi_U] + \mathbf{e}_{[34]}[\vec{b}_{AU-}\times\vec{V}^+ +$$
$$+\vec{e}_{AU-}\phi^+ + \vec{b}_{AU+}\times\vec{V}^- + \vec{e}_{AU+}\phi^-] + 2iq_1(\nabla\phi^+\phi^- +$$
$$\vec{\nabla}\cdot\vec{V}^+\cdot\vec{V}^- - \frac{\partial\vec{V}^+}{\partial t}\phi^- + \nabla\phi^-\phi^+ + \vec{\nabla}\cdot\vec{V}^-\vec{V}^+ - \frac{\partial\vec{V}^-}{\partial t}\phi^+$$
$$-\vec{\nabla}\cdot\vec{V}^+) - q_1^2(2\phi^+\phi^-\vec{A} + 2\vec{V}^+\cdot\vec{V}^-\vec{A} - \phi^+\phi_A\vec{V}^- - \vec{V}^+\cdot\vec{A}\vec{V}^-$$
$$-\phi^-\phi_A\vec{V}^- - (\vec{V}^-\cdot\vec{A})\vec{V}^+) \tag{115}$$

For simplicity we just study the photon equations. They show a nonlinear EM with polarization and magnetization described from first principles and the presence of spin interaction. Similarly this physics is extended for other quadruplet fields

5. Noether Theorem

Considering the three Noether's laws

$$\alpha\partial_\mu J^\mu + \partial_\nu\alpha\{\partial_\mu K^{\mu\nu} + J^\nu\} + \partial_\mu\partial_\nu\alpha K^{\mu\nu} = 0 \tag{116}$$

For antisymmetric sector:

$$\vec{\nabla}\cdot\left[4k_1(a_1+\beta_1)\vec{E}_A + 4k_2\left(a_2+\beta_2\right)\vec{E}_U + 2k_+\vec{E}_- + 2k_-\vec{E}_+ + 2\left(a_1k_1 + a_2k_2\right)\vec{\mathbf{e}}_+ -\right] = \rho_q^T \tag{117}$$

with

$$\rho_q^T \equiv -q\left\{4a_3\left\{(\Sigma^{\kappa\lambda})\vec{V}^+_{\kappa\lambda}\cdot\vec{E}_-\right\}+\left\{b_3(\Sigma^{\kappa\lambda})\vec{V}^+_{\kappa\lambda}\cdot[\vec{\mathbf{e}}_{-A}+\vec{\mathbf{e}}_{-U}]\right\}\right\} \tag{118}$$

and

$$\vec{\nabla}\times\left[4k_1(a_1+\beta_1)\vec{B}_A+4(a_2+\beta_2)\vec{B}_U+2k_+a_3\vec{B}_-+2k_-a_3\vec{B}_++2(a_1k_1+a_2k_2)\vec{\mathbf{b}}_{+-}\right]+ \\ -\frac{\partial}{\partial t}\left[4k_1(a_1+\beta_1)\vec{E}_A+4k_2(a_2+\beta_2)\vec{E}_U+2k_+a_3\vec{E}_-+2k_-a_3\vec{E}_-\right]=-\vec{j}_q^T \tag{119}$$

with

$$\vec{j}_q^T \equiv -q\{4a_3Im\{\vec{E}_-\cdot(\Sigma^{\kappa\lambda})\phi^+_{\kappa\lambda}+\vec{B}_-\times(\Sigma^{\kappa\lambda})\vec{V}^+_{\kappa\lambda}\}+4b_3Im\{\vec{\mathbf{e}}_{-A}(\Sigma^{\kappa\lambda})\phi^+_{\kappa\lambda}+ \\ \vec{\mathbf{b}}_{-A}\times(\Sigma^{\kappa\lambda})\vec{V}^+_{\kappa\lambda}\}\} \tag{120}$$

For longitudinal sector:

$$\frac{\partial}{\partial t}\{4(11k_1\rho_1+\frac{1}{2}k_2\rho_2\beta_1+11\rho_1\rho_2+\frac{1}{4}k_2\xi_{(12)})S^\alpha_{\alpha A}+4(11k_2\rho_2+\frac{1}{2}k_1\rho_1\beta_2+ \\ +11\rho_1\rho_2+\frac{1}{4}k_1\xi_{(12)})S^\alpha_{\alpha U}\}=\rho_q^L \tag{121}$$

with

$$\rho_q^L \equiv -q\{4Im\{\beta_+\beta_-(\Sigma^{\kappa\lambda})\vec{V}^+_{\kappa\lambda}\vec{S}_-+(16\rho_+\rho_-+\rho_+\beta_-+\rho_-\beta_+)(\Sigma^{\kappa\lambda})\phi^+_{\kappa\lambda}S^\alpha_{\alpha-}\}+ \\ Im\{\beta_+(\vec{s}_{-A}+\vec{s}_{-U})\cdot(\Sigma^{\kappa\lambda})\vec{V}^+_{\kappa\lambda}+(\beta_++17\rho_+)\left(s^\alpha_{\alpha-A}+s^\alpha_{\alpha-U}\right)(\Sigma^{\kappa\lambda})\phi^+_{\kappa\lambda\}\}} \tag{122}$$

and

$$\partial_i\{4(11k_1\rho_1+\frac{1}{2}k_2\rho_2\beta_1+11\rho_1\rho_2+\frac{1}{4}k_2\xi_{(12)})S^\alpha_{\alpha A}+4(11k_2\rho_2+\frac{1}{2}k_1\rho_1\beta_2+ \\ +11\rho_1\rho_2+\frac{1}{4}k_1\xi_{(12)})S^\alpha_{\alpha U}\}=j_{iq} \tag{123}$$

with

$$j^L_{iq}\equiv -q\{4Im\{\beta_+\beta_-(\Sigma^{\kappa\lambda})V^+_{j;\kappa\lambda}S_i^{j-}+(16\rho_+\rho_-+\rho_+\beta_-+\rho_-\beta_+)(\Sigma^{\kappa\lambda})V^+_{i;\kappa\lambda}S^\alpha_{\alpha-}\}+ \\ +4Im\{\beta_+\left(s^j_{i-A}\right)(\Sigma^{\kappa\lambda})V^+_{j;\kappa\lambda}+s^j_{i-U}(\Sigma^{\kappa\lambda})V^+_{j;\kappa\lambda}+(17\rho_++\beta_+)(s^\alpha_{\alpha-A} \\ +s^\alpha_{\alpha-U})(\Sigma^{\kappa\lambda})V^+_{i;\kappa\lambda}\}\} \tag{124}$$

Concluding, the Noether theorem provides two kinds of electric conserved charge. The transversal continuity equation obtained as

$$\frac{\partial\rho_q^T}{\partial t}+\vec{\nabla}\cdot\vec{j}_q^T=0 \tag{125}$$

is introducing the correspondent spin-1 charge. The electric dipole and magnetic moment interaction writen at eq. (1.15) are expressed at eqs (5.3) and (5.5)

Similarly, one gets the longitudinal continuity equation

$$\frac{\partial\rho_q^L}{\partial t}+\vec{\nabla}\vec{j}_q^L=0 \tag{126}$$

given by eqs (5.7) and (5.9)

Performing eq. (5.10), one gets.

$$\begin{aligned}
&iqIm\big\{-4q\vec{s}_{0j}\cdot(\vec{A}-\vec{U})[(A_j+U_j)V_0^+-(\phi_A+\phi_U)V_j^+]\\
&-4q\vec{s}_{0j}[(\phi_A-\phi_U)V_j^+(\phi_A+\phi_U)-(\phi_A-\phi_U)\phi^+(A_j+U_j)]\vec{V}^-\\
&+ie_{[34]}\big[(b_1\vec{E}_{A0j}+b_2\vec{E}_{U0j})\vec{s}_{0j}\cdot\vec{V}^+\phi^--(b_1\vec{B}_{Ajk}+b_2\vec{B}_{Ujk})\vec{s}_{jk}\cdot\vec{V}^-\phi^+\big]\\
&+ie_{[34]}\big[(b_1\vec{E}_{A0j}+b_2\vec{E}_{U0j})\vec{s}_{0j}\cdot\vec{V}^-\phi^+-(b_1\vec{B}_{Ajk}+b_2\vec{B}_{Ujk})\vec{s}_{jk}\cdot\vec{V}^+\phi^-\big]\\
&-ie_{[34]}\big[(b_1\vec{E}_{A0j}+b_2\vec{E}_{U0j})\cdot\vec{s}_{0j}\phi^+\phi^-+(b_1\vec{B}_{Ajk}+b_2\vec{B}_{Ujk})\cdot\vec{s}_{jk}\phi^-\phi^+\big]\\
&-ie_{[34]}\big[(b_1\vec{B}_{Akl}+b_2\vec{E}_{Ukl})\cdot\vec{s}_{kli}\phi^+\vec{V}^{i-}+(b_1\vec{B}_{Ajk}+b_2\vec{B}_{Ujk})\vec{S}_{jk}\cdot\vec{V}^+\phi^-\big]\\
&-ie_{[34]}\big[(b_1\vec{E}_{A0j}+b_2\vec{E}_{U0j})\cdot\vec{s}_{0j}\vec{V}^+\phi^-+b_3\vec{E}_{0j}(\vec{s}_{0j}\cdot(\vec{A}+\vec{U}))\phi^-\big]\\
&-ib_3e_{[34]}\{\vec{B}_{jk}^+\big[\vec{s}_{jk}\cdot(\vec{A}+\vec{U})+\vec{s}_{ki}^{j}(A_i+U_i)\big]\phi^-\vec{B}_{kl}^+\vec{s}_{kl}(\phi_A+\phi_U)\phi^-\}\\
&-b_3e_{[34]}\vec{B}_{jk}^+\big[(\phi_A+\phi_U)\vec{s}_{jk}\cdot\vec{V}^-\big]-ib_3e_{[34]}\vec{B}_{jk}\vec{B}_{jk}\cdot\vec{S}_{jk}(\phi_A+\phi_U)\\
&-ib_3e_{[34]}\big[\vec{E}_k^+\vec{s}_{kj}\cdot(\vec{A}+\vec{U})V^{j+}+\vec{B}_{kl}\vec{S}_{kl}\cdot(\vec{A}+\vec{U})V^{l+}\big]+\frac{\partial}{\partial t}\vec{V}^+\cdot[a_3\vec{E}^-+\\
&+b_3\vec{e}_{AU-}]-\vec{\nabla}\phi\cdot[a_3\vec{E}^-+b_3\vec{e}_{AU-}]\big\}
\end{aligned}\tag{127}$$

Communing eq (5.11), one derives

$$\begin{aligned}
&iq\big\{\phi^+\big[-4qe_{(34)}\vec{s}_{0j}\cdot(\vec{A}-\vec{U})V^{j-}(\phi_A+\phi_U)\big]+\\
&-4qe_{(34)}\vec{s}_{jk}\cdot(\vec{A}-\vec{U})V_k^-(\vec{A}^j+\vec{U}^j)-4i(\beta_1\vec{S}_1\cdot\vec{s}_{0j}\vec{V}^{j-}+\\
&+\beta_2\vec{S}_2\cdot\vec{s}_{0j}\vec{V}^{j-})-ie_{(12)}\beta_3\vec{S}^{ij-}\cdot\vec{s}_{jk}(\vec{A}^k+\vec{A}^k)+\\
&+24e_{(34)}(\rho_1S_{\alpha1}^{\alpha}+\rho_2S_{\alpha2}^{\alpha})\phi^-+24e_{(12)}\beta_3S_\alpha^{\alpha-}(\phi_A+\phi_U)\big]+\\
&\vec{V}^+\cdot\big[-4qe_{(34)}\vec{s}_{0j}(\phi_A+\phi_U)\vec{V}^{j-}(\phi_A-\phi_U)-4qe_{(34)}\vec{S}_{j0k}(\phi_A+\phi_U)\vec{V}^{k-}\\
&-4qe_{(34)}\vec{S}_{jkl}(\vec{A}^k+\vec{U}^k)\vec{V}^{l-}-2i(\beta1\vec{S}_{0j1}\cdot\vec{s}_{j0}\phi^-+\beta_2\vec{S}_2^{0j}\vec{s}_{j0}\phi^-)\\
&-2(\beta_1\vec{S}_1^{jk}\vec{S}_{kl}\vec{V}^{l-}+\beta_2\vec{S}_2^{jk}\vec{S}_{kl}\vec{V}^{l-})-ie_{(12)}\beta_3S^{00-}\vec{s}_{00j}(\vec{A}^j+\vec{U}^j)+\\
&-ie_{(12)}\beta_3\vec{S}^{0j-}\vec{s}_{0j}(\phi_A+\phi_U)-ie_{(12)}\beta_3\vec{S}^{jk-}\vec{S}_{jk}(\phi_A+\phi_U)\\
&-ie_{(12)}\beta_3\vec{S}^{jk-}\vec{S}_{jkl}(\vec{A}^l+\vec{U}^l)+24e_{(34)}(\rho_1S_{\alpha1}^{\alpha}+\rho_2S_{\alpha2}^{\alpha})\vec{V}^-+\\
&+24e_{(12)}\beta_3S_\alpha^{\alpha-}(\vec{A}+\vec{U})\big]+\frac{\partial}{\partial t}\phi^+\big[s_3^*S_\alpha^{\alpha-}+c_3^*e_\alpha^{(12-)\alpha}+
\end{aligned}\tag{128}$$

$$\vec{\nabla}\vec{V}^{i+}\big[s_3^*S_\alpha^{\alpha-}+c_3e_\alpha^{(12-)\alpha}\big]\big\}\tag{129}$$

6. Vectorial constitutive equations

Physics has to understand the determination of the EM flux. The fields set $\{A_\mu, U_\mu, V_\mu^\pm\}$ does not work as isolated fields. It provides a quadruplet with on interdependent fields dynamics. The four bosons electromagnetism is a physics based an fields conectivity. It gives, by coupling the Euler-Lagrangian and Noether equations the following vectorial constitutive equations.

A. Spin-1 sector

6. For A_μ photon field:

- Nonlinear spin-valued Gauss law:

$$\vec{\nabla}\cdot\{\bar{a}_1\vec{E}_A+\bar{b}_1(\vec{e}_{AU}+\vec{e}_{+-})\}+l^0_{AT}+c^0_{AT}=24_2m^2_U\phi_U+\rho_{AT} \tag{130}$$

where $\tilde{a}_I$ are multiplicative terms from a_I respecivaly.

Eq. (6.1) introduces a photon field dynamics beyond Maxwell. It contains a dynamics with granular and collective electric fields, masses terms from fields condensation into scalars writen at London and conglomerated terms, plus sources constituted by external masses and densities related to spin. Introducing sources that are beyond electric charge.

The London term is

$$\begin{aligned}
&l^0_{AT}=-2qa_3(b^2+ab)(\phi^+\phi^-+\vec{V}^+\vec{V}^-)\phi_U+\\
&-2qa_3(a^2+ab)(\phi^+\phi^-+\vec{V}^+\vec{V}^-)\phi_A\\
&\{\mathbf{e}^2_{[12]}[6(\phi_U\phi_U+\vec{U}\cdot\vec{U})+3(\phi^+\phi^-+\vec{V}^+\cdot\vec{V}^-)]\}\phi_A+\\
&-\{\mathbf{e}^2_{[12]}[2(\phi_A\phi_A+\vec{A}\cdot\vec{A})+3(\phi^+\phi^-+\vec{V}^+\cdot\vec{V}^-)]\}\phi_U+
\end{aligned} \tag{131}$$

Conglomerate term

$$\begin{aligned}
&c^0_{AT}=-6\mathbf{e}^2_{[12]}(\phi_U\phi_A+\vec{U}\cdot\vec{A})(\phi_U-\tilde{a}_2\phi_A)\\
&-3\mathbf{e}_{[12]}(\phi_A\phi^++\vec{A}\cdot V^+-\phi_U\phi^+-\vec{U}\cdot V^+)\phi^-\\
&+6\mathbf{e}_{[12]}(\mathbf{e}_{[34]}+1/2\mathbf{e}_{[12]})(\phi_U\phi^++\vec{U}\cdot V^+)\phi^-\\
&-6\mathbf{e}_{[12]}(\mathbf{e}_{[34]}+_2\mathbf{e}_{[12]})(\phi_U\phi^-+\vec{U}\cdot V^-)\phi^+\\
&+6_2\mathbf{e}_{[12]}\mathbf{e}_{[34]}[(\phi^+\phi_A+\vec{V}^+\cdot\vec{A})\phi^--\\
&-q_1^2[(\phi^+\phi_A+\vec{V}^+\cdot\vec{A})\phi^-+(\phi^-\phi_A+\vec{V}^-\cdot\vec{A})\phi^+]\\
&-2a_3q_2^2[(\phi^+\phi_U+\vec{V}^+\cdot\vec{U})\phi^-+(\phi^-\phi_U+\vec{V}^-\cdot\vec{U})\phi^+],
\end{aligned} \tag{132}$$

Density term is expressed as

$$\rho_{AT}=\rho^{ele}+\rho^{neut}+\rho^{spin-neut}+\rho^{spin-charge} \tag{133}$$

where the pure electric charge term is

$$\rho^{ele}=-q(2ia+a_3b)(\vec{E}^+\cdot\vec{V}^-+\vec{E}^-\cdot\vec{V}^+),$$

$$\begin{aligned}
&\rho^{neut}=+\mathbf{e}_{[12]}(b_1\vec{E}_A+b_2\vec{E}_U)\cdot(\vec{U}+\tilde{a}_2\vec{A})\\
&+b_3\mathbf{e}_{[34]}(\vec{E}_+\cdot\vec{V}^-+\vec{E}_-\cdot\vec{V}^+)
\end{aligned} \tag{134}$$

the pure spin term is

$$\begin{aligned}
&\rho^{spin} = i\mathbf{e}_{[12]}\{b_1[(\vec{E}_A\vec{s})_i^0(\tilde{a}_2\vec{A}+\vec{U}) + (\vec{B}_A\cdot\vec{S})_i^0({}_2\vec{A}+\vec{U})] \\
&+b_2[(\vec{E}_U\vec{s})_i^0({}_2\vec{A}+\vec{U}) + (\vec{B}_U\cdot\vec{S})_i^0 + ({}_2\vec{A}+\vec{U})]\} \\
&+i(1+\tilde{a}_2)\mathbf{e}_{[34]}[(\vec{E}_+\cdot\vec{s})_i^0\vec{V}^- + (\vec{B}_+\cdot\vec{S})_i^0\vec{V}^- + \\
&(\vec{E}_-\cdot\vec{s})_i^0\vec{V}^+ + (\vec{B}_-\cdot\vec{S})_i^0\vec{V}^+]
\end{aligned} \tag{135}$$

the charge-spin associated term

$$\rho^{spin-charge} = -q\left\{4a_3\left\{(\Sigma^{\kappa\lambda})\vec{V}^+_{\kappa\lambda}\cdot\vec{E}_-\right\} + \left\{b_3(\Sigma^{\kappa\lambda})\vec{V}^+_{\kappa\lambda}\cdot[\vec{\mathbf{e}}_{-A}+\vec{\mathbf{e}}_{-U}]\right\}\right\} \tag{136}$$

- Nonlinear spin-valued Ampère law:

$$\begin{aligned}
&\vec{\nabla}\times\{\bar{a}_1\vec{B}_A + \bar{b}_1(\vec{b}_{AU}+\vec{b}_{+-})\} - \partial_t\{\bar{a}_1\vec{E}_1 + \bar{b}_1(\vec{e}_{AU}+\vec{e}_{+-})\} + \vec{l}_{AT} + \vec{c}_{AT} = \\
&24\mathbf{m}_U^2\vec{U} + \vec{j}_{AT} + \vec{j}_{NT}
\end{aligned} \tag{137}$$

Similarly, one gets

$$\begin{aligned}
&\vec{l}_{AT} = -q^2 2a_3(b^2+ab)(\phi^+\phi^- + \vec{V}^+\cdot\vec{V}^-)\vec{U} + \\
&-q^2 2a_3(b^2+ab)(\phi^+\phi^- + \vec{V}^+\cdot\vec{V}^-)\vec{A} \\
&\{\mathbf{e}^2_{[12]}[6(\phi_U\phi_U + \vec{U}\cdot\vec{U}) + 3(\phi^+\phi^- + \vec{V}^+\cdot\vec{V}^-)]\}\vec{A} + \\
&-\{\mathbf{e}^2_{[12]}[{}_2(\phi_A\phi_A + \vec{A}\cdot\vec{A}) + 3(\phi^+\phi^- + \vec{V}^+\cdot\vec{V}^-)]\}\vec{U} +
\end{aligned} \tag{138}$$

and

$$\begin{aligned}
&\vec{c}_{AT} = -qa^2[(\phi^+\phi_A + \vec{V}^+\cdot\vec{A})\vec{V}^- + (\phi^-\phi_A + \vec{V}^-\cdot\vec{A})\vec{V}^+] \\
&-2qa_3b^2[(\phi^+\phi_U + \vec{V}^+\cdot\vec{U})\vec{V}^- + (\phi^-\phi_U + \vec{V}^-\cdot\vec{U})\vec{V}^+] \\
&-6\mathbf{e}^2_{[12]}(\phi_U\phi_A + \vec{U}\cdot\vec{A})(\phi_U -_2 \phi_A) \\
&-3\mathbf{e}_{[12]}(\phi_A\phi^+ + \vec{A}\cdot V^+ - \phi_U\phi^+ - \vec{U}\cdot V^+)\vec{V}^- \\
&+6\mathbf{e}_{[12]}(\mathbf{e}_{[34]} + 1/2\mathbf{e}_{[12]})(\phi_U\phi^+ + \vec{U}\cdot V^+)\vec{V}^- \\
&-6\mathbf{e}_{[12]}(\mathbf{e}_{[34]} +_2 \mathbf{e}_{[12]})(\phi_U\phi^- + \vec{U}\cdot V^-)\vec{V}^+ \\
&+6\tilde{a}_2\mathbf{e}_{[12]}\mathbf{e}_{[34]}[(\phi^+\phi_A + \vec{V}^+\cdot\vec{A})\vec{V}^-
\end{aligned} \tag{139}$$

The current term $\vec{j}_{AT}$ is decomposed as

$$\vec{j}_{AT} = \vec{j}^{ele} + \vec{j}^{neut} + \vec{j}^{spin-neut} + \vec{j}^{spin-charge} \tag{140}$$

with

$$\vec{j}^{ele}_{AT} = q(2ia + a_3b)[(\vec{E}^+\phi^- + \vec{E}^-\phi^+) + (\vec{B}_+\times\vec{V}^- + \vec{B}_-\times\vec{V}^+)] \tag{141}$$

$$\vec{j}_{AT}^{neut} = \mathbf{e}_{[12]}[(b_1\vec{E}_A + b_2\vec{E}_U)(\phi_U + \tilde{a}_2\phi_A) + (b_1\vec{B}_A +$$
$$+b_2\vec{B}_U) \times (\vec{U} + \tilde{a}_2\vec{A})] + b_3\mathbf{e}_{[34]}(\vec{E}_+\phi^- + \vec{E}_-\phi^+) +$$
$$+b_3\mathbf{e}_{[34]}(\vec{B}_+ \times \vec{V}^- + \vec{B}_- \times \vec{V}^+) \tag{142}$$

$$\vec{j}_{AT}^{spin-neut} = i\mathbf{e}_{[12]}\{b_1[(\vec{E}_A\vec{s})^i_0({}_2\phi_A + \phi_U) + (\vec{E}_A\vec{s})^i_j(\tilde{a}_2\vec{A} + \vec{U})$$
$$(\vec{B}_A \cdot \vec{S})^i_0({}_2\phi_A + \phi_U) + (\vec{B}_A \cdot \vec{S})^i_j({}_2\vec{A} + \vec{U})] +$$
$$b_2[(\vec{E}_U\vec{s})^0_i({}_2\vec{A} + \vec{U}) + (\vec{B}_U \cdot \vec{S})^0_i({}_2\vec{A} + \vec{U})]\}$$
$$+i(1 + \tilde{a}_2)\mathbf{e}_{[34]}[(\vec{E}_+ \cdot \vec{s})^i_0\phi^- + (\vec{E}_+ \cdot \vec{s})^i_j\vec{V}^-$$
$$+(\vec{B}_+ \cdot \vec{S})^i_0\phi^- + (\vec{B}_+ \cdot \vec{S})^i_j\vec{V}^- + (\vec{E}_- \cdot \vec{s})^i_0\phi^+$$
$$+(\vec{E}_- \cdot \vec{s})^i_j\vec{V}^+ + (\vec{B}_- \cdot \vec{S})^i_0\phi^+ + (\vec{B}_- \cdot \vec{S})^i_j\vec{V}^+] \tag{143}$$

and

$$\vec{j}^{spin-charge} \equiv -q\{4a_3Im\{\vec{E}_- \cdot (\Sigma^{\kappa\lambda})\phi^+_{\kappa\lambda} + \vec{B}_- \times (\Sigma^{\kappa\lambda})\vec{V}^+_{\kappa\lambda}\} + 4b_3Im\{\vec{\mathbf{e}}_{-A}(\Sigma^{\kappa\lambda})\phi^+_{\kappa\lambda} +$$
$$\vec{\mathbf{b}}_{-A} \times (\Sigma^{\kappa\lambda})\vec{V}^+_{\kappa\lambda}\}\} \tag{144}$$

The nonlinear above equations are expliciting that the associated currents are beyond the electric charge current

An enlargemente to the electromagnetic phenomena is obtained. Gauss and Ampère laws are obtained where the correspondent EM fiels are no more a consequenceof eletric charge. Fields generate fields and electric charge is extended to modulated electric charges, neutral charges, spin charges and electric-spin charges.

6. For massive U^μ:

- Nonlinear spin-valued Ampère law:

$$\vec{\nabla} \cdot \{\bar{a}_2\vec{E}_U + \bar{b}_2(\vec{e}_{AU} + \vec{e}_{+-})\} + l^0_{UT} + c^0_{UT} - 2_2m^2_U\phi_U = \rho_{UT} \tag{145}$$

Similarly,

$$l^0_{UT} = -2qa_3(a^2 + ab)(\phi^+\phi^- + \vec{V}^+\vec{V}^-)\phi_U$$
$$-2qa_3(a^2 + ab)(\phi^+\phi^- + \vec{V}^+\vec{V}^-)\phi_A +$$
$$\{\mathbf{e}^2_{[12]}[6(\phi_A\phi_A + \vec{A} \cdot \vec{A}) + 3(\phi^+\phi^- + \vec{V}^+ \cdot \vec{V}^-)]\}\phi_U +$$
$$-\{\mathbf{e}^2_{[12]}[{}_1(\phi_U\phi_U + \vec{U} \cdot \vec{U}) + 3(\phi^+\phi^- + \vec{V}^+ \cdot \vec{V}^-)]\}\phi_A \tag{146}$$

$$\begin{aligned}
&c^0_{UT} = -2q^2a_3b^2[(\phi^+\phi_U + \vec{V}^+\cdot\vec{U})\phi^- + (\phi^-\phi_U + \vec{V}^-\cdot\vec{U})\phi^+]\\
&-q^2a^2[(\phi^+\phi_A + \vec{V}^+\cdot\vec{A})\phi^- + (\phi^-\phi_A + \vec{V}^-\cdot\vec{A})\phi^+]\\
&-6\mathbf{e}^2_{[12]}(\phi_U\phi_A + \vec{U}\cdot\vec{A})(\phi_A - \tilde{a}_1\phi_U)\\
&-3\mathbf{e}_{[12]}(\phi_U\phi^+ + \vec{U}\cdot V^+ - \phi_A\phi^+ - \vec{A}\cdot V^+)\phi^-\\
&+6\mathbf{e}_{[12]}(\mathbf{e}_{[34]} + 1/2\mathbf{e}_{[12]})(\phi_A\phi^+ + \vec{A}\cdot V^+)\phi^-\\
&-6\mathbf{e}_{[12]}(\mathbf{e}_{[34]} + \tilde{a}_1\mathbf{e}_{[12]})(\phi_A\phi^- + \vec{A}\cdot V^-)\phi^+\\
&+6\tilde{a}_2\mathbf{e}_{[12]}\mathbf{e}_{[34]}[(\phi^+\phi_U + \vec{V}^+\cdot\vec{U})\phi^-
\end{aligned} \tag{147}$$

and

$$\rho_{UT} = \rho^{elec}_{UT} + \rho^{neut}_{UT} + \rho^{spin}_{UT} \tag{148}$$

where

$$\rho^{elec}_{UT} = -q(2ia + a_3b)(\vec{E}^+\cdot\vec{V}^- + \vec{E}^-\cdot\vec{V}^+) \tag{149}$$

$$\rho^{neut}_{UT} = \mathbf{e}_{[12]}(b_1\vec{E}_A + b_2\vec{E}_U)\cdot(\vec{A} + \tilde{a}_1\vec{U}) + b_3\mathbf{e}_{[34]}(\vec{E}_+\cdot\vec{V}^- + \vec{E}_-\cdot\vec{V}^+) \tag{150}$$

$$\begin{aligned}
&\rho^{spin}_{UT} = i\mathbf{e}_{[12]}\{b_1[(\vec{E}_A\vec{s})^0_i(\tilde{a}_1\vec{U} + \vec{A}) + (\vec{B}_A\cdot\vec{S})^0_i(\tilde{a}_1\vec{U} + \vec{A})] +\\
&b_2[(\vec{E}_U\vec{s})^0_i(\tilde{a}_1\vec{U} + \vec{A}) + (\vec{B}_U\cdot\vec{S})^0_i(\tilde{a}_1\vec{U} + \vec{A})]\} + i(1 + \tilde{a}_1)\mathbf{e}_{[34]}[(\vec{E}_+\cdot\vec{s})^0_i\vec{V}^- +\\
&+(\vec{B}_+\cdot\vec{S})^0_i\vec{V}^- + (\vec{E}_-\cdot\vec{s})^0_i\vec{V}^+ + (\vec{B}_-\cdot\vec{S})^0_i\vec{V}^+]
\end{aligned} \tag{151}$$

and

$$\rho^{spin-charge} = -q\left\{4a_3\left\{(\Sigma^{\kappa\lambda})\vec{V}^+_{\kappa\lambda}\cdot\vec{E}_-\right\} + \left\{b_3(\Sigma^{\kappa\lambda})\vec{V}^+_{\kappa\lambda}\cdot[\vec{\mathbf{e}}_{-A} + \vec{\mathbf{e}}_{-U}]\right\}\right\} \tag{152}$$

- Nonlinear spin-valued Ampère law:

$$\begin{aligned}
&\vec{\nabla}\times\{\bar{a}_2\vec{B}_U + \bar{b}_2(\vec{b}_{AU} + \vec{b}_{+-})\} - \partial_t\{\bar{a}_2\vec{E}_U + \bar{b}_2(\vec{e}_{AU} + \vec{e}_{+-})\}\\
&-2\mathbf{m}^2_U\vec{U} + \vec{l}_{UT} + \vec{c}_{UT} = \vec{j}_{UT} + \tilde{a}_1\vec{j}_{AU}
\end{aligned} \tag{153}$$

where

$$\begin{aligned}
&\vec{l}_{UT} = -2q^2a_3(a^2 + ab)(\phi^+\phi^- + \vec{V}^+\vec{V}^-)\vec{A} +\\
&-2q^2a_3(a^2 + ab)(\phi^+\phi^- + \vec{V}^+\vec{V}^-)\vec{U}\\
&\{\mathbf{e}^2_{[12]}[6(\phi_A\phi_U + \vec{U}\cdot\vec{U}) + 3(\phi^+\phi^- + \vec{V}^+\cdot\vec{V}^-)]\}\vec{U} +\\
&-\{\mathbf{e}^2_{[12]}[\tilde{a}_1(\phi_U\phi_U + \vec{U}\cdot\vec{U}) + 3(\phi^+\phi^- + \vec{V}^+\cdot\vec{V}^-)]\}\vec{A}
\end{aligned} \tag{154}$$

$$
\begin{aligned}
&\vec{c}_{UT} = -q^2b^2[(\phi^+\phi_U + \vec{V}^+\cdot\vec{U})\vec{V}^- + (\phi^-\phi_U + \vec{V}^-\cdot\vec{U})\vec{V}^+] \\
&-2a_3q^2a^2[(\phi^+\phi_A + \vec{V}^+\cdot\vec{A})\vec{V}^- + (\phi^-\phi_A + \vec{V}^-\cdot\vec{A})\vec{V}^+] \qquad (155)\\
&-6\mathbf{e}^2_{[12]}(\phi_U\phi_A + \vec{U}\cdot\vec{A})(\phi_A - \tilde{a}_1\phi_U) \\
&-3\mathbf{e}_{[12]}(\phi_A\phi^+ + \vec{A}\cdot V^+ - \phi_U\phi^+ - \vec{U}\cdot V^+)\vec{V}^- \\
&+6\mathbf{e}_{[12]}(\mathbf{e}_{[34]} + 1/2\mathbf{e}_{[12]})(\phi_U\phi^+ + \vec{U}\cdot V^+)\vec{V}^- \\
&-6\mathbf{e}_{[12]}(\mathbf{e}_{[34]} +_2 \mathbf{e}_{[12]})(\phi_U\phi^- + \vec{U}\cdot V^-)\vec{V}^+ \\
&+6\tilde{a}_1\mathbf{e}_{[12]}\mathbf{e}_{[34]}[(\phi^+\phi_U + \vec{V}^+\cdot\vec{U})\vec{V}^- \qquad (156)
\end{aligned}
$$

$$\vec{j}_{UT} = \vec{j}^{elec}_{UT} + \vec{j}^{neut} + \vec{j}^{spin-neut} + \vec{j}^{spin-charge} \qquad (157)$$

$$\vec{j}^{elec}_{UT} = -q(2ib + a_3a)[(\vec{E}^+\phi - + \vec{E}^-\phi^+) + (\vec{B}_+\times\vec{V}^- + \vec{B}_-\times\vec{V}^+)] \qquad (158)$$

$$
\begin{aligned}
&\vec{j}^{neut}_{UT} = \mathbf{e}_{[12]}[(b_1\vec{E}_A + b_2\vec{E}_U)(\phi_A + \tilde{a}_1\phi_U) + (b_1\vec{B}_A + \\
&+b_2\vec{B}_U)\times(\vec{A} + \tilde{a}_1\vec{U})] + b_3\mathbf{e}_{[34]}(\vec{E}_+\phi^- + \vec{E}_-\phi^+) + \\
&+b_3\mathbf{e}_{[34]}(\vec{B}_+\times\vec{V}^- + \vec{B}_-\times\vec{V}^+) \qquad (159)
\end{aligned}
$$

$$
\begin{aligned}
&\vec{j}^{spin}_{UT} = i\mathbf{e}_{[12]}\{b_1[(\vec{E}_A\vec{s})^i_0(_2\phi_A + \phi_U) + (\vec{E}_A\vec{s})^i_j(\tilde{a}_1\vec{U} + \vec{A}) \\
&(\vec{B}_A\cdot\vec{S})^i_0(\tilde{a}_1\phi_U + \phi_A) + (\vec{B}_A\cdot\vec{S})^i_j(\tilde{a}_1\vec{U} + \vec{A})] + \\
&b_2[(\vec{E}_U\vec{s})^0_i(\tilde{a}_1\vec{U} + \vec{A}) + (\vec{B}_U\cdot\vec{S})^0_i(\tilde{a}_1\vec{U} + \vec{A})]\} \\
&+i(1 + \tilde{a}_1)\mathbf{e}_{[34]}[(\vec{E}_+\cdot\vec{s})^i_0\phi^- + (\vec{E}_+\cdot\vec{s})^i_j\vec{V}^- \\
&+(\vec{B}_+\cdot\vec{S})^i_0\phi^- + (\vec{B}_+\cdot\vec{S})^i_j\vec{V}^- + (\vec{E}_-\cdot\vec{s})^i_0\phi^+ \\
&+(\vec{E}_-\cdot\vec{s})^i_j\vec{V}^+ + (\vec{B}_-\cdot\vec{S})^i_0\phi^+ + (\vec{B}_-\cdot\vec{S})^i_j\vec{V}^+] \qquad (160)
\end{aligned}
$$

and

$$
\begin{aligned}
&\vec{j}^{spin-charge} \equiv -q\{4a_3Im\{\vec{E}_-\cdot(\Sigma^{\kappa\lambda})\phi^+_{\kappa\lambda} + \vec{B}_-\times(\Sigma^{\kappa\lambda})\vec{V}^+_{\kappa\lambda}\} + 4b_3Im\{\vec{\mathbf{e}}_{-A}(\Sigma^{\kappa\lambda})\phi^+_{\kappa\lambda} + \\
&\vec{\mathbf{b}}_{-A}\times(\Sigma^{\kappa\lambda})\vec{V}^+_{\kappa\lambda}\}\} \qquad (161)
\end{aligned}
$$

6. For charged photons $V^{\mu\pm}$:

- Nonlinear spin-valued Gauss law:

$$\vec{\nabla}\cdot\{(6a_3 - 4\beta_3)\vec{E}^\pm + 3a_3\vec{e}_{AU\pm}\} - 12\mathbf{m}_V\phi^\pm + l^{0\pm}_{VT} + c^{0\pm}_{VT} = \rho^\pm_{VT} \qquad (162)$$

where

$$
\begin{aligned}
&l^{0\pm}_{VT} = -q^2a^2\{\phi_A\phi_A + \vec{A}\cdot\vec{A}\}\phi^\pm - q^2b^2\{\phi_U\phi_U + \vec{U}\cdot\vec{U}\}\phi^\pm \\
&6\mathbf{e}^2_{[34]}\{(\phi^+\phi^- + \vec{V}^+\vec{V}^-)\phi^\pm - (\phi^-\phi^\pm + \vec{V}^-\vec{V}^\pm)\phi^+\} \\
&+(\phi^+\phi^\pm + \vec{V}^+\cdot\vec{V}^\pm)\phi^-, \qquad (163)
\end{aligned}
$$

$$c_{VT}^{0\pm} = q^2b^2\{\phi^\pm\phi_U + \vec{V}^\pm\cdot\vec{U}\}\phi_U + 6\mathbf{e}_{[34]}^2\{(\phi^\pm\phi_U + \tag{164}$$

$$\vec{V}^\pm\cdot\vec{U})\phi_A - (\phi^\pm\phi_A + \vec{V}^\pm\cdot\vec{A})\phi_U\} +$$
$$+3\mathbf{e}_{[34]}\mathbf{e}_{[12]}\{(\phi^\pm\phi_U + \vec{V}^\pm\cdot\vec{U})\phi_A - (\phi^\pm\phi_A + \vec{V}^\pm\cdot\vec{A})\phi_A\}$$
$$+3\mathbf{e}_{[12]}\mathbf{e}_{[34]}\{(\phi_A\phi^\pm + \vec{A}\cdot\vec{V}^\pm)(\phi_A - \phi_U) - (\phi^\pm\phi_U + \vec{V}^\pm\cdot\vec{U})(\phi_A -$$
$$-\phi_U\} + q_1^2\{\phi^\pm\phi_A + \vec{V}^\pm\cdot\vec{A}\}\phi_A, \tag{165}$$

and

$$\rho_{VT}^{elect\pm} = -2iqa\vec{E}^\pm\cdot\vec{A} + 2iqb\vec{E}^\pm\cdot\vec{U} \tag{166}$$

$$\vec{j}_{VT}^{neut\pm} = (\vec{B}^\pm\cdot\vec{S})_i^0(\vec{A}+\vec{U})] + \mathbf{e}_{[34]}b_3\vec{E}^\pm\cdot(\vec{A}+\vec{U}) \tag{167}$$

and

$$\rho_{VT}^{spin\pm} = -i\mathbf{e}_{[34]}\{b_1[(\vec{E}_A\cdot\vec{s})_i^0\vec{V}^\pm + (\vec{B}_A\cdot\vec{S})_i^0\vec{V}^\pm] +$$
$$+b_2[(\vec{E}_U\cdot\vec{s})_i^0\vec{V}^\pm + (\vec{B}_U\cdot\vec{S})_i^0\vec{V}^\pm\} + \mathbf{e}_{[34]}b_1(\vec{E}_A + b_2\vec{E}_U)\vec{V}^\pm + i\mathbf{e}_{[34]}b_3[(\vec{E}^\pm\cdot\vec{s})_i^0(\vec{A}+$$
$$-\vec{U}) \tag{168}$$

- Nonlinear spin-valued Ampère law:

$$\vec{\nabla}\times\{(6a_3-4\beta_3)\vec{B}_\pm + 3a_3\vec{b}_{AU\pm}\} - \partial_t\{(6a_3-4\beta_3)\vec{E}^\pm + 3a_3\vec{e}_{AU\pm}\}$$
$$-12\mathbf{m}^2\vec{V}^\pm + \vec{l}_{VT}^\pm = \vec{j}_{VT}^\pm \tag{169}$$

$$\vec{l}_{VT}^\pm = -q^2a^2\{\phi_A\phi_A + \vec{A}\cdot\vec{A}\}\vec{V}^\pm - q_2^2\{\phi_U\phi_U + \vec{U}\cdot\vec{U}\}\vec{V}^\pm$$
$$6\mathbf{e}_{[34]}^2[(\phi^+\phi^\pm + \vec{V}^+\cdot\vec{V}^\pm)\vec{V}^- - (\phi^-\phi^\pm + \vec{V}^-\cdot\vec{V}^\pm)\vec{V}^+], \tag{170}$$

$$\vec{c}_{VT}^\pm = q^2a^2\{\phi^\pm\phi_A + \vec{V}^\pm\cdot\vec{A}\}\vec{A} + q^2b^2\{\phi^\pm\phi_U + \vec{V}^\pm\cdot\vec{U}\}\vec{U}$$
$$+3\mathbf{e}_{[34]}\mathbf{e}_{[12]}\{(\phi^\pm\phi_U + \vec{V}^\pm\cdot\vec{U})\vec{A} - (\phi^\pm\phi_A + \vec{V}^\pm\cdot\vec{A})\vec{U}\} +$$
$$+3\mathbf{e}_{[34]}\mathbf{e}_{[12]}\{(\phi_A\phi^\pm + \vec{A}\cdot\vec{V}^\pm)(\vec{A}-\vec{U}) + (\phi_U\phi^\pm + \vec{U}\cdot\vec{V}^\pm)(\vec{A}-$$
$$-\vec{U}) - (\phi_A\phi_A + \vec{A}\cdot\vec{A})\vec{V}^\pm - (\phi_U\phi_U + \vec{U}\cdot\vec{U})\vec{V}^\pm\}$$
$$6\mathbf{e}_{[34]}^2[(\phi^\pm\phi_U + \vec{V}^\pm\cdot\vec{U}\vec{A}) - (\phi^\pm\phi_A + \vec{V}^\pm\cdot\vec{A})\vec{U}], \tag{171}$$

and

$$\vec{j}_{VT}^\pm = \vec{j}_{VT}^{elec\pm} + \vec{j}_{VT}^{neut\pm} + \vec{j}_{VT}^{spin\pm} \tag{172}$$

$$\vec{j}_{VT}^\pm = -2iqb(\vec{B}^\pm\times\vec{U} + \vec{E}^\pm\phi_A) \tag{173}$$

$$\vec{j}_{VT}^{neut\pm} = \mathbf{e}_{[34]}b_3[\vec{B}_\pm\times(\vec{A}+\vec{U}) + \vec{E}^\pm(\phi_A+\phi_U)] \tag{174}$$

and

$$\vec{j}^{spin\pm}_{VT} = i\mathbf{e}_{[34]}\{b_1[(\vec{E}_A \cdot \vec{s})^i_0\phi^{\pm} + (\vec{E}_A \cdot \vec{s})^i_j\vec{V}^{\pm} +$$
$$+(\vec{B}_A \cdot \vec{S})^i_0\phi^{\pm}(\vec{B}_A \cdot \vec{S})^i_j\vec{V}^{\pm}] + b_2[(\vec{E}_U \cdot \vec{s})^i_0\phi^{\pm} +$$
$$+(\vec{E}_U \cdot \vec{s})^i_j\vec{V}^{\pm}+\} + (\vec{B}_U \cdot \vec{S})^i_0\phi^{\pm}] + \mathbf{e}_{[34]}[(b_1\vec{B}_A +$$
$$+b_2\vec{B}_U) \times \vec{V}^{\pm} + (b_1\vec{E}_A + b_2\vec{E}_U)\phi^{\pm}] +$$
$$\mathbf{e}_{[34]}b_3[(\vec{E}^{\pm} \cdot \vec{s})^i_0(\phi_A + \phi_U) + (\vec{E}^{\pm}) \cdot \vec{s})^i_j(\vec{A} + \vec{U}) +$$
$$+(\vec{E}^{\pm} \cdot \vec{s})^i_0(\phi_A + \phi_U) + (\vec{B}^{\pm} \cdot \vec{s})^i_0(\phi_A + \phi_U) +$$
$$+(\vec{B}^{\pm} \cdot \vec{s})^i_j(\vec{A} + \vec{U})] \tag{175}$$

Thus, while at Maxwell laws, electric charge appears with the function of attracting and repelling bodies, at four bosons electromagnetism it assembles fields. This is the picture shown by the above equations. The quadruplet is assembled beyond electric charge.

6. Continuity equations

An electromagnetic flux is constituted by the quadruplet.Continuity equations are developed. All above equations contain a respective continuity equation. The difference here is that they enlarge the electric charge meaning. It holds the generic expression

$$\frac{\partial}{\partial t}\rho_{IT} + \vec{\nabla} \cdot \vec{j}_{IT} = 0 \tag{176}$$

where

$$\rho^T_I = -l^0_{IT} - c^0_I + 24m^2_I\phi_I + \rho_{IT} + \rho_{NT} \tag{177}$$
$$\vec{j}_{IT} = -\vec{l}_{IT} - \vec{c}_{IT} + 24m_I\vec{A}_I + \vec{j}_{IT} + \vec{J}_{NT} \tag{178}$$

7. Scalar constitutive equations.

B. Spin-0 sector

7. For photon spin-valued scalar field:

- Time evolution.

$$\partial^0\{(s_1 * +\tilde{s}_1)S^{\alpha}_{\alpha 1} + (c_1 * +\tilde{c}_1)(s^{\alpha}_{\alpha AA} + s^{\alpha}_{\alpha AU} + s^{\alpha}_{\alpha UU} + s^{+-\alpha}_{\alpha})\}$$
$$+l^0_{AL} + c^0_{AL} = 48(k_1 * +\tilde{M}_1)\mathbf{m}^2_U U^0 + \tilde{t}_1 j^0_{AL} + \tilde{l}_1 J^0_{UL} - j^0_{NL} \tag{179}$$

with

$$\begin{aligned} & l_{AL}^0 = (-4\mathbf{e}_{(22)}\mathbf{e}_{(11)} - 8\mathbf{e}_{(22)}^2 + 96\tilde{l}_2\mathbf{e}_{(11)}\mathbf{e}_{(22)}) + (U_0U^0 + U_iU^i)U_0 \\ & +(200\mathbf{e}_{(22)}\mathbf{e}_{(11)} - 8\mathbf{e}_{(22)}^2 + 98\tilde{l}_2\mathbf{e}_{(22)}\mathbf{e}_{(11)})(U_0U^0 + U_iU^i)A_0 + \\ & (-12\mathbf{e}_{(22)\mathbf{e}_{(12)}} - 8\mathbf{e}_{(22)}\mathbf{e}_{(12)} + 106\tilde{l}_2\mathbf{e}_{(12)}\mathbf{e}_{(11)})(A_0A^0 + A_iA^i)A^0 \\ & (-8\mathbf{e}_{(22)}\mathbf{e}_{(11)} + 96\mathbf{e}_{(12)}^2)(A_0A^0 + A_iA^i)U^0, \end{aligned} \tag{180}$$

$$\begin{aligned} & c_{AL}^0 = (-2\mathbf{e}_{(11)}^2 - 8\mathbf{e}_{(11)}\mathbf{e}_{(12)} - 2\tilde{l}_1\mathbf{e}_{(11)}\mathbf{e}_{(22)} - 2\tilde{l}_1\mathbf{e}_{(12)}\mathbf{e}_{(22)})(U_0A^0 + U_iA^i)U^0 \\ & +(-10\mathbf{e}_{(11)}\mathbf{e}_{(12)} - 4\mathbf{e}_{(11)}^2 - 2\tilde{l}_1 - 8\tilde{l}_1\mathbf{e}_{(22)}\mathbf{e}_{(12)})(A_0U^0 + A_iU^i)A^0 \\ & +(-2\mathbf{e}_{(11)}\mathbf{e}_{(34)} - \tilde{l}_1\mathbf{e}_{(22)}\mathbf{e}_{(34)})[(A_0V^{0+} + A_iV^{i+})V^{0-} + (A_0V^{0-} + A_iV^{i-})V^{0+}] \\ & +(-2\mathbf{e}_{(11)}\mathbf{e}_{(34)} - \tilde{l}_1\mathbf{e}_{(22)}\mathbf{e}_{(34)})[(U_0V^{0+} + U_iV^{i+})V^{0-} + (U_0V^{0-} + U_iV^{i-})V^{0+}] \\ & +(-2\mathbf{e}^2 - 8\tilde{l}_1\mathbf{e}_{(12)}^2 + \tilde{l}_1\mathbf{e}_{(12)}\mathbf{e}_{(11)})\{[(A_0 + U_0)V^{0-} + (A_i + U_i)V^{i-}]V^{0+} + \\ & +[(A_0 + U_0)V^{0+} + (A_i + U_i)V^{i+}]V^{0-}\} \end{aligned} \tag{181}$$

and

$$\begin{aligned} & j_{AL}^0 = 2(1 + \tilde{l}_1)\mathbf{e}_{(12)}[(\beta_1S_{i1}^0 + \beta_2S_{i2}^0)(A^i + U^i) - (\beta_1S_{\alpha1}^\alpha + \beta_2S_{\alpha2}^\alpha)(A^0 + U^0)] + \\ & +2(1 + \tilde{l}_1)\beta_3\mathbf{e}_{(12)}(S_i^{0+}V^{i-} + S_i^{0-}V^{i+}) + 48(1 + \tilde{l}_1)\mathbf{e}_{(12)}(\rho_1S_{\alpha1}^\alpha + \rho_2S_\alpha^\alpha)(A^0 + U^0) \\ & 24(1 + \tilde{l}_1)\mathbf{e}_{(12)}\rho_3(S_\alpha^{\alpha+}V^{0-} + S_\alpha^{\alpha-}V^{0+}) \end{aligned} \tag{182}$$

- Space dynamics:

$$\begin{aligned} & \partial^i\{(s_1 * + \tilde{s}_1)S_{\alpha1}^\alpha + (c_1 * + \tilde{c}_1)(s_{\alpha AA}^\alpha + s_{\alpha AU}^\alpha + s_{\alpha UU}^\alpha + s_\alpha^{+-\alpha})\} \\ & +l_{AL}^i + c_{AL}^i = 48(k_1 * + \tilde{M}_1)\mathbf{m}_U^2U^0 + \tilde{t}_1j_{AL}^i + \tilde{l}_1J_{UL}^i - j_{NL}^i \end{aligned} \tag{183}$$

with

$$\begin{aligned} & l_{AL}^i = (-4\mathbf{e}_{(22)}\mathbf{e}_{(11)} - 8\mathbf{e}_{(22)}^2 + 96\tilde{l}_2\mathbf{e}_{(11)}\mathbf{e}_{(22)}) + (U_0U^0 + U_iU^i)U^i \\ & +(200\mathbf{e}_{(22)}\mathbf{e}_{(11)} - 8\mathbf{e}_{(22)}^2 + 98\tilde{l}_2\mathbf{e}_{(22)}\mathbf{e}_{(11)})(U_0U^0 + U_iU^i)A^i + \\ & (-12\mathbf{e}_{(22)\mathbf{e}_{(12)}} - 8\mathbf{e}_{(22)}\mathbf{e}_{(12)} + 106\tilde{l}_2\mathbf{e}_{(12)}\mathbf{e}_{(11)})(A_0A^0 + A_iA^i)A^i \\ & (-8\mathbf{e}_{(22)}\mathbf{e}_{(11)} + 96\mathbf{e}_{(12)}^2)(A_0A^0 + A_iA^i)U^i, \end{aligned} \tag{184}$$

$$\begin{aligned} & c_{AL}^i = (-2\mathbf{e}_{(11)}^2 - 8\mathbf{e}_{(11)}\mathbf{e}_{(12)} - 2\tilde{l}_1\mathbf{e}_{(11)}\mathbf{e}_{(22)} - 2\tilde{l}_1\mathbf{e}_{(12)}\mathbf{e}_{(22)})(U_0A^0 + U_iA^i)U^i \\ & +(-10\mathbf{e}_{(11)}\mathbf{e}_{(12)} - 4\mathbf{e}_{(11)}^2 - 2\tilde{l}_1 - 8\tilde{l}_1\mathbf{e}_{(22)}\mathbf{e}_{(12)})(A_0U^0 + A_iU^i)A^i \\ & +(-2\mathbf{e}_{(11)}\mathbf{e}_{(34)} - \tilde{l}_1\mathbf{e}_{(22)}\mathbf{e}_{(34)})[(A_0V^{0+} + A_iV^{i+})V^{i-} + (A_0V^{0-} + A_iV^{i-})V^{i+}] \\ & +(-2\mathbf{e}_{(11)}\mathbf{e}_{(34)} - \tilde{l}_1\mathbf{e}_{(22)}\mathbf{e}_{(34)})[(U_0V^{0+} + U_iV^{i+})V^{i-} + (U_0V^{0-} + U_iV^{i-})V^{i+}] \\ & +(-2\mathbf{e}^2 - 8\tilde{l}_1\mathbf{e}_{(12)}^2 + \tilde{l}_1\mathbf{e}_{(12)}\mathbf{e}_{(11)})\{[(A_0 + U_0)V^{0-} + (A_i + U_i)V^{i-}]V^{i+} + \\ & +[(A_0 + U_0)V^{0+} + (A_i + U_i)V^{i+}]V^{i-}\} \end{aligned} \tag{185}$$

and

$$j^i_{AL} = 2(1+\tilde{l}_1)\mathbf{e}_{(12)}[(\beta_1 S^i_{01} + \beta_2 S^i_{02})(A^0 + U^0) + (\beta_1 S^i_{j1} + \beta_2 S^i_{j2})(A^j + U^j)$$
$$-(\beta_1 S^\alpha_{\alpha 1} + \beta_2 S^\alpha_{\alpha 2})(A^i + U^i)] + +2(1+\tilde{l}_1)\beta_3 \mathbf{e}_{(12)}[(S^{i+}_0 V^{0-} + S^{i-}_0 V^{0+}) +$$
$$+(S^{i+}_j V^{j-} + S^{i-}_j V^{j+})] + 48(1+\tilde{l}_1)\mathbf{e}_{(12)}(\rho_1 S^\alpha_{\alpha 1} + \rho_2 S^\alpha_\alpha)(A^i + U^i)$$
$$24(1+\tilde{l}_1)\mathbf{e}_{(12)}\rho_3(S^{\alpha+}_\alpha V^{i-} + S^{\alpha-}_\alpha V^{i+}) \tag{186}$$

7. For massive photon scalar fields:

- Time evolution.

$$\partial^0\{(s_2 * + \tilde{s}_2)S^\alpha_{\alpha 2} + (c_2 * + \tilde{c}_2)(s^\alpha_{\alpha AA} + s^\alpha_{\alpha AU} + s^\alpha_{\alpha UU} + s^{+-\alpha}_\alpha)\}$$
$$+l^0_{UL} + c^0_{UL} - 48\mathbf{m}^2_U U^0 = j^0_{UL} + \tilde{l}_2 J^0_{AL} - j^0_{NL} \tag{187}$$

with

$$l^0_{UL} = (-4\mathbf{e}_{(22)}\mathbf{e}_{(11)} - 8\mathbf{e}^2_{(11)} + 96\tilde{l}_2\mathbf{e}_{(11)}\mathbf{e}_{(22)}) + (A_0A^0 + A_iA^i)A_0$$
$$+(200\mathbf{e}_{(22)}\mathbf{e}_{(11)} - 8\mathbf{e}^2_{(11)} + 98\tilde{l}_2\mathbf{e}_{(22)}\mathbf{e}_{(11)})(A_0A^0 + A_iA^i)U_0 +$$
$$(-12\mathbf{e}_{(11)\mathbf{e}_{(12)}} - 8\mathbf{e}_{(11)}\mathbf{e}_{(12)} + 106\tilde{l}_2\mathbf{e}_{(12)}\mathbf{e}_{(22)})(U_0U^0 + U_iU^i)U^0$$
$$(-8\mathbf{e}_{(22)}\mathbf{e}_{(11)} + 96\mathbf{e}^2_{(12)})(U_0U^0 + U_iU^i)A^0, \tag{188}$$

$$c^0_{UL} = (-2\mathbf{e}^2_{(22)} - 8\mathbf{e}_{(22)}\mathbf{e}_{(12)} - 2\tilde{l}_2\mathbf{e}_{(11)}\mathbf{e}_{(22)} - 2\tilde{l}_2\mathbf{e}_{(12)}\mathbf{e}_{(11)})(U_0A^0 + U_iA^i)A^0$$
$$+(-10\mathbf{e}_{(22)}\mathbf{e}_{(12)} - 4\mathbf{e}^2_{(22)} - 2\tilde{l}_2 - 8\tilde{l}_2\mathbf{e}_{(11)}\mathbf{e}_{(12)})(A_0U^0 + A_iU^i)U^0$$
$$+(-2\mathbf{e}_{(22)}\mathbf{e}_{(34)} - \tilde{l}_2\mathbf{e}_{(11)}\mathbf{e}_{(34)})[(U_0V^{0+} + U_iV^{i+})V^{0-} + (U_0V^{0-} + U_iV^{i-})V^{0+}]$$
$$+(-2\mathbf{e}_{(22)}\mathbf{e}_{(34)} - \tilde{l}_2\mathbf{e}_{(11)}\mathbf{e}_{(34)})[(A_0V^{0+} + A_iV^{i+})V^{0-} + (U_0V^{0-} + U_iV^{i-})V^{0+}]$$
$$+(-2\mathbf{e}^2 - 8\tilde{l}_2\mathbf{e}^2_{(12)} + \tilde{l}_1\mathbf{e}_{(12)}\mathbf{e}_{(22)})\{[(A_0 + U_0)V^{0-} + (A_i + U_i)V^{i-}]V^{0+} +$$
$$+[(A_0 + U_0)V^{0+} + (A_i + U_i)V^{i+}]V^{0-}\} \tag{189}$$

and

$$j^0_{UL} = 2(1+\tilde{l}_2)\mathbf{e}_{(12)}[(\beta_1 S^0_{i1} + \beta_2 S^0_{i2})(A^i + U^i) - (\beta_1 S^\alpha_{\alpha 1} + \beta_2 S^\alpha_{\alpha 2})(A^0 + U^0)] +$$
$$+2(1+\tilde{l}_2)\beta_3\mathbf{e}_{(12)}(S^{0+}_i V^{i-} + S^{0-}_i V^{i+}) + 48(1+\tilde{l}_2)\mathbf{e}_{(12)}(\rho_1 S^\alpha_{\alpha 1} + \rho_2 S^\alpha_\alpha)(A^0 + U^0)$$
$$24(1+\tilde{l}_2)\mathbf{e}_{(12)}\rho_3(S^{\alpha+}_\alpha V^{0-} + S^{\alpha-}_\alpha V^{0+}) \tag{190}$$

- Space dynamics.

$$\partial^i\{(s_2 * + \tilde{s}_2)S^\alpha_{\alpha 2} + (c_2 * + \tilde{c}_2)(s^\alpha_{\alpha AA} + s^\alpha_{\alpha AU} + s^\alpha_{\alpha UU} + s^{+-\alpha}_\alpha)\}$$
$$+l^i_{UL} + c^i_{UL} - 48\mathbf{m}^2_U U^0 + j^i_{UL} + \tilde{l}_2 J^i_{UL} - j^i_{NL} \tag{191}$$

with

$$l^i_{UL} = (-4\mathbf{e}_{(22)}\mathbf{e}_{(11)} - 8\mathbf{e}^2_{(11)} + 96\tilde{l}_2\mathbf{e}_{(11)}\mathbf{e}_{(22)}) + (A_0A^0 + A_iA^i)A^i$$
$$+(200\mathbf{e}_{(11)}\mathbf{e}_{(22)} - 8\mathbf{e}^2_{(11)} + 98\tilde{l}_1\mathbf{e}_{(22)}\mathbf{e}_{(11)})(A_0A^0 + A_iA^i)U^i +$$
$$(-12\mathbf{e}_{(11)\mathbf{e}_{(12)}} - 8\mathbf{e}_{(11)}\mathbf{e}_{(12)} + 106\tilde{l}_2\mathbf{e}_{(12)}\mathbf{e}_{(22)})(U_0U^0 + U_iU^i)U^i$$
$$(-8\mathbf{e}_{(22)}\mathbf{e}_{(11)} + 96\mathbf{e}^2_{(12)})(U_0U^0 + U_iU^i)A^i, \tag{192}$$

$$c^i_{UL} = (-2\mathbf{e}^2_{(22)} - 8\mathbf{e}_{(22)}\mathbf{e}_{(12)} - 2\tilde{l}_2\mathbf{e}_{(11)}\mathbf{e}_{(22)} - 2\tilde{l}_2\mathbf{e}_{(12)}\mathbf{e}_{(11)})(U_0A^0 + U_iA^i)A^i$$
$$+(-10\mathbf{e}_{(22)}\mathbf{e}_{(12)} - 4\mathbf{e}^2_{(22)} - 2\tilde{l}_2 - 8\tilde{l}_2\mathbf{e}_{(11)}\mathbf{e}_{(12)})(A_0U^0 + A_iU^i)U^i$$
$$+(-2\mathbf{e}_{(22)}\mathbf{e}_{(34)} - \tilde{l}_2\mathbf{e}_{(11)}\mathbf{e}_{(34)})[(U_0V^{0+} + U_iV^{i+})V^{i-} + (U_0V^{0-} + U_iV^{i-})V^{i+}]$$
$$+(-2\mathbf{e}_{(22)}\mathbf{e}_{(34)} - \tilde{l}_2\mathbf{e}_{(11)}\mathbf{e}_{(34)})[(A_0V^{0+} + A_iV^{i+})V^{i-} + (A_0V^{0-} + A_iV^{i-})V^{i+}]$$
$$+(-2\mathbf{e}^2 - 8\tilde{l}_2\mathbf{e}^2_{(12)} + \tilde{l}_2\mathbf{e}_{(12)}\mathbf{e}_{(22)})\{[(A_0 + U_0)V^{0-} + (A_i + U_i)V^{i-}]V^{i+} +$$
$$+[(A_0 + U_0)V^{0+} + (A_i + U_i)V^{i+}]V^{i-}\} \tag{193}$$

and

$$j^i_{UL} = 2(1 + \tilde{l}_2)\mathbf{e}_{(12)}[(\beta_1S^i_{01} + \beta_2S^i_{02})(A^0 + U^0) + (\beta_1S^i_{j1} + \beta_2S^i_{j2})(A^j + U^j)$$
$$-(\beta_1S^\alpha_{\alpha1} + \beta_2S^\alpha_{\alpha2})(A^i + U^i)] + 2(1 + \tilde{l}_2)\beta_3\mathbf{e}_{(12)}[(S^{i+}_0V^{0-} + S^{i-}_0V^{0+}) +$$
$$+(S^{i+}_jV^{j-} + S^{i-}_jV^{j+})] + 48(1 + \tilde{l}_2)\mathbf{e}_{(12)}(\rho_1S^\alpha_{\alpha1} + \rho_2S^\alpha_\alpha)(A^i + U^i)$$
$$24(1 + \tilde{l}_2)\mathbf{e}_{(12)}\rho_3(S^{\alpha+}_\alpha V^{i-} + S^{\alpha-}_\alpha V^{i+}) \tag{194}$$

7. For spin-valued charged photons scalar fields

- Time evolution.

$$\partial^0\{s *_3 S^{\alpha\pm}_\alpha + c_3 * s^\alpha_{\alpha AU\pm}\} + l^{0\pm}_{VL} + c^{0\pm}_{VL} - 24m^2_VV^{0\pm} = j^{0\pm}_{VL} \tag{195}$$

with

$$l^{0\pm}_{VL} = -4\mathbf{e}^2_{(34)}(V^+_0V^{0\pm} + V^+_iV^{i\pm})V^{0-} - 4\mathbf{e}^2_{(34)}(V^+_0V^{0\pm} + V^+_iV^{i\pm})V^{0+} +$$
$$+4\mathbf{e}_{(34)}\mathbf{e}_{(11)}(A_0A^0 + A_iA^i)V^{0\pm} + 4\mathbf{e}_{(34)}\mathbf{e}_{(22)}(U_0U^0 + U_iU^i)V^{0\pm}$$
$$4\mathbf{e}^2_{(34)}(V^+_0V^{0-} + V^+_iV^{i-})V^{0\pm} - 2\mathbf{e}^2_{(12)}(A_0A^0 + A_iA^i)V^{0\pm}$$
$$-2\mathbf{e}^2_{(12)}(U_0U^0 + U_iU^i)V^{0\pm} + 48\mathbf{e}_{(11)}\mathbf{e}_{(34)}(A_0A^0 + A_iA^i)V^{0\pm}$$
$$+48\mathbf{e}_{(34)}\mathbf{e}_{(22)}(U_0U^0 + U_iU^i)V^{0\pm} + 48\mathbf{e}^2_{(34)}(V^+_0V^{0-} + V^+_iV^{i-})V^{0\pm} \tag{196}$$

$$c^0_{VL} = -4\mathbf{e}_{(34)}\mathbf{e}_{(11)}(A_0V^{0\pm} + A_iV^{i\pm})A^0 - 4\mathbf{e}_{(34)}\mathbf{e}_{(22)}(U_0V^{0\pm} + U_iV^{i\pm})U^0$$
$$-4\mathbf{e}_{(34)}\mathbf{e}_{(12)}[(U_0V^{0\pm} + U_iV^{i\pm})A^0 + (A_0V^{0\pm} + A_iV^{i\pm})U^0 + 4\mathbf{e}_{(34)}\mathbf{e}_{(12)}(A_0U^0 +$$
$$+A_iU^i)V^{0\pm}] - 2\mathbf{e}^2_{(12)}[(V^\pm_0A^0 + V^\pm_iA^i)(A^0 + U^0) + (V^\pm_0U^0 + V^\pm_iU^i)(A^0 + U^0)] +$$
$$+48\mathbf{e}_{(34)}\mathbf{e}_{(12)}(A_0U^0 + A_iU^i)V^{0\pm} \tag{197}$$

and

$$j^{0\pm}_{VL} = 2[\beta_1(S^0_{01}V^{0\pm} + S^0_{i1})V^{i\pm}) + \beta_2(S^0_{02}V^{0\pm} + S^0_{i2}V^{i\pm})] +$$
$$\beta_3\mathbf{e}_{(12)}[S^{0\pm}_0(A^0 + U^0) + S^{0\pm}_i(A^i + U^i)] + 24\mathbf{e}_{(34)}[(\rho_1 S^\alpha_{\alpha 1} +$$
$$\rho_2 S^\alpha_{\alpha 2})V^{0\pm} + 24\mathbf{e}_{(12)}\beta_3 S^{\alpha\pm}_\alpha(A^0 + U^0) + 2(\beta_1 S^\alpha_{\alpha 1} + \beta_2 S^\alpha_{\alpha 2})V^{0\pm}] \tag{198}$$

- Space dynamics

$$\vec{\nabla} \times \{\bar{a}_2\vec{B}_U + \bar{b}_2(\vec{b}_{AU} + \vec{b}_{+-})\} - \partial_t\{\bar{a}_2\vec{E}_U + \bar{b}_2(\vec{e}_{AU} + \vec{e}_{+-})\} + \vec{l}_{AT} + \vec{c}_{AT} +$$
$$-24\mathbf{m}^2_U\vec{U} = \vec{j}_{UT} + \vec{j}_{NT} \tag{199}$$

with

$$l^{i\pm}_{VL} = -4\mathbf{e}^2_{(34)}(V^+_0V^{0\pm} + V^+_iV^{i\pm})V^{i-} - 4\mathbf{e}^2_{(34)}(V^+_0V^{0\pm} + V^+_iV^{i\pm})V^{i+} +$$
$$+4\mathbf{e}_{(34)}\mathbf{e}_{(11)}(A_0A^0 + A_iA^i)V^{i\pm} + 4\mathbf{e}_{(34)}\mathbf{e}_{(22)}(U_0U^0 + U_iU^i)V^{i\pm}$$
$$4\mathbf{e}^2_{(34)}(V^+_0V^{0-} + V^+_iV^{i-})V^{i\pm} - 2\mathbf{e}^2_{(12)}(A_0A^0 + A_iA^i)V^{i\pm}$$
$$-2\mathbf{e}^2_{(12)}(U_0U^0 + U_iU^i)V^{i\pm} + 48\mathbf{e}_{(11)}\mathbf{e}_{(34)}(A_0A^0 + A_iA^i)V^{i\pm}$$
$$+48\mathbf{e}_{(34)}\mathbf{e}_{(22)}(U_0U^0 + U_iU^i)V^{i\pm} + 48\mathbf{e}^2_{(34)}(V^+_0V^{0-} + V^+_iV^{i-})V^{i\pm} \tag{200}$$

$$c^i_{VL} = -4\mathbf{e}_{(34)}\mathbf{e}_{(11)}(A_0V^{0\pm} + A_iV^{i\pm})A^i - 4\mathbf{e}_{(34)}\mathbf{e}_{(22)}(U_0V^{0\pm} + U_iV^{i\pm})U^i$$
$$-4\mathbf{e}_{(34)}\mathbf{e}_{(12)}[(U_0V^{0\pm} + U_iV^{i\pm})A^i + (A_0V^{0\pm} + A_iV^{i\pm})U^i + 4\mathbf{e}_{(34)}\mathbf{e}_{(12)}(A_0U^0 +$$
$$+A_iU^i)V^{i\pm}] - 2\mathbf{e}^2_{(12)}[(V^\pm_0A^0 + V^\pm_iA^i)(A^i + U^i) + (V^\pm_0U^i + V^\pm_iU^i)(A^i + U^i)] +$$
$$+48\mathbf{e}_{(34)}\mathbf{e}_{(12)}(A_0U^0 + A_iU^i)V^{i\pm} \tag{201}$$

and

$$j^{i\pm}_{VL} = 2\beta_1(S^i_{01}V^{0\pm} + S^i_jV^{j\pm} + S^\alpha_{\alpha 1}V^{i\pm}) + 2\beta_2(S^j_{02}V^{0\pm} + S^i_{j2}V^{j\pm} +$$
$$+S^\alpha_{\alpha 2}V^{j\pm}) + \beta_3\mathbf{e}_{(12)}[S^{i\pm}_0(A^0 + U^0) + S^{i\pm}_j(A^j + U^j) + S^{\alpha\pm}_\alpha(A^i + U^i)]$$
$$+24\mathbf{e}_{(34)}(\rho_1 S^\alpha_{\alpha 1} + \rho_2 S^\alpha_{\alpha 2})V^{i\pm} + 24\mathbf{e}_{(12)}\beta_3 S^{\alpha\pm}_\alpha(A^i + U^i) \tag{202}$$

8. Bianchi identities

A. Spin-1 sector:

A new result from four bosons electromagnetism is Bianchi identities with composite sources.

8. Antisymmetric granular identity

The granular electromagnetic fields remains the same.

$$\vec{\nabla} \times \vec{E}_I + \frac{\partial \vec{B}_I}{\partial t} = 0$$
$$\vec{\nabla} \cdot \vec{B}_I = 0, \tag{203}$$

8. For antisymmetric neutral collective fields

The collective fields introduce sources. It yields,

$$\vec{\nabla} \times \vec{e}_{AU} + \partial_t \vec{b}_{AU} = \mathbf{e}_{[12]}\{(\vec{E}_U \cdot \vec{s})_{0j}\vec{A} + (\vec{E}_U \cdot \vec{s})_{ij}\phi_A + (\vec{B}_U \cdot \vec{S})_{0j}\vec{A}$$
$$(\vec{B_U \cdot S})_{ij}\phi_A - (\vec{E}_A \cdot \vec{s})_{0j}\vec{U} - (\vec{E}_A \cdot \vec{s})_{ij}\phi_U + (\vec{B}_A \cdot \vec{S})_{0j}\vec{U} - \vec{B_A \cdot S})_{ij}\phi_U - \} \tag{204}$$

and

$$\vec{\nabla} \cdot \vec{b}_{AU} = \mathbf{e}_{[12]}\{(\vec{E}_U \cdot \vec{s})_{ij}\vec{A} + (\vec{B}_U \cdot \vec{S})_{ij} \cdot \vec{A} - (\vec{E}_A \cdot \vec{s})_{ij}\vec{U} - (\vec{B}_A \cdot \vec{S})_{ij} \cdot \vec{U}\} \tag{205}$$

8. For antisymmetric charged collective fields

$$\vec{\nabla} \times \vec{e}_{+-} + \partial_t \vec{b}_{+-} = \mathbf{e}_{[34]}\{(\vec{E}_+ \cdot \vec{s})_{0j}\vec{V}^- + (\vec{E}_+ \cdot \vec{s})_{ij}\phi^- + (\vec{B}_+ \cdot \vec{S})_{0j}\vec{V}^-$$
$$(\vec{B_+ \cdot S})_{ij}\phi^- - (\vec{E}_- \cdot \vec{s})_{0j}\vec{V}^+ - (\vec{E}_- \cdot \vec{s})_{ij}\phi^+ + (\vec{B}_- \cdot \vec{S})_{0j}\vec{V}^+ - \vec{B_- \cdot S})_{ij}\phi^+\} \tag{206}$$

and

$$\vec{\nabla} \cdot \vec{b}_{+-} = \mathbf{e}_{[34]}\{(\vec{E}_+ \cdot \vec{s})_{ij}\vec{V}^- + (\vec{B}_+ \cdot \vec{S})_{ij} \cdot \vec{V}^- - (\vec{E}_- \cdot \vec{s})_{ij}\vec{V}^+ - (\vec{B}_- \cdot \vec{S})_{ij} \cdot \vec{V}^+\} \tag{207}$$

Eqs. (7.1-7.3) are showing composite magnetic poles. All sources are spin dependent Spin-valued sources coupling granular fields strength with EM potential fields.

B. spin-0 sector:

8. Neutral collective fields

$$\partial^i(s^{0j}_{AA} + s^{0j}_{AU} + s^{0j}_{UU}) + \partial^j(s^{i0}_{AA} + s^{i0}_{AU} + s^{i0}_{UU}) + \partial^i(s^{0j}_{AA} + s^{0j}_{AU} +$$
$$s^{0j}_{UU}) + \partial^0(s^{ij}_{AA} + s^{ij}_{AU} + s^{ij}_{UU}) = \mathbf{e}_{(11)}\{A_i S_{0j1} + A_j S_{i01} + A_0 S_{ij1}\}$$
$$+i\mathbf{e}_{(11)}\{g^{i0}A_\alpha\partial_j A^\alpha + g^{ji}A_\alpha\partial_0 A^\alpha + g^{0j}A_\alpha\partial_i A^\alpha\} + \mathbf{e}_{(22)}\{U_i S_{0j2} +$$
$$U_j S_{i02} + U_0 S_{ij2}\} + i\mathbf{e}_{(22)}\{g^{i0}U_\alpha\partial_j U^\alpha + g^{ji}U_\alpha\partial_0 U^\alpha + g^{0j}U_\alpha\partial_i U^\alpha\} +$$
$$+\mathbf{e}_{(12)}\{A_i S_{0j2} + A_j S_{i02} + A_0 S_{ij2} + U_i S_{0j1} + U_j S_{i01} + U_0 S_{ij1}\} +$$
$$+i\mathbf{e}_{(12)}\{g^{i0}(A_\alpha\partial_j A^\alpha + U_\alpha\partial_j U^\alpha) + g^{ji}(A_\alpha\partial_0 A^\alpha + U_\alpha\partial_0 U^\alpha)$$
$$+g^{0j}(A_\alpha\partial_i A^\alpha + U_\alpha\partial_i U^\alpha\} \tag{208}$$

and

$$\begin{aligned}
&\partial^0\{s^{0i}_{AA}+s^{0i}_{UU}+s^{0i}_{AU}\}=\mathbf{e}_{(11)}\{2A_0S_{0i1}+A_iS_{001}\}+\\
&+\mathbf{e}_{(12)}\{2(A_0S_{0i2}+U_0S_{0i1})+A_iS_{002}+U_iS_{001}\}+\\
&+\mathbf{e}_{(22)}\{2U_0S_{0i2}+U_iS_{002}\}+i\mathbf{e}_{(11)}\{g^{00}A_\alpha\partial_iA^\alpha+\\
&+2g^{i0}A_\alpha\partial_0A^\alpha\}+i\mathbf{e}_{(22)}\{g^{00}U_\alpha\partial_iU^\alpha+2g^{i0}U_\alpha\partial_0U^\alpha\}\\
&+i\mathbf{e}_{(12)}\{g^{00}U_\alpha\partial_iA^\alpha+g^{00}A_\alpha\partial_iA^\alpha+2g^{i0}(A_\alpha\partial_0U^\alpha+\\
&+U_\alpha\partial_0A^\alpha)\}
\end{aligned} \tag{209}$$

8. Charged collective fields

$$\begin{aligned}
&\partial^is^{0j}_{+-}+\partial^js^{i0}_{+-}+\partial^0s^{ij}_{+-}=\mathbf{e}_{(34)}\{V^{i+}S^{0j-}+V^{j+}S^{i0-}+\\
&+V^{i-}S^{0j+}+V^{j-}S^{i0+}+V^{0-}S^{ij+}\}+iq_1\{A_iV^+_0+V^+_jA_iV^-_0\\
&+V^+_iA_0V^-_0\}+iq_2\{U_iV^+_0+V^+_jU_iV^-_0+V^+_iU_0V^-_0\}
\end{aligned} \tag{210}$$

and

$$\begin{aligned}
&\partial^0s^{0i}_{+-}+\partial^is^{00}_{+-}=i\mathbf{e}_{(34)}\{V^{0+}S_{0i-}+V^{i+}S^{00-}+V^{0-}S^{0i+}+V^{i+}S^{00-}\}\\
&+iq_1\{V^{0+}A^0V^{i-}+V^{0+}A^iV^{0-}+V^{i+}A^0V^0-\}+iq_2\{V^{0+}U^0V^{i-}+\\
&V^{0+}U^iV^{0-}+V^{i+}U^0V^{0-}\}
\end{aligned} \tag{211}$$

9. Conclusion

Spin is being studied for one century. Its electromagnetic interaction with charge and uncharged particles developed the spintronics. A research mainly focused on the electron. A subject exploring univoque localized spins, atomics sites in crystals, semiconductor quantum dots, neutrino anomalous EM moment. For qubits quantum computing, spin transport, spin dynamics in macroscopic systems, couplings between spin transport and spin dynamics in many ways. However, the spintronics referred to spin-1 is not a developed subject.

Our objective here is to introduce spin-1 effects by studying a spin-valued four bosons electromagnetism. Although the spintronics literature for spin-1 is not much developed [16], new feautures appear to be considered. Given eq. (2.1), one gets new terms in the Lagrangian, new couplings between potential fields and external EM fields strengths (granular and collective), spin charges.

An electromagnetism enlargement is proposed by the four bosons electromagnetism. The electric charge $\{+,0,-\}$ transmission by a quadruplet $\{A_\mu,U_\mu,V^\pm_\mu\}$ reinterprets the meaning of EM in terms of charges and fields. While in Maxwell EM fields are consequence from electric charge the four bosons EM associates each field at quadruplet. They create ther own pair of EM fields $\vec{E}_I-\vec{B}_I$ and collective fields due to the fields interdependence composition. A four-four EM is generated [17].

Electric charge is no more the isolated coupling for EM effects. The Noether theorem and the constitutive equations are showing continuity equations based on electric charge, modulated electric charges, neutral and spin charges. Spin charges are due the interactions of two types of spin vectors, $\vec{S}$ and $\vec{s}$, with magnetic and electric fields respectively. These relationships are registered by constitutive equations and Bianchi identities.

Given these results one should consider on spintronics-1. There is a short literature due to the spin-1 bosons are in general unstable. Decay in a short time and not establish stationary currents. On other hand, the interaction between a magnetic field and the photon is known since 1845 with Faraday [18]. An effect which Maxwell equations did not provide. It requires to introduce a term $gF^{\alpha\beta}(\Sigma^{\alpha\beta})_{\mu}^{\nu}A_{\nu}$ as eq. (1.8). It appears at sources eqs. (3.14, 3.19) and Bianchi identity eq. (3.38).

Thus, spin effects are introduced by enlarging the EM by four intermediate bosons. While in Maxwell, fields are generated from electric charge the EM microscopic approach reverts the approach. The EM origin is in the quadruplet $\{A_{\mu}, U_{\mu}, V_{\mu}^{\pm}\}$. Based on eq. (2.1), it redefines what EM is. From this primitive fields set physics is constituted. Constitutive and Bianchi equations are generated with masses depending as fields: Spin interactions appear from first principle.

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