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**Non-Abelian Constructivist Lagrangian**

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**Abstract**

A whole Yang-Mills symmetry is proposed. A grouping physics is constituted. It consists in inserting a given Yang-Mills field $A_\mu^a$ in a fields set $\{A_{\mu I}^a\}$ constituted by other fields families, $I = 1, \ldots, N$. Each field becomes part of a whole. A set action physics happens preserving the Yang-Mills symmetry. However the usual properties of an isolated field are extended to antireductionist properties.

An associative physics is formed. A Yang-Mills whole quantum system is constituted. A whole Yang-Mills physics is obtained. The quantum corresponding to a specific $A_{\mu I}^a$ field inserted in a whole develops features depending on the fields set $\{A_{\mu I}^a\}$ associativity. Properties established from a so-called constructivist gauge theory are identified. Usual YM interactions are enlarged to YM interrelationships.

Classical equations are studied under set action. A Yang-Mills whole unity is constituted by a constructivist Lagrangian. The reductionist approach substituted by constructivism. Physics under set transformations. A cause and effect relationship is expressed based on whole unity. The whole is that moves to future. Minimal action principle, Noether theorem, Bianchi identities are derived. A fields set with diversity, interdependence, nonlinearity, chance is expressed.

## 1. Introduction

Quantum has been considered the smallest energy packet. Classically, it was introduced by Planck-Einstein radiation. Quantically, developed by wave-particle complementarity and relativity. Nevertheless physics is not constituted only by a reductionist quantum behaviour. Nature contains a grouping performance. Principles as associativity, confinement, complexity are ruling for a grouping physics. Associativity as the nature basic behaviour, confinement by cutting the reductionist route, complexity as considering that physical properties are depending more on the context than due to isolated parts.

Thus, a non abelian grouping physics is proposed. It provides different fields families transforming under a same symmetry group. The usual Yang-Mills fields $A_\mu^a$ are incorporated in a fields set $\{A_{\mu I}^a\}$, where $I$ means the flavour index expressing the number of involved families. The Yang-Mills symmetry is preserved and amplified for

$$A_{\mu I}{}' = U A_{\mu I} U^{-1} + \frac{i}{g_I} \partial_\mu U . U^{-1} \tag{1.1}$$

where $A_{\mu I} = A_{\mu I}^a t_a$ and $U = e^{i\omega_a t_a}$ correspond to the $SU(N)$ symmetry group.

A new physics is expected. Quantum physics based in conglomerate. Part inserted in the whole. A Yang-Mills whole unity is constituted. A constructivist lagrangian will be performed. It considers the fields set $\{A^a_{\mu I}\}$ under a same symmetry group. The corresponding quantum is called as non abelian whole quantum. It introduces new properties beyond usual Yang-Mills theory [1]-[6].

The investigation is to consider the field $A^a_\mu$ under set action. Study an isolated field inserted at fields set $\{A^a_{\mu I}\} = \{A^a_\mu, B^a_\mu, \dots N^a_\mu\}$ [7-15]. Identify the physical difference between an isolated field and in a grouping expressed through the non abelian antireductionist gauge theory. The grouping physics develops conglomerates under associativity and set. The quantum passage from interaction to interrelationship. Features are developed under the whole quantum. Set, diversity, interdependence, nonlinearity, chance. A set determinism is derived. A whole quantum system is expressed with evolution, emergence, complexity.

The classical whole Yang Mills physics will be studied. A non abelian constructivist physics performed. Classical equations are considered. $2N$ equations of motion related to spin-1 and spin-0 are derived. Induction laws given by $N+3$ Bianchi identities are expressed. Three Noether identities obtained from gauge symmetry. They state a whole non-abelian quantum physics.

## 2. Fields strengths

A non abelian constructivist lagrangian will be studied. It is constituted by fields strengths. They contain granular and collective natures. Producing antisymmetric and symmetric terms. Eq. (1.1) symmetry is studied at Appendix A through a field basis $\{D_\mu, X^i_\mu\}$. There $D_\mu$ transforms inhomogeneously while fields $X^i_\mu$ covariantly. The constructor basis $\{D_\mu, X^i_\mu\}$ works to establish covariant field strengths.

### 1. Antisymmetric granular sector $Z_{[\mu\nu]}$

The antisymmetric granular tensor is defined at Appendix A in terms of the tensors $D_{\mu\nu}$ and $X^i_{[\mu\nu]}$, according to:

$$Z_{[\mu\nu]} = dD_{\mu\nu} + \alpha_i X^i_{[\mu\nu]} \tag{2.1}$$

where $d$ and $\alpha_i$ are free parameters. The physical basis $\{G^a_{\mu I}\}$ is expressed in terms of the $\Omega$ matrix [16]-[18]. From Appendix A, one writes

$$D_\mu = \Omega_{1I} G^I_\mu, \quad X_{\mu i} = \Omega_{iI} G^I_\mu, \quad i = 2 \dots N \tag{2.2}$$

From $D_{\mu\nu} = \partial_\mu D_\nu - \partial_\nu D_\mu - ig[D_\mu, D_\nu]$, one rewrites

$$D_{\mu\nu} = \Omega_{1,I}(\partial_\mu G^I_\nu - \partial_\nu G^I_\mu - ig\Omega_{1,J}[G^I_\mu, G^J_\nu]), \tag{2.3}$$

where $I, J \in \{1, 2, \dots, N\}$.

From $X^i_{[\mu\nu]} = \partial_\mu X^i_\nu - \partial_\nu X^i_\mu - ig([D_\mu, X^i_\nu] - [D_\nu, X^i_\mu])$, one rewrites

$$X^i_{[\mu\nu]} = \Omega_{i,I}(\partial_\mu G^I_\nu - \partial_\nu G^I_\mu) - ig(\Omega_{1,I}\Omega_{i,J} + \Omega_{1,J}\Omega_{i,I})[G^I_\mu, G^J_\nu]. \tag{2.4}$$

Using the above equations, it yields

$$Z_{[\mu\nu]} = a_I(\partial_\mu G^I_\nu - \partial_\nu G^I_\mu) - ige_{(IJ)}[G^I_\mu, G^J_\nu] \tag{2.5}$$

where

$$a_I \equiv d\Omega_{1,I} + \alpha_i \Omega_{i,I}, \quad e_{(IJ)} \equiv d\Omega_{1,I}\Omega_{1,J} + \alpha_i \Omega_{1,I}\Omega_{i,J} + \alpha_i \Omega_{1,J}\Omega_{i,I}.$$

The antisymmetric collective tensor is defined at Appendix A as

$$z_{[\mu\nu]} = a_{(ij)}[X^i_\mu, X^j_\nu] + b_{[ij]}\{X^i_\mu, X^j_\nu\} + \gamma_{[ij]} X^i_\mu X^j_\nu. \tag{2.6}$$

Using the relation $X^i_\mu = \Omega_{i,J} G^J_\mu$, it yields

$$z_{[\mu\nu]} = a_{(IJ)}[G^I_\mu, G^J_\nu] + b_{[IJ]}\{G^I_\mu, G^J_\nu\} + \gamma_{[IJ]} G^I_\mu G^J_\nu \tag{2.7}$$

where

$$a_{(IJ)} \equiv a_{(ij)}\Omega_{i,I}\Omega_{j,J}, \quad b_{[IJ]} \equiv b_{[ij]}\Omega_{i,I}\Omega_{j,J}, \quad \gamma_{[IJ]} \equiv \gamma_{[ij]}\Omega_{i,I}\Omega_{j,J}.$$

## 2. Symmetric granular sector $Z_{(\mu\nu)}$

The symmetric granular tensor is defined in terms of the tensor $X^i_{(\mu\nu)}$ according to:

$$Z_{(\mu\nu)} = \beta_i X^i_{(\mu\nu)} + \delta_i g_{\mu\nu} X^{\alpha)i}_{(\alpha} \tag{2.8}$$

where

$$X^i_{(\mu\nu)} = \partial_\mu X^i_\nu + \partial_\nu X^i_\mu - ig([D_\mu, X^i_\nu] + [D_\nu, X^i_\mu]). \tag{2.9}$$

Replacing the above definitions in terms of physical fields we have

$$X^i_{(\mu\nu)} = \Omega_{i,I}\Big(\partial_\mu G_\nu + \partial_\nu G^I_\mu - ig\Omega_{1,J}([G^I_\mu, G^J_\nu] + [G^I_\nu, G^J_\mu])\Big). \tag{2.10}$$

Considering $X^i_{(\mu\nu)} = \Omega_{i,I} G^I_{(\mu\nu)}$, one gets

$$G^I_{(\mu\nu)} \equiv \partial_\mu G^I_\nu + \partial_\nu G^I_\mu + ig\Omega_{1,J}([G^I_\mu, G^J_\nu] + [G^I_\nu, G^J_\mu]). \tag{2.11}$$

Also

$$X^{\alpha)i}_{(\alpha} = \Omega_{i,I} G^{\alpha)I}_{(\alpha}. \tag{2.12}$$

Thus,

$$Z_{(\mu\nu)} = \beta_i \Omega_{i,I} G^I_{(\mu\nu)} + \delta_i \Omega_{i,I} g_{\mu\nu} G^{\alpha)I}_{(\alpha} \tag{2.13}$$

which can be written as

$$Z_{(\mu\nu)} = \beta_I G^I_{(\mu\nu)} + \delta_I g_{\mu\nu} G^{\alpha)I}_{(\alpha} \tag{2.14}$$

where $\beta_I \equiv \beta_i \Omega_{i,I}$ and $\delta_I \equiv \delta_i \Omega_{i,I}$.

The symmetric granular tensor is first defined at $\{D, X_i\}$ basis. It gives

$$z_{(\mu\nu)} = b_{[ij]}[X^i_\mu, X^j_\nu] + c_{(ij)}\{X^i_\mu, X^j_\nu\} + u_{[ij]} g_{\mu\nu}[X^i_\alpha, X^{\alpha j}] + v_{(ij)} g_{\mu\nu}\{X^i_\alpha, X^{\alpha j}\} \tag{2.15}$$

which yields

$$z_{(\mu\nu)} = b_{[IJ]}[G^I_\mu, G^J_\nu] + c_{(IJ)}\{G^I_\mu, G^J_\nu\} + u_{[IJ]} g_{\mu\nu}[G^I_\alpha, G^{\alpha J}] + v_{(IJ)} g_{\mu\nu}\{G^I_\alpha, G^{\alpha J}\} \tag{2.16}$$

where $b_{[IJ]} \equiv b_{[ij]}\Omega_{i,I}\Omega_{j,J}$, $c_{(IJ)} \equiv c_{(ij)}\Omega_{i,I}\Omega_{j,J}$, $u_{[IJ]} \equiv u_{[ij]}\Omega_{i,I}\Omega_{j,J}$, $v_{(IJ)} \equiv v_{(ij)}\Omega_{i,I}\Omega_{j,J}$.

**3. Symmetry of difference consistency**

Eq. (1.1) is showing different fields under a common SU(N) gauge parameter. It yields a symmetry difference. A quanta pluriformity is obtained. Diverse quanta are expressed under a common symmetry. Next we should analyse on the consistency of this symmetry.

Physics does not depend on the fields basis. The symmetry results obtained in the constructor basis can be transposed to the physical basis $\{G^a_{\mu I}\}$. A first consistency that the above fields strengths are also preserving the covariance property

$$\begin{aligned} Z_{\mu\nu}(G_I) &\to Z_{\mu\nu}(G_I)' = U Z_{\mu\nu}(G_I) U^{-1}, \\ z_{\mu\nu}(G_I) &\to z_{\mu\nu}(G_I)' = U z_{\mu\nu}(G_I) U^{-1}. \end{aligned} \tag{2.17}$$

A further consistency on how such non-abelian symmetry of difference is implemented at SU(N) gauge group. Analyzing through the constructor basis one derives eight aspects attached to group generators and gauge parameters. They are from group generators: algebra closure through Jacobi identities and Bianchi identities. From gauge parameters: Noether theorem, gauge fixing, BRST symmetry, global transformations (BRST, ghost scale, gauge global); charges algebra, covariant equations of motion plus Poincaré lemma [19].

As example to express the consistency for introducing gauge families, let us take the Jacobi identity unification. Taking the infinitesimal transformation from eq. (1.1)

$$\begin{aligned} \delta G^a_{\mu I} &= \partial_\mu \omega^a(x) + g f^a_{bc} G^b_{\mu I} \omega^c \omega^c(x) \\ &= [D_{\mu I} \omega(x)]^a \end{aligned} \tag{2.18}$$

one has to verify Jacobi identity acting on the field $G^a_{\mu I}$

$$\{[\delta_1, [\delta_2, \delta_3]] + [\delta_3, [\delta_1, \delta_2]] + [\delta_2, [\delta_3, \delta_1]]\} G^a_{\mu I} = 0 \tag{2.19}$$

Given that,

$$[\delta_1, \delta_2] G^a_{\mu I} = g f^{abc} \Big( (D_{\mu I} \alpha_2)^b \alpha_1^c - (D_{\mu I} \alpha_1)^b \alpha_2^c \Big) = g f^a_{bc} D_{\mu I} (\alpha_2^b \alpha_1^c) \tag{2.20}$$

one verifies eq.(2.18) similarly to the YM case.

Thus, eq. (1.1) is saying that it is possible to derive a Lagrangian where the number of potential fields is not necessarily equal to the number of group generators as ruled by Yang-Mills theory. The introduction of Yang-Mills families in a same SU(N) group becomes a realistic whole physics to be understood. Gauge theory does not follow group theory [20-21].

**3. Constructivist Yang-Mills Lagrangian**

We are going to consider a family of fields with a Yang-Mills type structure given by an additional internal index $a$. Introduce the fields set $\{G^I_\mu\}$ where $G^I_\mu \equiv G^I_{\mu a} t^a$. Given the $SU(N)$ group, $t^a$ is a set of matrices that satisfy the following relations:

$$[t^a, t^b] = if_{abc}t^a, \quad f_{abc} \text{ are constants,} \tag{3.1}$$

$$\{t^a, t^b\} = \frac{1}{N}\delta_{ab}\mathbf{1}_{N\times N} + d_{abc}t^c, \quad d_{abc} \text{ are constants,} \tag{3.2}$$

$$(t^a) = 0, \tag{3.2}$$

$$(t^a t^b) = N\delta_{ab}, \tag{3.4}$$

$$(t^a t^b t^c) = i\frac{N}{2}f_{abc}, \tag{3.5}$$

$$(t^a t^b t^c t^d) = \delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc} + \frac{N}{4}(d_{abf}d_{cdf} - d_{acf}d_{bdf} + d_{adf}d_{bcf}), \tag{3.6}$$

where $f_{abcd}$ is completely antisymmetric at every pair of adjacent indices: $f_{abc} = -f_{acb} = f_{cab} = -f_{cba} = f_{bca} = -f_{bac}$. $d_{abc}$ is completely symmetric at every pair of adjacent indices.

The non abelian constructivist lagrangian will be formed through the fields strengths. It gives

$$L = L^A + L^S + L^M \tag{3.7}$$

where

$$L^A = \lambda_1\Big(Z_{[\mu\nu]}Z^{[\mu\nu]}\Big) + \lambda_2\Big(z_{[\mu\nu]}z^{[\mu\nu]}\Big) + \lambda_3\Big(Z_{[\mu\nu]}z^{[\mu\nu]}\Big), \tag{3.8}$$

$$L^S = \xi_1\Big(Z_{(\mu\nu)}Z^{(\mu\nu)}\Big) + \xi_2\Big(z_{(\mu\nu)}z^{(\mu\nu)}\Big) + \xi_3\Big(Z_{(\mu\nu)}z^{(\mu\nu)}\Big). \tag{3.9}$$

The Yang-Mills extension becomes realistic. Eq.(3.7) underly the basis for deriving a whole non-abelian gauge theory. It also contains scalar and vector sectors to be understood. This fact is already predicted from Lorentz Group representation $(\frac{1}{2}, \frac{1}{2})$. A next step is to open it through the fields variables.

### 1. $L^A$ sector

Taking into account the general properties of trace, $(AB) = (BA)$, $(A + B) = (A) + (B)$, $(aA) = a(A)$, where $A$ and $B$ are matrices and $a$ is a scalar, we obtain the following results:

$$\begin{aligned}\Big(Z_{[\mu\nu]}Z^{[\mu\nu]}\Big) &= a_I a_J\Big((\partial_\mu G^I_\nu - \partial_\nu G^I_\mu)(\partial^\mu G^{\nu J} - \partial^\nu G^{\mu J})\Big)\\ &\quad - 2iga_I e_{(KL)}\Big((\partial_\mu G^I_\nu - \partial_\nu G^I_\mu)[G^{\mu K}, G^{\nu L}]\Big)\\ &\quad - g^2 e_{(IJ)}e_{(KL)}\Big([G^I_\mu, G^J_\nu][G^{\mu K}, G^{\nu L}]\Big),\end{aligned}$$

$$\begin{aligned}\Big(z_{[\mu\nu]}z^{[\mu\nu]}\Big) =& a_{(IJ)}a_{(KL)}\Big([G^I_\mu, G^J_\nu][G^{\mu K}, G^{\nu L}]\Big)\\ &+ b_{[IJ]}b_{[KL]}\Big(\{G^I_\mu, G^J_\nu\}\{G^{\mu K}, G^{\nu L}\}\Big)\\ &+ \gamma_{[IJ]}\gamma_{[KL]}\Big(G^I_\mu G^J_\mu G^{\mu K}G^{\nu L}\Big)\\ &+ 2a_{(IJ)}b_{[KL]}\Big([G^I_\mu, G^J_\nu]\{G^{\mu K}, G^{\nu L}\}\Big)\\ &+ 2a_{(IJ)}\gamma_{[KL]}\Big([G^I_\mu, G^J_\nu]G^{\mu K}G^{\nu L}\Big)\\ &+ 2b_{[IJ]}\gamma_{[KL]}\Big(\{G^I_\mu, G^J_\nu\}G^{\mu K}G^{\nu L}\Big)\end{aligned}$$

$$
\begin{aligned}
\Big(Z_{[\mu\nu]}z^{[\mu\nu]}\Big) =& a_I a_{(KL)}\Big((\partial_\mu G^I_\nu - \partial_\nu G^I_\mu)[G^{\mu K}, G^{\nu L}]\Big)\\
&+ a_I b_{[KL]}\Big((\partial_\mu G^I_\nu - \partial_\nu G^I_\mu)\{G^{\mu K}, G^{\nu L}\}\Big)\\
&+ a_I \gamma_{[KL]}\Big((\partial_\mu G^I_\nu - \partial_\nu G^I_\mu)G^{\mu K} G^{\nu L}\Big)\\
&- ige_{(IJ)}a_{(KL)}\Big([G^I_\mu, G^J_\nu][G^{\mu K}, G^{\nu L}]\Big)\\
&- ige_{(IJ)}b_{[KL]}\Big([G^I_\mu, G^J_\nu]\{G^{\mu K}, G^{\nu L}\}\Big)\\
&- ige_{(IJ)}\gamma_{[KL]}\Big([G^I_\mu, G^J_\nu]G^{\mu K} G^{\nu L}\Big)
\end{aligned}
$$

Splitting the lagrangian in kinetic, trilinear and quadrilinear terms, one gets

$$
L^A = L^A_K + L^A_3 + L^A_4, \tag{3.10}
$$

where

$$
L^A_K = 2\lambda_1 a_I a_K\Big((\partial_\mu G^I_\nu)(\partial^\mu G^{\nu K}) - (\partial_\mu G^I_\nu)(\partial^\nu G^{\mu K})\Big), \tag{3.11}
$$

$$
\begin{aligned}
L^A_3 =& 2a_I(\lambda_3 a_{(KL)} - 2ig\lambda_1 e_{(KL)})\Big((\partial_\mu G^I_\nu)[G^{\mu K}, G^{\nu L}]\Big)\\
&+ 2\lambda_3 a_I b_{[KL]}\Big((\partial_\mu G^I_\nu)\{G^{\mu K}, G^{\nu L}\}\Big)\\
&+ 2\lambda_3 a_I \gamma_{[KL]}\Big((\partial_\mu G^I_\nu)G^{\mu K} G^{\nu L}\Big),
\end{aligned} \tag{3.12}
$$

$$
\begin{aligned}
L^A_4 =& (-\lambda_1 g^2 e_{(IJ)}e_{(KL)} + [\lambda_2 a_{(IJ)} - \lambda_3 ige_{(IJ)}]a_{(KL)})\Big([G^I_\mu, G^J_\nu][G^{\mu K}, G^{\nu L}]\Big)\\
&+ \lambda_2 b_{[IJ]}b_{[KL]}\Big(\{G^I_\mu, G^J_\nu\}\{G^{\mu K}, G^{\nu L}\}\Big)\\
&+ \lambda_2 \gamma_{[IJ]}\gamma_{[KL]}\Big(G^I_\mu G^J_\nu G^{\mu K} G^{\nu L}\Big)\\
&+ (2\lambda_2 a_{(IJ)} - ig\lambda_3 e_{(IJ)})b_{[KL]}\Big([G^I_\mu, G^J_\nu]\{G^{\mu K}, G^{\nu L}\}\Big)\\
&+ (2\lambda_2 a_{(IJ)} - ig\lambda_3 e_{(IJ)})\gamma_{[KL]}\Big([G^I_\mu, G^J_\nu]G^{\mu K} G^{\nu L}\Big)\\
&+ 2\lambda_2 b_{[IJ]}\gamma_{[KL]}\Big(\{G^I_\mu, G^J_\nu\}G^{\mu K} G^{\nu L}\Big).
\end{aligned} \tag{3.13}
$$

Working out the above equations, one gets for the antisymmetric kinetic sector:

$$
L^A_K = l_{IK}\Big((\partial_\mu G^I_{\nu a})(\partial^\mu G^{\nu K}_a) - (\partial_\mu G^I_{\nu a})(\partial^\nu G^{\mu K}_a)\Big), \tag{3.14}
$$

where

$$
l_{IK} = 2N\lambda_1 a_I a_K. \tag{3.15}
$$

For antisymmetric trilinear sector:

$$
L^A_3 = \theta_{IKL,abc}(\partial_\mu G^I_{\mu a})G^{\mu K}_b G^{\nu L}_c, \tag{3.16}
$$

where

$$
\begin{aligned}
\theta_{IKL,abc} \equiv& 2Na_I(i\lambda_3 a_{(KL)} + 2g\lambda_1 e_{(KL)})f_{abc}\\
&+ 2Nd_{abc}\lambda_3 a_I b_{[KL]} + iN\lambda_3 a_I \gamma_{[KL]}f_{abc}.
\end{aligned} \tag{3.17}
$$

For antisymmetric quadrilinear sector:

$$L_4^A = \theta_{IJKL,abcd} G_{\mu a}^I G_{\nu b}^J G_c^{\mu K} G_d^{\nu L} + \omega_{IJKL} G_{\mu a}^I G_{\nu a}^J G_b^{\mu K} G_b^{\nu L}, \tag{3.18}$$

where

$$\begin{aligned}\theta_{IJKL,abcd} \equiv\, & N\big(\lambda_1 g^2 e_{(IJ)} e_{(KL)} - \lambda_2 a_{(IJ)} a_{(KL)} \\ & + i\lambda_3 g e_{(IJ)} a_{(KL)}\big) f_{abm} f_{cdm} \\ & + N\lambda_2 b_{[IJ]} b_{[KL]} d_{abm} d_{cdm} \\ & + \frac{N}{4} \gamma_{[IJ]} \gamma_{[KL]} \big(d_{abm} d_{cdm} - d_{acm} d_{bdm} + d_{adm} d_{bcm}\big) \\ & + N\big(2ig\lambda_2 a_{(IJ)} + g\lambda_3 e_{(IJ)}\big) b_{[KL]} f_{abm} d_{cdm} \\ & - \frac{N}{2} \big(2g\lambda_2 a_{(IJ)} - ig\lambda_3 e_{(IJ)}\big) \gamma_{[KL]} f_{abm} f_{cdm} \\ & + iN\lambda_2 b_{[IJ]} \gamma_{[KL]} d_{abm} f_{cdm},\end{aligned} \tag{3.19}$$

$$\begin{aligned}\omega_{IJKL} \equiv\, & \frac{1}{N} \lambda_2 b_{[IJ]} b_{[KL]} + \lambda_2 \big(\gamma_{[IJ]} \gamma_{[KL]} + \gamma_{[IL]} \gamma_{[KJ]}\big) \\ & + 2\lambda_2 b_{[IJ]} \gamma_{[KL]}.\end{aligned} \tag{3.20}$$

**2. $L^S$ sector**

Working out $L^S$, one gets

$$\begin{aligned}\Big(Z_{(\mu\nu)} Z^{(\mu\nu)}\Big) =\, & \beta_I \beta_K \Big( (\partial_\mu G_\nu^I + \partial_\nu G_\mu^I)(\partial^\mu G^{\nu K} + \partial^\nu G^{\mu K}) \Big) \\ & - \beta_I \beta_K g_J g_L \Big( ([G_\mu^I, G_\nu^J] + [G_\nu^I, G_\mu^J])([G^{\mu K}, G^{\nu L}] + [G^{\nu K}, G^{\mu L}]) \Big) \\ & + 4\delta_I \delta_K \Big( g_{\mu\nu} g^{\mu\nu} (\partial_\alpha G^{\alpha I})(\partial_\beta G^{\beta K}) \Big) \\ & - 4\delta_I \delta_K g_J g_L \Big( g_{\mu\nu} g^{\mu\nu} [G_\alpha^I, G^{\alpha J}][G_\beta^K, G^{\beta L}] \Big) \\ & + 2i\beta_I \beta_K g_L \Big( (\partial_\mu G_\nu^I + \partial_\nu G_\mu^I)([G^{\mu K}, G^{\nu L}][G^{\nu K}, G^{\mu L}]) \Big) \\ & + 4\beta_I \delta_K \Big( g^{\mu\nu} (\partial_\mu G_\nu^I + \partial_\nu G_\mu^I)(\partial_\alpha G^{\alpha K}) \Big) \\ & + 4i\beta_I \delta_K g_L \Big( g^{\mu\nu} (\partial_\mu G_\nu^I + \partial_\nu G_\mu^I)[G_\alpha^K, G^{\alpha L}] \Big) \\ & + 4i\beta_I \delta_K g_J \Big( g^{\mu\nu} ([G_\mu^I, G_\nu^J] + [G_\nu^I, G_\mu^J])(\partial_\alpha G^{\alpha K}) \Big) \\ & - 4\beta_I \delta_K g_J \Big( g^{\mu\nu} ([G_\mu^I, G_\nu^J] + [G_\nu^I, G_\mu^J])[G_\alpha^K, G^{\alpha L}] \Big) \\ & + 8i\delta_I \delta_K g_L \Big( g_{\mu\nu} g^{\mu\nu} (\partial_\alpha G^{\alpha I})[G_\beta^K, G^{\beta L}] \Big),\end{aligned} \tag{3.21}$$

$$
\begin{aligned}
\Big(z_{(\mu\nu)}z^{(\mu\nu)}\Big) &= b_{[IJ]}b_{[KL]}\Big([G^I_\mu, G^J_\nu][G^{\mu K}, G^{\nu L}]\Big)\\
&+ c_{(IJ)}c_{(KL)}\Big(\{G^I_\mu, G^J_\nu\}\{G^{\mu K}, G^{\nu L}\}\Big)\\
&+ u_{[IJ]}u_{[KL]}\Big(g_{\mu\nu}g^{\mu\nu}[G^I_\alpha, G^{\alpha J}][G^K_\beta, G^{\beta L}]\Big)\\
&+ v_{(IJ)}v_{(KL)}\Big(g_{\mu\nu}g^{\mu\nu}\{G^I_\alpha, G^{\alpha J}\}\{G^K_\beta, G^{\beta L}\}\Big)\\
&+ 2b_{[IJ]}c_{(KL)}\Big([G^I_\mu, G^J_\nu]\{G^{\mu K}, G^{\nu L}\}\Big)\\
&+ 2b_{[IJ]}u_{[KL]}\Big(g^{\mu\nu}[G^I_\mu, G^J_\nu][G^K_\beta, G^{\beta L}]\Big)\\
&+ 2b_{[IJ]}v_{(KL)}\Big(g^{\mu\nu}[G^I_\mu, G^J_\nu]\{G^K_\beta, G^{\beta L}\}\Big)\\
&+ 2c_{(IJ)}u_{[KL]}\Big(g^{\mu\nu}\{G^I_\mu, G^J_\nu\}[G^K_\beta, G^{\beta L}]\Big)\\
&+ 2c_{(IJ)}v_{(KL)}\Big(g^{\mu\nu}\{G^I_\mu, G^J_\nu\}\{G^K_\beta, G^{\beta L}\}\Big)\\
&+ 2u_{[IJ]}v_{(KL)}\Big(g_{\mu\nu}g^{\mu\nu}[G^I_\alpha, G^{\alpha J}]\{G^K_\beta, G^{\beta L}\}\Big),
\end{aligned}
\tag{3.22}
$$

$$
\begin{aligned}
\Big(Z_{(\mu\nu)}z^{(\mu\nu)}\Big) &= \beta_I b_{[KL]}\Big((\partial_\mu G^I_\nu + \partial_\nu G^I_\mu)[G^{\mu K}, G^{\nu L}]\Big)\\
&+ \beta_I c_{(KL)}\Big((\partial_\mu G^I_\nu + \partial_\nu G^I_\mu)\{G^{\mu K}, G^{\nu L}\}\Big)\\
&+ \beta_I u_{[KL]}\Big(g^{\mu\nu}(\partial_\mu G^I_\nu + \partial_\nu G^I_\mu)[G^K_\beta, G^{\beta L}]\Big)\\
&+ \beta_I v_{(KL)}\Big(g^{\mu\nu}(\partial_\mu G^I_\nu + \partial_\nu G^I_\mu)\{G^K_\beta, G^{\beta L}\}\Big)\\
&+ i\beta_I g_J b_{[KL]}\Big(([G^I_\mu, G^J_\nu] + [G^I_\nu, G^J_\mu])[G^{\mu K}, G^{\nu L}]\Big)\\
&+ i\beta_I g_J c_{(KL)}\Big(([G^I_\mu, G^J_\nu] + [G^I_\nu, G^J_\mu])\{G^{\mu K}, G^{\nu L}\}\Big)\\
&+ i\beta_I g_J u_{[KL]}\Big(g^{\mu\nu}([G^I_\mu, G^J_\nu] + [G^I_\nu, G^J_\mu])[G^K_\beta, G^{\beta L}]\Big)\\
&+ i\beta_I g_J v_{(KL)}\Big(g^{\mu\nu}([G^I_\mu, G^J_\nu] + [G^I_\nu, G^J_\mu])\{G^K_\beta, G^{\beta L}\}\Big)\\
&+ 2\delta_I b_{[KL]}\Big(g_{\mu\nu}(\partial_\alpha G^{\alpha I})[G^{\mu K}, G^{\nu L}]\Big)\\
&+ 2\delta_I c_{(KL)}\Big(g_{\mu\nu}(\partial_\alpha G^{\alpha I})\{G^{\mu K}, G^{\nu L}\}\Big)\\
&+ 2\delta_I u_{[KL]}\Big(g_{\mu\nu}g^{\mu\nu}(\partial_\alpha G^{\alpha I})[G^K_\beta, G^{\beta L}]\Big)\\
&+ 2\delta_I v_{(KL)}\Big(g_{\mu\nu}g^{\mu\nu}(\partial_\alpha G^{\alpha I})\{G^K_\beta, G^{\beta L}\}\Big)\\
&+ 2i\delta_I g_J b_{[KL]}\Big(g_{\mu\nu}[G^I_\alpha, G^{\alpha J}][G^{\mu K}, G^{\nu L}]\Big)\\
&+ 2i\delta_I g_J c_{(KL)}\Big(g_{\mu\nu}[G^I_\alpha, G^{\alpha J}]\{G^{\mu K}, G^{\nu L}\}\Big)\\
&+ 2i\delta_I g_J u_{[KL]}\Big(g_{\mu\nu}g^{\mu\nu}[G^I_\alpha, G^{\alpha J}][G^K_\beta, G^{\beta L}]\Big)\\
&+ 2i\delta_I g_J v_{(KL)}\Big(g_{\mu\nu}g^{\mu\nu}[G^I_\alpha, G^{\alpha J}]\{G^K_\beta, G^{\beta L}\}\Big).
\end{aligned}
\tag{3.23}
$$

Rearranging the terms, we obtain the following form for $L^S$:

$$L^S = L^S_K + L^S_3 + L^S_4, \tag{3.24}$$

where

$$\begin{aligned} L_K^S &= 2\xi_1\beta_I\beta_K\Big((\partial_\mu G_\nu^I)(\partial^\mu G^{\nu K}) + (\partial_\mu G_\nu^I)(\partial^\nu G^{\mu K})\\ &+ \xi_1\delta_I\delta_K\Big(g_{\mu\nu}g^{\mu\nu}(\partial_\alpha G^{\alpha I})(\partial_\beta G^{\beta K})\Big)\\ &+ 8\xi_1\beta_I\delta_K\Big(g^{\mu\nu}(\partial_\mu G_\nu^I)(\partial_\alpha G^{\alpha K})\Big), \end{aligned} \tag{3.25}$$

$$\begin{aligned} L_3^S &= 4i\xi_1\beta_I\beta_K g_L\Big((\partial_\mu G_\nu^I)[G^{\mu K}, G^{\nu L}]\Big)\\ &+ 4i\xi_1\beta_I\beta_K g_L\Big((\partial_\mu G_\nu^I)[G^{\mu K}, G^{\nu L}]\Big)\\ &+ (8i\xi_1\beta_I\delta_K g_L + 2\xi_3\beta_I u_{[KL]})\Big(g^{\mu\nu}(\partial_\mu G_\nu^I)[G_\alpha^K, G^{\alpha L}]\\ &+ 8i\xi_1\beta_I\delta_K g_J\Big(g^{\mu\nu}[G_\mu^I, G_\nu^J](\partial_\alpha G^{\alpha K})\Big)\\ &+ (8i\xi_1\beta_I\delta_K g_L + 2\xi_3\beta_I u_{[KL]})\Big(g_{\mu\nu}g^{\mu\nu}(\partial_\alpha G^{\alpha I})[G_\beta^K, G^{\beta L}]\Big)\\ &+ 2\xi_3\beta_I b_{[KL]}\Big((\partial_\mu G_\nu^I)[G^{\mu K}, G^{\nu L}]\Big)\\ &+ 2\xi_3\beta_I c_{(KL)}\Big((\partial_\mu G_\nu^I)\{G^{\mu K}, G^{\nu L}\}\Big)\\ &+ 2\xi_3\beta_I v_{(KL)}\Big(g^{\mu\nu}(\partial_\mu G_\nu^I)\{G_\beta^K, G^{\beta L}\}\Big)\\ &+ 2\xi_3\delta_I b_{[KL]}\Big(g_{\mu\nu}(\partial_\alpha G^{\alpha I})[G^{\mu K}, G^{\nu L}]\Big)\\ &+ 2\xi_3\delta_I c_{(KL)}\Big(g_{\mu\nu}(\partial_\alpha G^{\alpha I})\{G^{\mu K}, G^{\nu L}\}\Big)\\ &+ 2\xi_3\delta_I v_{(KL)}\Big(g_{\mu\nu}g^{\mu\nu}(\partial_\alpha G^{\alpha I})\{G_\beta^K, G^{\beta L}\}\Big), \end{aligned} \tag{3.26}$$

$$\begin{aligned}
L_4^S &= -2\xi_1\beta_I\beta_K g_J g_L\Big([G_\mu^I, G_\nu^J][G^{\mu K}, G^{\nu L}]\Big)\\
&= -2\xi_1\beta_I\beta_K g_J g_L\Big([G_\mu^I, G_\nu^J][G^{\nu K}, G^{\mu L}]\Big)\\
&+ (-4\xi\delta_I\delta_K g_J g_L + \xi_2 u_{[IJ]}u_{[KL]})\Big(g_{\mu\nu}g^{\mu\nu}[G_\alpha^I, G^{\alpha J}][G_\beta^K, G^{\beta L}]\Big)\\
&+ (-8\xi\beta_I\delta_K g_J g_L + 2i\xi_3 g_I g_J u_{[KL]})\Big(g^{\mu\nu}[G_\mu^I, G_\nu^J][G_\alpha^K, G^{\alpha L}]\Big)\\
&+ \xi_2 b_{[IJ]}b_{[KL]}\Big([G_\mu^I, G_\nu^J][G^{\mu K}, G^{\nu L}]\Big)\\
&+ \xi_2 c_{(IJ)}c_{(KL)}\Big(\{G_\mu^I, G_\nu^J\}\{G^{\mu K}, G^{\nu L}\}\Big)\\
&+ \xi_2 v_{(IJ)}v_{(KL)}\Big(g_{\mu\nu}g^{\mu\nu}\{G_\alpha^I, G^{\alpha J}\}\{G_\beta^K, G^{\beta L}\}\\
&+ 2\xi_2 b_{[IJ]}c_{(KL)}\Big([G_\mu^I, G_\nu^J]\{G^{\mu K}, G^{\nu L}\}\Big)\\
&+ (2\xi_2 b_{[IJ]}u_{[KL]} + 2i\xi_3\delta_K g_L b_{[IJ]})\Big(g^{\mu\nu}[G_\mu^I, G_\nu^J][G_\beta^K, G^{\beta L}]\Big)\\
&+ (2\xi_2 b_{[IJ]}v_{(KL)} + 2i\xi_3\delta_K g_L c_{(IJ)})\Big(g^{\mu\nu}[G_\mu^I, G_\nu^J]\{G_\beta^K, G^{\beta L}\}\Big)\\
&+ 2\xi_2 c_{(IJ)}u_{[KL]}\Big(g^{\mu\nu}\{G_\mu^I, G_\nu^J\}[G_\beta^K, G^{\beta L}]\Big)\\
&+ 2\xi_2 c_{(IJ)}v_{(KL)}\Big(g^{\mu\nu}\{G_\mu^I, G_\nu^J\}\{G_\beta^K, G^{\beta L}\}\Big)\\
&+ (2\xi_2 u_{[IJ]}v_{(KL)} + 2i\xi_3\delta_I g_J v_{(KL)})\Big(g_{\mu\nu}g^{\mu\nu}[G_\alpha^I, G^{\alpha J}]\{G_\beta^K, G^{\beta L}\}\Big)\\
&+ 2i\xi_3\beta_I g_J b_{[KL]}\Big([G_\mu^I, G_\nu^J][G^{\mu K}, G^{\nu L}]\Big)\\
&+ 2i\xi_3\beta_I g_J c_{(KL)}\Big([G_\mu^I, G_\nu^J]\{G^{\mu K}, G^{\nu L}\}\Big)\\
&+ 2i\xi_3\beta_I g_J v_{(KL)}\Big(g^{\mu\nu}[G_\mu^I, G_\nu^J]\{G_\beta^K, G^{\beta L}\}\Big)\\
&+ 2i\xi_3\delta_I g_J u_{[KL]}\Big(g_{\mu\nu}g^{\mu\nu}[G_\alpha^I, G^{\alpha J}]\{G_\beta^K, G^{\beta L}\}\Big)
\end{aligned} \tag{3.27}$$

After some algebraic manipulations and simplifications, we obtain the following results. For symmetric kinetic sector:

$$\begin{aligned}
L_K^S =& m_{IK}\Big((\partial_\mu G_{\nu a}^I)(\partial^\mu G_a^{\nu K}) + (\partial_\mu G_{\nu a}^I)(\partial^\nu G_a^{\mu K})\Big)\\
&+ n_{IK}(\partial_\alpha G_a^{\alpha I})(\partial_\beta G_a^{\beta K}).
\end{aligned} \tag{3.28}$$

where

$$m_{IK} = 2N\xi_1\beta_I\beta_K, \tag{3.29}$$

$$n_{IK} = 8N\xi_1(2\delta_I\delta_K + \beta_I\delta_K). \tag{3.30}$$

For symmetric trilinear sector:

$$L_3^S = \omega_{IKL,abc}(\partial_\mu G_{\nu a}^I)G_b^{\mu K}G_c^{\nu L} + \sigma_{IKL,abc}(\partial_\mu G_a^{\mu I})G_{\nu b}^K G_c^{\nu L}, \tag{3.31}$$

where

$$\begin{aligned}
\omega_{IKL,abc} =& 4N\xi_1\beta_I(-\beta_K g_L + \beta_L g_K)f_{abc} + 2iN\xi_3\beta_I b_{[KL]}f_{abc}\\
&+ 2N\xi_3\beta_I c_{(KL)}d_{bca},
\end{aligned} \tag{3.32}$$

$$\begin{aligned}\sigma_{IKL,abc} = {} & 2N(-4\xi_1\beta_I\delta_K g_L + i\xi_3\beta_I u_{[KL]}))d_{bca} - 8N\xi_1\beta_K\delta_I g_L f_{abc} \\ & + 8N(-4\xi_1\delta_I\delta_K g_L + i\xi_3)\delta_I u_{[KL]})f_{abc} \\ & + 2N\xi_3(\beta_I v_{(KL)} + \delta_I c_{(KL)} + 4\delta_I v_{(KL)})d_{bca} \\ & + 2iN\xi_3\delta_I b_{[KL]} f_{abc}. \end{aligned} \tag{3.33}$$

For symmetric quadrilinear sector:

$$\begin{aligned} L_4^S = {} & \rho_{IJKL,abcd} G^I_{\mu a} G^J_{\nu b} G^{\mu K}_c G^{\nu L}_d + \chi_{IJKL} G^I_{\mu a} G^J_{\nu a} G^{\mu K}_b G^{\nu L}_b \\ & + \sigma_{IJKL} G^I_{\mu a} G^{\mu J}_a G^K_{\nu b} G^{\nu L}_b, \end{aligned} \tag{3.34}$$

where

$$\begin{aligned}\rho_{IJKL,abcd} = {} & 2N\xi_1\beta_I\beta_K g_J g_L f_{abm} f_{cdm} - 2N\xi_1\beta_I\beta_L g_J g_K f_{abm} f_{cdm} \\ & - N\xi_2 b_{[IJ]} b_{[KL]} f_{abm} f_{cdm} + N\xi_2 c_{(IJ)} c_{(KL)} d_{abm} d_{cdm} \\ & + 2iN\xi_2 b_{[IJ]} c_{(KL)} f_{abm} d_{cdm} - 2iN\xi_3\beta_I g_J b_{[KL]} f_{abm} f_{cdm} \\ & - 2N\xi_3\beta_I g_J c_{(KL)} f_{abm} f_{cdm} + 4N\xi_2 \nu_{(IK)} \nu_{(JL)} d_{acm} d_{bdm} \\ & + 4N(4\xi_1\delta_I\delta_J g_K g_L - \xi_2 u_{[IK]} u_{[JL]}) f_{acm} f_{bdm} \\ & + 2N(4\xi_1\beta_I\delta_J g_K g_L - i\xi_3\beta_I g_K u_{[JL]}) f_{acm} f_{bdm} \\ & + 2N(\xi_2 b_{[IK]} u_{[JL]} + i\xi_3\delta_J g_L b_{[IK]}) d_{acm} d_{bdm} \\ & + 2iN(\xi_2 b_{[IK]} \nu_{(JL)} + i\xi_3\delta_J g_L c_{(IK)}) f_{acm} d_{bdm} \\ & + 2iN\xi_2 c_{(IK)} u_{[JL]} d_{acm} f_{bdm} + 2N\xi_2 c_{(IK)} \nu_{(JL)} d_{acm} d_{bdm} \\ & + 8iN(\xi_2 u_{[IK]} \nu_{(JL)} + i\xi_3\delta_I g_K \nu_{(JL)}) f_{acm} d_{bdm} \\ & - 2N\xi_3\beta_I g_K \nu_{(JL)} f_{acm} d_{bdm} - 8iN\delta_I g_K u_{[JL]} f_{acm} f_{bdm}, \end{aligned} \tag{3.35}$$

$$\chi_{IJKL} \equiv \frac{1}{N}\xi_2 c_{(IJ)} c_{(KL)}, \tag{3.35}$$

$$\begin{aligned}\sigma_{IJKL} \equiv {} & \frac{4}{N}\xi_2 \nu_{(IJ)} \nu_{(KL)} + \frac{2}{N}(\xi_2 b_{[IJ]} u_{[KL]} + i\xi_3\delta_K g_L b_{[IJ]}) \\ & + \frac{2}{N}\xi_2 c_{(IJ)} \nu_{(KL)}. \end{aligned} \tag{3.37}$$

### 3. Mass sector

From Appendix A, one gets

$$L_M = m^2_{ij}\, X^i_\mu\, X^{\mu j} \tag{3.38}$$

which, transposed to the physical basis,

$$L_M = m^2_{II}\, G^I_\mu\, G^{\mu I} \tag{3.39}$$

where $m^2_{II} = \Omega_{iI}\Omega_{jI}\, m^2_{ij}$.

Eq. (3.39) diversely from Higgs mechanism [22-26] introduces mass as energy.

## 4. Equations of Motion

There is a wholeness principle to be expressed classically. A non abelian constructivist lagrangian is obtained. It contains the fields set $\{G^a_{\mu I}\}$ association. An extension to the usual Yang-Mills is considered. The first step is to understand how it provides a whole interconnected dynamics.

The corresponding Euler-Lagrange system of equations of motion is

$$
\begin{aligned}
&\lambda_1(4a_I\partial_\nu Z^{[\mu\nu]}t_a - 4iga_{(IJ)}G^{Jb}_\nu Z^{[\mu\nu]}[t_a,t_b]) \\
&+\lambda_2(4b_{(IJ)}G^{Jb}_\nu z^{[\mu\nu]}[t_a,t_b] + 4c_{[IJ]}G^{Jb}_\nu z^{[\mu\nu]}\{t_a,t_b\} + 2\gamma_{[IJ]}G^{Jb}_\nu z^{[\mu\nu]}[t_a,t_b]) \\
&+\lambda_3\begin{pmatrix} 2a_I\partial_\nu z^{[\mu\nu]}t_a - 2iga_{(IJ)}G^{Jb}_\nu z^{[\mu\nu]}[t_a,t_b] + 2b_{(IJ)}G^{Jb}_\nu Z^{[\mu\nu]}[t_a,t_b] \\ +2c_{[IJ]}G^{Jb}_\nu Z^{[\mu\nu]}\{t_a,t_b\} + 2\gamma_{[IJ]}G^{Jb}_\nu Z^{[\mu\nu]}\{t_a,t_b\} \end{pmatrix} \\
&+\xi_1\begin{pmatrix} -4\beta_I\partial_\nu Z^{(\mu\nu)}t_a - 4\rho_I g_{\rho\sigma}\partial^\mu Z^{(\rho\sigma)}t_a + \\ +4i(g_I\beta_J - g_J\beta_I)G^{Jb}_\nu Z^{(\mu\nu)}[t_a,t_b] + \\ +4i(g_I\rho_J - g_J\rho_I)g_{\rho\sigma}G^{\mu Jb}Z^{(\rho\sigma)}[t_a,t_b] \end{pmatrix} \\
&+\xi_2\begin{pmatrix} 4b_{[IJ]}G^{Jb}_\nu z^{(\mu\nu)}[t_a,t_b] + 4c_{(IJ)}G^{Jb}_\nu z^{(\mu\nu)}\{t_a,t_b\} \\ +4u_{[IJ]}G^{\mu Jb}g_{\rho\sigma}z^{(\rho\sigma)}[t_a,t_b] + 4u_{(IJ)}G^{\mu Jb}g_{\rho\sigma}z^{(\rho\sigma)}\{t_a,t_b\} \end{pmatrix} \\
&+\xi_3\begin{pmatrix} -2\beta_I\partial_\nu z^{(\mu\nu)}t_a - 2\rho_I\partial^\mu g_{\rho\sigma}z^{(\rho\sigma)}t_a + 2b_{[IJ]}G^{Jb}_\nu Z^{(\mu\nu)}[t_a,t_b] + \\ +2c_{(IJ)}G^{Jb}_\nu Z^{(\mu\nu)}\{t_a,t_b\} + 2u_{[IJ]}G^{\mu Jb}g_{\rho\sigma}Z^{(\rho\sigma)}[t_a,t_b] + \\ +2v_{(IJ)}G^{\mu Jb}g_{\rho\sigma}Z^{(\rho\sigma)}\{t_a,t_b\} + 2i(g_I\beta_J - g_J\beta_I)G^{Jb}_\nu z^{(\mu\nu)}[t_a,t_b] + \\ +2i(g_I\rho_J - g_J\rho_I)G^{\mu Jb}g_{\rho\sigma}z^{(\rho\sigma)}[t_a,t_b] \end{pmatrix} \\
&= 0.
\end{aligned} \tag{4.1}
$$

Eq. (4.1) shows the systemic dynamics involving the fields set determinism $\{G^a_{\mu I}\}$ at space-time. Notice that the term $\lambda_1$ rewrites the usual Yang-Mills for $N$-potential fields. Nevertheless the other terms are adding new contributions. They are showing that SU($N$) symmetry should not be limited to Yang-Mills theory. Beyond the inclusion of other fields, it develops an expression in terms of group generators.

Taking the trace in the above equations, one gets

$$
\begin{aligned}
&\lambda_1(2a_I\partial_\nu Z^{[\mu\nu]a} + 2ga_{(IJ)}f_{abc}G_\nu^{Jb}Z^{[\mu\nu]c})+\\
&+\lambda_2(2ib_{(IJ)}f_{abc}G_\nu^{Jb}z^{[\mu\nu]c} + 2c_{[IJ]}d_{abc}G_\nu^{Jb}z^{[\mu\nu]c} + \gamma_{[IJ]}d_{abc}G_\nu^{Jb}z^{[\mu\nu]c})+\\
&+\lambda_3\left(\begin{array}{c} a_I\partial_\nu z^{[\mu\nu]a} + ga_{(IJ)}f_{abc}G_\nu^{Jb}z^{[\mu\nu]c} + ib_{(IJ)}f_{abc}G_\nu^{Jb}Z^{[\mu\nu]c}+\\ +c_{[IJ]}d_{abc}G_\nu^{Jb}Z^{[\mu\nu]c} + \gamma_{[IJ]}d_{abc}G_\nu^{Jb}Z^{[\mu\nu]c}\end{array}\right)+\\
&+\xi_1\left(\begin{array}{c} \beta_I\partial_\nu Z^{(\mu\nu)a} - 2\rho_I\partial^\mu Z_{(\nu}{}^{\nu)a} - 2(g_I\beta_J + g_J\beta_I)f_{abc}G_\nu^{Jb}Z^{(\mu\nu)c}+\\ -2(g_I\rho_J + g_J\rho_I)f_{abc}G^{\mu Jb}z_{(\nu}{}^{\nu)c}\end{array}\right)\\
&+\xi_2\left(\begin{array}{c} 2ib_{[IJ]}f_{abc}G_\nu^{Jb}z^{(\mu\nu)c} + 2c_{(IJ)}d_{abc}G_\nu^{Jb}z^{(\mu\nu)c}+\\ +2iu_{[IJ]}f_{abc}G^{\mu Jb}z_{(\nu}{}^{\nu)c} + 2u_{(IJ)}d_{abc}G^{\mu Jb}z_{(\nu}{}^{\nu)c}\end{array}\right)+\\
&+\xi_3\left(\begin{array}{c} -\beta_I\partial_\nu z^{(\mu\nu)a} - (g_I\beta_J - g_J\beta_I)f_{abc}G_\nu^{Jb}z^{(\mu\nu)c}\\ -\rho_I\partial^\mu z_{(\nu}{}^{\nu)a} - (g_I\rho_J - g_J\rho_I)f_{abc}G^{\mu Jb}z_{(\nu}{}^{\nu)c}\\ +ib_{[IJ]}f_{abc}G_\nu^{Jb}Z^{(\mu\nu)c} + c_{(IJ)}d_{abc}G_\nu^{Jb}Z^{(\mu\nu)c}\\ +iu_{[IJ]}f_{abc}G^{\mu Jb}Z_{(\nu}{}^{\nu)c} + v_{(IJ)}d_{abc}G^{\mu Jb}Z_{(\nu}{}^{\nu)c}\end{array}\right)\\
&= 0.
\end{aligned}
\tag{4.2}
$$

Comparing with the usual Yang-Mills dynamics, eq. (4.2) is showing how from SU($N$) symmetry one can enlarge the dynamics. As consequence, we would say that just from symmetry one can not defines physics. It is necessary to implement a physical principle qualifying the number of intermediate gauge bosons.

Multiplying by $t_k$ and taking the trace one derives the final relationship for the whole equations of motion system

$$\begin{aligned}
&\lambda_I\Big(a_I(d_{aek}-if_{aek})\partial_\nu Z^{[\mu\nu]e}+ga_{(IJ)}f_{abc}(d_{ekc}-if_{ekc})G^{Jb}_\nu Z^{[\mu\nu]e}\Big)+\\
&+\lambda_2\begin{pmatrix} b_{(IJ)}f_{abc}(id_{eck}-f_{eck})G^{Jb}_\nu z^{[\mu\nu]e}+\frac{2}{N}c_{[IJ]}G^{Ja}_\nu z^{[\mu\nu]k}+\\ +c_{[IJ]}d_{abc}(d_{eck}+if_{eck})G^{Jb}_\nu z^{[\mu\nu]e}+\frac{1}{N}\gamma_{[IJ]}G^{Ja}_\nu z^{[\mu\nu]k}+\\ +\frac{1}{2}\gamma_{[IJ]}d_{abc}(d_{eck}+if_{eck})G^{Jb}_\nu z^{[\mu\nu]e}\end{pmatrix}+\\
&+\lambda_3\begin{pmatrix} \frac{1}{2}a_I(d_{aek}-if_{aek})\partial_\nu z^{[\mu\nu]e}+\frac{g}{2}a_{(IJ)}f_{abc}(d_{ekc}-if_{ekc})G^{Jb}_\nu z^{[\mu\nu]e}+\\ +\frac{1}{2}b_{(IJ)}f_{abc}(id_{eck}-f_{eck})G^{Jb}_\nu Z^{[\mu\nu]e}+\frac{1}{N}c_{[IJ]}G^{Ja}_\nu Z^{[\mu\nu]k}\\ +\frac{1}{2}c_{[IJ]}d_{abc}(d_{eck}+if_{eck})G^{Jb}_\nu Z^{[\mu\nu]e}+\frac{1}{N}\gamma_{[IJ]}G^{Ja}_\nu Z^{[\mu\nu]k}+\\ +\frac{1}{2}\gamma_{[IJ]}d_{abc}(d_{eck}+if_{cek})G^{Jb}_\nu Z^{[\mu\nu]e}\end{pmatrix}\\
&+\xi_1\begin{pmatrix} \beta_I(if_{aek}-d_{aek})\partial_\nu Z^{(\mu\nu)e}+\rho_I(if_{aek}-d_{aek})\partial^\mu Z_{(\nu}{}^{\nu)e}+\\ +(g_I\beta_J+g_J\beta_I)f_{abc}(if_{aek}-d_{aek})G^{Jb}_\nu Z^{(\mu\nu)e}+\\ +(g_I\rho_J+g_J\rho_I)f_{abc}(if_{aek}-d_{aek})G^{\mu Jb}Z_{(\mu}{}^{\mu)e}\end{pmatrix}+\\
&+\xi_2\begin{pmatrix} b_{[IJ]}f_{abc}(id_{ekc}+f_{ekc})G^{Jb}_\nu z^{(\mu\nu)e}+\frac{2}{N}c_{(IJ)}G^{Ja}_\nu z^{(\mu\nu)k}+\\ +c_{(IJ)}d_{abc}(d_{ekc}-if_{ekc})G^{Jb}_\nu z^{(\mu\nu)e}+\\ +u_{[IJ]}f_{abc}(id_{ekc}+f_{ekc})G^{\mu Jb}z_{(\nu}{}^{\nu)e}+\frac{2}{N}v_{(IJ)}G^{\mu Ja}z_{(\nu}{}^{\nu)k}\\ +v_{(IJ)}d_{abc}(d_{ekc}-if_{ekc})G^{\mu Jb}z_{(\nu}{}^{\nu)e}\end{pmatrix}+\\
&+\xi_3\begin{pmatrix} \frac{1}{2}\beta_I(if_{aek}-d_{aek})\partial_\nu z^{(\mu\nu)e}+\frac{1}{2}\rho_I(if_{aek}-d_{aek})\partial^\mu z_{(\nu}{}^{\nu)e}+\\ +\frac{1}{2}(g_I\beta_J+g_J\beta_I)f_{abc}(if_{aek}-d_{aek})G^{Jb}_\nu z^{(\mu\nu)e}+\\ +\frac{1}{2}(g_I\rho_J+g_J\rho_I)f_{abc}(if_{aek}-d_{aek})G^{\mu Jb}z_{(\nu}{}^{\nu)e}+\\ +\frac{1}{2}b_{[IJ]}f_{abc}(id_{ekc}+f_{ekc})G^{Jb}_\nu Z^{(\mu\nu)e}+\frac{1}{N}c_{(IJ)}G^{Jb}_\nu Z^{(\mu\nu)k}+\\ +\frac{1}{2}c_{(IJ)}d_{abc}(d_{ekc}-if_{ekc})G^{Jb}_\nu Z^{(\mu\nu)e}+\\ +\frac{1}{2}u_{[IJ]}f_{abc}(id_{ekc}+f_{ekc})G^{\mu Jb}z_{(\nu}{}^{\nu)e}+\frac{1}{N}v_{(IJ)}G^{\mu Jb}Z_{(\nu}{}^{\nu)k}+\\ +\frac{1}{2}v_{(IJ)}d_{abc}(d_{ekc}-if_{ekc})G^{\mu Jb}Z_{(\nu}{}^{\nu)e}\end{pmatrix}=0.
\end{aligned} \tag{4.3}$$

## 5. Constructivist semitopological Yang-Mills type sector

A more general Yang-Mills type Lagrangian has the following form:

$$L=\Big\{(Z_{\mu\nu}+z_{\mu\nu})(Z^{\mu\nu}+z^{\mu\nu})\Big\}+\eta\Big\{(Z_{\mu\nu}+z_{\mu\nu})(\widetilde{Z}^{\mu\nu}+\widetilde{z}^{\mu\nu})\Big\}, \tag{5.1}$$

where the dual field strengths $\widetilde{Z}^{\mu\nu}$ and $\widetilde{z}^{\mu\nu}$ are defined as

$$\widetilde{Z}^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} Z_{\rho\sigma}, \quad \widetilde{z}^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} z_{\rho\sigma}, \tag{5.2}$$

where $\epsilon^{\mu\nu\rho\sigma}$ is the Levi-Civita tensor of rank 4.

The additional term is called the semitopological sector:

$$L^{ST} = \eta\Big\{(Z_{\mu\nu} + z_{\mu\nu})(\widetilde{Z}^{\mu\nu} + \widetilde{z}^{\mu\nu})\Big\}. \tag{5.3}$$

Expanding the above expression and simplifying, we obtain:

$$L^{ST} = \eta(Z_{\mu\nu}\,\widetilde{Z}^{\mu\nu}) + \eta(z_{\mu\nu}\,\widetilde{z}^{\mu\nu}) + 2\eta(z_{\mu\nu}\,\widetilde{Z}^{\mu\nu}). \tag{5.4}$$

This so-called semitopological Lagrangian [ ] is composed of three sub-sectors. The first with only granular terms, the second with only collective terms and the third with mixed terms:

$$L^{ST} = L^{ST}_{gr} + L^{ST}_{co} + L^{ST}_{mix}. \tag{5.5}$$

### 1. Granular semitopological sector

$$L^{ST}_{gr} = \eta(Z_{\mu\nu}\,\widetilde{Z}^{\mu\nu}) \tag{5.6}$$

Now, making $Z_{\mu\nu} = Z_{[\mu\nu]} + Z_{(\mu\nu)}$, we get

$$L^{ST}_{gr} = \eta\varepsilon^{\mu\nu\rho\sigma}(Z_{[\mu\nu]}\,Z_{[\rho\sigma]}),$$

and, after some algebraic operations, we obtain:

$$\begin{aligned} L^{ST}_{gr} &= 4a_I a_K \eta\varepsilon^{\mu\nu\rho\sigma}\Big((\partial_\mu G^I_\nu)(\partial_\rho G^K_\sigma)\Big) \\ &- g^2 a_{(IJ)} a_{(KL)} \eta\varepsilon^{\mu\nu\rho\sigma}\Big([G^I_\mu, G^J_\nu][G^K_\rho, G^L_\sigma]\Big) \\ &- 4g a_I a_{(KL)} \eta\varepsilon^{\mu\nu\rho\sigma}\Big((\partial_\mu G^I_\nu)[G^K_\rho, G^L_\sigma]\Big), \end{aligned}$$

which yields,

$$\begin{aligned} L^{ST}_{gr} &= 4N a_I a_K \eta\varepsilon^{\mu\nu\rho\sigma}(\partial_\mu G^I_{\nu a})(\partial_\rho G^K_{\sigma a}) \\ &- 4iNg a_I a_{(KL)} f_{abc} \eta\varepsilon^{\mu\nu\rho\sigma}(\partial_\mu G^I_{\nu a}) G^K_{\rho b} G^L_{\sigma c} \\ &+ 4Ng^2 a_{(IJ)} a_{(KL)} f_{abm} f_{cdm} \eta\varepsilon^{\mu\nu\rho\sigma} G^I_{\mu a}\, G^J_{\nu b}\, G^K_{\rho c}\, G^L_{\sigma d}. \end{aligned} \tag{5.7}$$

### 2. Collective semitopological sector

Similarly to the previous case, we obtain

$$L^{ST}_{co} = \eta\varepsilon^{\mu\nu\rho\sigma}(z_{[\mu\nu]}\,z_{[\rho\sigma]}).$$

After some algebraic manipulations, we obtain:

$$\begin{aligned} L_{co}^{ST} &= a_{(IJ)}a_{(KL)}\eta\varepsilon^{\mu\nu\rho\sigma}\Big([G_{\mu}^{I}, G_{\nu}^{J}][G_{\rho}^{K}, G_{\sigma}^{L}]\Big) \\ &+ b_{[IJ]}b_{[KL]}\eta\varepsilon^{\mu\nu\rho\sigma}\Big(\{G_{\mu}^{I}, G_{\nu}^{J}\}\{G_{\rho}^{K}, G_{\sigma}^{L}\}\Big) \\ &+ \gamma_{[IJ]}\gamma_{[KL]}\eta\varepsilon^{\mu\nu\rho\sigma}\Big(G_{\mu}^{I}\, G_{\nu}^{J}\, G_{\rho}^{K}\, G_{\sigma}^{L}\Big) \\ &+ 2a_{(IJ)}b_{[KL]}\eta\varepsilon^{\mu\nu\rho\sigma}\Big([G_{\mu}^{I}, G_{\nu}^{J}]\{G_{\rho}^{K}, G_{\sigma}^{L}\}\Big) \\ &+ 2a_{(IJ)}\gamma_{[KL]}\eta\varepsilon^{\mu\nu\rho\sigma}\Big([G_{\mu}^{I}, G_{\nu}^{J}]G_{\rho}^{K}\, G_{\sigma}^{L}\Big) \\ &+ 2b_{(IJ)}\gamma_{[KL]}\eta\varepsilon^{\mu\nu\rho\sigma}\Big(\{G_{\mu}^{I}, G_{\nu}^{J}\}G_{\rho}^{K}\, G_{\sigma}^{L}\Big), \end{aligned}$$

which yields,

$$\begin{aligned} L_{co}^{ST} = \Big\{ &- Na_{(IJ)}a_{(KL)}f_{abm}f_{cdm} + Nb_{[IJ]}b_{[KL]}d_{abm}d_{cdm} \\ &+ \frac{N}{4}\gamma_{[IJ]}\gamma_{[KL]}(d_{abm}d_{cdm} - d_{acm}d_{bdm} + d_{adm}d_{bcm} \\ &+ 2iNa_{(IJ)}b_{[KL]}f_{abm}d_{cdm} - Na_{(IJ)}\gamma_{[KL]}f_{abm}f_{cdm} \\ &+ iNb_{[IJ]}\gamma_{[KL]}d_{abm}f_{cdm}\Big\}\eta\varepsilon^{\mu\nu\rho\sigma}G_{\mu a}^{I}\, G_{\nu b}^{J}\, G_{\rho c}^{K}\, G_{\sigma d}^{L} \\ &+ \Big\{\frac{1}{N}b_{[IJ]}b_{[KL]} + \gamma_{[IJ]}\gamma_{[KL]} - \gamma_{[IL]}\gamma_{[KJ]} \\ &+ 2b_{[IJ]}\gamma_{[KL]}\Big\}\eta\varepsilon^{\mu\nu\rho\sigma}G_{\mu a}^{I}\, G_{\nu a}^{J}\, G_{\rho b}^{K}\, G_{\sigma b}^{L}. \end{aligned} \tag{5.8}$$

### 3. Mixed semitopological sector

Similarly to the two previous cases, we obtain

$$L_{mix}^{ST} = 2\eta\varepsilon^{\mu\nu\rho\sigma}(z_{[\mu\nu]}\, Z_{[\rho\sigma]}).$$

Using the known expressions for $z^{\mu\nu}$ and $Z^{\mu\nu}$ in the above equation, and after some algebraic operations, we have:

$$\begin{aligned} L_{mix}^{ST} &= 4a_{(IJ)}a_{K}\eta\varepsilon^{\mu\nu\rho\sigma}\Big([G_{\mu}^{I}, G_{\nu}^{J}](\partial_{\rho}G_{\sigma}^{K})\Big) \\ &- 2iga_{(IJ)}\eta\varepsilon^{\mu\nu\rho\sigma}\Big([G_{\mu}^{I}, G_{\nu}^{J}][G_{\rho}^{K}, G_{\sigma}^{L}]\Big) \\ &+ 4b_{[IJ]}a_{K}\varepsilon^{\mu\nu\rho\sigma}\Big(\{G_{\mu}^{I}, G_{\nu}^{J}\}(\partial_{\rho}G_{\sigma}^{K})\Big) \\ &- 2igb_{[IJ]}a_{(KL)}\eta\varepsilon^{\mu\nu\rho\sigma}\Big(\{G_{\mu}^{I}, G_{\nu}^{J}\}[G_{\rho}^{K}, G_{\sigma}^{L}]\Big) \\ &+ 4\gamma_{[IJ]}a_{K}\eta\varepsilon^{\mu\nu\rho\sigma}\Big(G_{\mu}^{I}\, G_{\nu}^{J}(\partial_{\rho}G_{\sigma}^{K})\Big) \\ &- 2ig\gamma_{[IJ]}a_{(KL)}\eta\varepsilon^{\mu\nu\rho\sigma}\Big(G_{\mu}^{I}\, G_{\nu}^{J}[G_{\rho}^{K}, G_{\sigma}^{L}]\Big), \end{aligned}$$

which yields,

$$\begin{aligned} L_{mix}^{ST} = \Big\{&4iNa_{I}a_{(JK)}f_{bca} + 4Na_{I}b_{[JK]}d_{bca} \\ &+ 2iNa_{I}\gamma_{[JK]}f_{bca}\Big\}\eta\varepsilon^{\mu\nu\rho\sigma}(\partial_{\mu}G_{\nu a}^{I})G_{\rho b}^{J}\, G_{\sigma c}^{K} \\ &+ \Big\{2iNga_{(IJ)}a_{(KL)}f_{abm}f_{cdm} + 2Ngb_{[IJ]}a_{(KL)}d_{abm}f_{cdm} \\ &+ iN\gamma_{[IJ]}a_{(KL)}f_{abm}f_{cdm}\Big\}\eta\varepsilon^{\mu\nu\rho\sigma}G_{\mu a}^{I}\, G_{\nu b}^{J}\, G_{\rho c}^{K}\, G_{\sigma d}^{L}. \end{aligned} \tag{5.9}$$

## 6. Euler-Lagrange on potential fields

Now we consider the Euler-Lagrange equations corresponding to the eq. (5.1) entire Lagrangian. The corresponding equations of motion are of the form:

$$\partial_\alpha \frac{\delta L}{\delta \partial_\alpha G^M_{\beta f}} = \frac{\delta L}{\delta G^M_{\beta f}}. \tag{6.1}$$

Then, just for clarity we are going to show the corresponding result in respective pieces. It gives,

### 1. Equations of Motion for $L^A_K$

Applying equations (6.1) to the Lagrangian term given by eq. (3.14), we get the following equations of motion for $L^A_K$:

$$2l_{MI}\left[\partial_\alpha(\partial^\alpha G^{\beta I}_f) - \partial_\alpha(\partial^\beta G^{\alpha I}_f)\right] = 0. \tag{6.2}$$

where coefficient $l_{MI}$ is given by eq. (3.15).

### 2. Equations of Motion for $L^S_K$

Applying equations (6.1) to the Lagrangian term given by eq. (3.28), we get the following equations of motion for $L^S_K$:

$$2m_{MI}\left[\partial_\alpha(\partial^\alpha G^{\beta I}_f) + \partial_\alpha(\partial^\beta G^{\alpha I}_f)\right] + 2n_{MI}\partial_\alpha(g^{\alpha\beta}\,\partial_\mu G^{\mu I}_f) = 0. \tag{6.3}$$

where coefficients $m_{MI}$ and $n_{MI}$ are given by eqs. (3.29) and (3.30).

### 3. Equations of Motion for $L^A_3$

Applying equations (6.1) to the Lagrangian term given by eq. (3.16), we get the following equations of motion for $L^A_3$:

$$\theta_{MKL,fab}\partial_\alpha(G^{\alpha K}_a G^{\beta L}_b) = \theta_{KML,afb}(\partial^\beta G^K_{\mu a})G^{\mu L}_b + \theta_{KLM,abf}(\partial_\mu G^{\beta K}_a)G^{\mu L}_b. \tag{6.4}$$

where coefficient $\theta_{MKL,fab}$ is given by eq. (3.17).

### 4. Equations of Motion for $L^S_3$

Applying equations (6.1) to the Lagrangian term given by eq. (3.31), we get the following equations of motion for $L^S_3$:

$$\begin{aligned} &\omega_{MKL,fab}\partial_\alpha(G^{\alpha K}_a G^{\beta L}_b) + \sigma_{MKL,fab}\partial_\alpha(g^{\alpha\beta}G^K_{\nu a}G^{\nu L}_b) \\ &= \omega_{KML,afb}(\partial^\beta G^K_{\mu a})G^{\mu L}_b + \omega_{KLM,abf}(\partial_\mu G^{\beta K}_a)G^{\mu L}_b \\ &+ [\sigma_{KLM,abf} + \sigma_{KML,afb}](\partial_\mu G^{\mu K}_a)G^{\beta L}_b. \end{aligned} \tag{6.5}$$

where coefficients $\omega_{MKL,fab}$ and $\sigma_{MKL,fab}$ are given by eqs. (3.32) and (3.33).

## 5. Equations of Motion for $L_3^{ST}$

Considering the semi-topological sector treated previously, the trilinear part of this sector is given by:

$$\begin{aligned} L_3^{ST} &= b^{S_g}_{IJK,abc}\eta\varepsilon^{\mu\nu\rho\sigma}(\partial_\mu G^I_{\nu a})G^J_{\rho b}G^K_{\sigma c} \\ &+ a^{S_m}_{IJK,abc}\eta\varepsilon^{\mu\nu\rho\sigma}(\partial_\mu G^I_{\nu a})G^J_{\rho b}G^K_{\sigma c} \\ &= [b^{S_g}_{IJK,abc} + a^{S_m}_{IJK,abc}]\eta\varepsilon^{\mu\nu\rho\sigma}(\partial_\mu G^I_{\nu a})G^J_{\rho b}G^K_{\sigma c}. \end{aligned} \tag{6.6}$$

The corresponding equations of motion for $L_3^{ST}$ are:

$$\begin{aligned} &[b^{S_g}_{MKL,fab} + a^{S_m}_{MKL,fab}]\eta\varepsilon^{\alpha\beta\rho\sigma}\partial_\alpha(G^K_{\rho a}G^L_{\sigma b}) = \\ &= [b^{S_g}_{KLM,abf} + a^{S_m}_{KLM,abf} - b^{S_g}_{KML,afb} - b^{S_m}_{KML,afb}]\eta\varepsilon^{\mu\nu\rho\beta}(\partial_\mu G^K_{\nu a})G^L_{\rho b} \end{aligned} \tag{6.7}$$

.

## 6. Equations of Motion for $L_4^A$

Applying equations (6.1) to the Lagrangian term given by eq. (3.18), we get the following equations of motion for $L_4^A$

$$\begin{aligned} 0 &= [\theta_{MJKL,fabc} + \theta_{JMLK,afcb} + \theta_{KLMJ,bcfa} + \theta_{LKJM,cbaf}]G^J_{\mu a}G^{\beta K}_b G^{\mu L}_c \\ &+ [\omega_{MJKL} + \omega_{JMLK} + \omega_{KLMJ} + \omega_{LKJM}]G^J_{\mu f}G^{\beta K}_a G^{\mu L}_a. \end{aligned} \tag{6.8}$$

where the coefficients $\theta_{MJKL,fabc}$ and $\omega_{MJKL}$ are given by eqs. (3.19) and (3.20).

## 7. Equations of Motion for $L_4^S$

Applying equations (6.1) to the Lagrangian term given by eq. (3.34), we get the following equations of motion for $L_4^S$

$$\begin{aligned} 0 &= [\rho_{MJKL,fabc} + \rho_{JMLK,afcb} + \rho_{KLMJ,bcfa} + \rho_{LKJM,cbaf}]G^J_{\mu a}G^{\beta K}_b G^{\mu L}_c \\ &+ [\chi_{MJKL} + \chi_{JMLK} + \chi_{KLMJ} + \chi_{LKJM}]G^J_{\mu f}G^{\beta K}_a G^{\mu L}_a \\ &+ [\sigma_{MJKL} + \sigma_{JMKL} + \sigma_{KLMJ} + \sigma_{LKJM}]G^{\beta J}_f G^K_{\mu a}G^{\mu L}_a. \end{aligned} \tag{6.9}$$

where the coefficients $\rho_{MJKL,fabc}$, $\chi_{MJKL}$ and $\sigma_{MJKL}$ are given by eqs. (3.35), (3.35) and (3.37).

## 8. Equations of Motion for $L_4^{ST}$

Considering the semi-topological sector treated previously, the quadrilinear part of this sector is given by:

$$\begin{aligned} L_4^{ST} &= c^{S_g}_{IJKL,abcd}\eta\varepsilon^{\mu\nu\rho\sigma}G^I_{\mu a}G^J_{\nu b}G^K_{\rho c}G^L_{\sigma d} \\ &+ a^{S_c}_{IJKL,abcd}\eta\varepsilon^{\mu\nu\rho\sigma}G^I_{\mu a}G^J_{\nu b}G^K_{\rho c}G^L_{\sigma d} \\ &+ b^{S_c}_{IJKL}\eta\varepsilon^{\mu\nu\rho\sigma}G^I_{\mu a}G^J_{\nu a}G^K_{\rho b}G^L_{\sigma b} \\ &+ b^{S_m}_{IJKL,abcd}\eta\varepsilon^{\mu\nu\rho\sigma}G^I_{\mu a}G^J_{\nu b}G^K_{\rho c}G^L_{\sigma d} \\ &= [c^{S_g}_{IJKL,abcd} + a^{S_c}_{IJKL,abcd} + b^{S_m}_{IJKL,abcd}]\eta\varepsilon^{\mu\nu\rho\sigma}G^I_{\mu a}G^J_{\nu b}G^K_{\rho c}G^L_{\sigma d} \\ &+ b^{S_c}_{IJKL}\eta\varepsilon^{\mu\nu\rho\sigma}G^I_{\mu a}G^J_{\nu a}G^K_{\rho b}G^L_{\sigma b}. \end{aligned} \tag{6.10}$$

The corresponding equations of motion for $L_4^{ST}$ are:

$$\begin{aligned}
0 = &[c^{S_g}_{MJKL,fabc} - c^{S_g}_{JMKL,afbc} + c^{S_g}_{JKML,abfc} - c^{S_g}_{JKLM,abcf} \\
&+ a^{S_c}_{MJKL,fabc} - a^{S_c}_{JMKL,afbc} + a^{S_c}_{JKML,abfc} - a^{S_c}_{JKLM,abcf} \\
&+ b^{S_m}_{MJKL,fabc} - b^{S_m}_{JMKL,afbc} + b^{S_m}_{JKML,abfc} - b^{S_m}_{JKLM,abcf}]\eta\varepsilon^{\beta\mu\nu\rho} G^J_{\mu a} G^K_{\nu b} G^L_{\rho c} \\
&+ [b^{S_c}_{MJKL} + b^{S_c}_{JMLK} + b^{S_c}_{KLMJ} + b^{S_c}_{LKJM}]\eta\varepsilon^{\beta\mu\nu\rho} G^J_{\mu f} G^K_{\nu a} G^L_{\rho a}.
\end{aligned} \tag{6.11}$$

## 9. Complete Equations of Motion

In total, it gives:

$$\begin{aligned}
&2l_{MI}\partial_\alpha(\partial^\alpha G^{\beta I}_f) - 2l_{MI}\partial_\alpha(\partial^\beta G^{\alpha I}_f) \\
&+ 2m_{MI}\partial_\alpha(\partial^\alpha G^{\beta I}_f) + 2m_{MI}\partial_\alpha(\partial^\beta G^{\alpha I}_f) + 2n_{MI}\partial_\alpha(g^{\alpha\beta}\,\partial_\mu G^{\mu I}_f) \\
&+ \theta_{MKL,fab}\partial_\alpha(G^{\alpha K}_a G^{\beta L}_b) + \omega_{MKL,fab}\partial_\alpha(G^{\alpha K}_a G^{\beta L}_b) + \sigma_{MKL,fab}\partial_\alpha(g^{\alpha\beta} G^K_{\nu a} G^{\nu L}_b) \\
&+ [b^{S_g}_{MKL,fab} + a^{S_m}_{MKL,fab}]\eta\varepsilon^{\alpha\beta\rho\sigma}\partial_\alpha(G^K_{\rho a} G^L_{\sigma b}) \\
&= \theta_{KML,afb}(\partial^\beta G^K_{\mu a}) G^{\mu L}_b + \theta_{KLM,abf}(\partial_\mu G^{\beta K}_a) G^{\mu L}_b + \omega_{KML,afb}(\partial^\beta G^K_{\mu a}) G^{\mu L}_b \\
&+ \omega_{KLM,abf}(\partial_\mu G^{\beta K}_a) G^{\mu L}_b + [\sigma_{KLM,abf} + \sigma_{KML,afb}](\partial_\mu G^{\mu K}_a) G^{\beta L}_b \\
&+ [b^{S_g}_{KLM,abf} + a^{S_m}_{KLM,abf} - b^{S_g}_{KML,afb} - b^{S_m}_{KML,afb}]\eta\varepsilon^{\mu\nu\rho\beta}(\partial_\mu G^K_{\nu a}) G^L_{\rho b} \\
&+ [\theta_{MJKL,fabc} + \theta_{JMLK,afcb} + \theta_{KLMJ,bcfa} + \theta_{LKJM,cbaf}] G^J_{\mu a} G^{\beta K}_b G^{\mu L}_c \\
&+ [\omega_{MJKL} + \omega_{JMLK} + \omega_{KLMJ} + \omega_{LKJM}] G^J_{\mu f} G^{\beta K}_a G^{\mu L}_a \\
&+ [\rho_{MJKL,fabc} + \rho_{JMLK,afcb} + \rho_{KLMJ,bcfa} + \rho_{LKJM,cbaf}] G^J_{\mu a} G^{\beta K}_b G^{\mu L}_c \\
&+ [\chi_{MJKL} + \chi_{JMLK} + \chi_{KLMJ} + \chi_{LKJM}] G^J_{\mu f} G^{\beta K}_a G^{\mu L}_a \\
&+ [\sigma_{MJKL} + \sigma_{JMKL} + \sigma_{KLMJ} + \sigma_{LKJM}] G^{\beta J}_f G^K_{\mu a} G^{\mu L}_a \\
&+ [c^{S_g}_{MJKL,fabc} - c^{S_g}_{JMKL,afbc} + c^{S_g}_{JKML,abfc} - c^{S_g}_{JKLM,abcf} \\
&+ a^{S_c}_{MJKL,fabc} - a^{S_c}_{JMKL,afbc} + a^{S_c}_{JKML,abfc} - a^{S_c}_{JKLM,abcf} \\
&+ b^{S_m}_{MJKL,fabc} - b^{S_m}_{JMKL,afbc} + b^{S_m}_{JKML,abfc} - b^{S_m}_{JKLM,abcf}]\eta\varepsilon^{\beta\mu\nu\rho} G^J_{\mu a} G^K_{\nu b} G^L_{\rho c} \\
&+ [b^{S_c}_{MJKL} + b^{S_c}_{JMLK} + b^{S_c}_{KLMJ} + b^{S_c}_{LKJM}]\eta\varepsilon^{\beta\mu\nu\rho} G^J_{\mu f} G^K_{\nu a} G^L_{\rho a}.
\end{aligned} \tag{6.12}$$

Rearranging the above expression, we have:

$$\begin{aligned}
&\alpha_{MI}\partial_\alpha(\partial^\alpha G^{\beta I}_f) + \beta_{MI}\partial_\alpha(\partial^\beta G^{\alpha I}_f) + 2n_{MI}\partial_\alpha(g^{\alpha\beta}\,\partial_\mu G^{\mu I}_f) \\
&+ \gamma_{MKL,fab}\partial_\alpha(G^{\alpha K}_a G^{\beta L}_b) + \sigma_{MKL,fab}\partial_\alpha(g^{\alpha\beta} G^K_{\nu a} G^{\nu L}_b) \\
&+ \kappa_{MKL,fab}\varepsilon^{\alpha\beta\rho\sigma}\partial_\alpha(G^K_{\rho a} G^L_{\sigma b}) \\
&= A_{KML,afb}(\partial^\beta G^K_{\mu a}) G^{\mu L}_b + B_{KLM,abf}(\partial_\mu G^{\beta K}_a) G^{\mu L}_b \\
&+ C_{KLM,abf}(\partial_\mu G^{\mu K}_a) G^{\beta L}_b + D_{KLM,abf}\varepsilon^{\mu\nu\rho\beta}(\partial_\mu G^K_{\nu a}) G^L_{\rho b} \\
&+ E_{MJKL,fabc} G^J_{\mu a} G^{\beta K}_b G^{\mu L}_c + F_{MJKL} G^J_{\mu f} G^{\beta K}_a G^{\mu L}_a \\
&+ G_{MJKL} G^{\beta J}_f G^K_{\mu a} G^{\mu L}_a + H_{MJKL,fabc}\varepsilon^{\beta\mu\nu\rho} G^J_{\mu a} G^K_{\nu b} G^L_{\rho c} \\
&+ I_{MJKL}\varepsilon^{\beta\mu\nu\rho} G^J_{\mu f} G^K_{\nu a} G^L_{\rho a},
\end{aligned} \tag{6.13}$$

where

$$\begin{aligned}
\alpha_{MI} &= 2l_{MI} + 2m_{MI} \\
\beta_{MI} &= -2l_{MI} + 2m_{MI} \\
\gamma_{MKL,fab} &= \theta_{MKL,fab} + \omega_{MKL,fab} \\
\kappa_{MKL,fab} &= [b^{S_g}_{MKL,fab} + a^{S_m}_{MKL,fab}]\eta
\end{aligned}$$

and

$$\begin{aligned}
A_{KML,afb} &= \theta_{KML,afb} + \omega_{KML,afb} \\
B_{KLM,abf} &= \theta_{KLM,abf} + \omega_{KLM,abf} \\
C_{KLM,abf} &= \sigma_{KLM,abf} + \sigma_{KML,afb} \\
D_{KLM,abf} &= [b^{S_g}_{KLM,abf} + a^{S_m}_{KLM,abf} - b^{S_g}_{KML,afb} - b^{S_m}_{KML,afb}]\eta \\
E_{MJKL,fabc} &= \theta_{MJKL,fabc} + \theta_{JMLK,afcb} + \theta_{KLMJ,bcfa} + \theta_{LKJM,cbaf} \\
&\quad + \rho_{MJKL,fabc} + \rho_{JMLK,afcb} + \rho_{KLMJ,bcfa} + \rho_{LKJM,cbaf} \\
F_{MJKL} &= \omega_{MJKL} + \omega_{JMLK} + \omega_{KLMJ} + \omega_{LKJM} \\
&\quad + \chi_{MJKL} + \chi_{JMLK} + \chi_{KLMJ} + \chi_{LKJM} \\
G_{MJKL} &= \sigma_{MJKL} + \sigma_{JMKL} + \sigma_{KLMJ} + \sigma_{LKJM} \\
H_{MJKL,fabc} &= [c^{S_g}_{MJKL,fabc} - c^{S_g}_{JMKL,afbc} + c^{S_g}_{JKML,abfc} - c^{S_g}_{JKLM,abcf} \\
&\quad + a^{S_c}_{MJKL,fabc} - a^{S_c}_{JMKL,afbc} + a^{S_c}_{JKML,abfc} - a^{S_c}_{JKLM,abcf} \\
&\quad + b^{S_m}_{MJKL,fabc} - b^{S_m}_{JMKL,afbc} + b^{S_m}_{JKML,abfc} - b^{S_m}_{JKLM,abcf}]\eta \\
I_{MJKL} &= [b^{S_c}_{MJKL} + b^{S_c}_{JMLK} + b^{S_c}_{KLMJ} + b^{S_c}_{LKJM}]\eta.
\end{aligned}$$

Thus the constructivist non-abelian model provides a nonlinear dynamics. Each field $A^I_{\mu a}$ at eq. (6.13) contains a corresponding set of distinct quantum numbers. A diversity is obtained. It can be separated in spin-1 and spin-0 sectors. For instance, every field carries its own mass.

## 7. Bianchi and Noether identities

The granular Bianchi identities are

$$D_\rho G_{[\mu\nu]I} + D_\nu G_{[\rho\mu]I} + D_\mu G_{[\nu\rho]I} = 0. \tag{7.1}$$

where $D_\mu = \partial_\mu i g_I G_{\mu I}$

The antisymmetric collectives ones are

$$\partial_\mu z_{[\nu\rho]} + \partial_\nu z_{[\rho\mu]} + \partial_\rho z_{[\mu\nu]} = \gamma_{[IJ]} G^I_\nu G^J_{[\mu\rho]} + \gamma_{[IJ]} G^I_\rho G^J_{[\nu\mu]} + \gamma_{[IJ]} G^I_\mu G^J_{[\rho\nu]}. \tag{7.2}$$

The collective symmetric Bianchi identity is

$$\partial_\mu z_{(\nu\rho)} + \partial_\nu z_{(\rho\mu)} + \partial_\rho z_{(\mu\nu)} = \gamma_{(IJ)} G^I_\mu G^J_{(\nu\rho)} + \gamma_{(IJ)} G^I_\nu G^J_{(\rho\mu)} + \gamma_{(IJ)} G^I_\rho G^J_{(\mu\nu)} \tag{7.3}$$

and

$$\partial_\mu z_{(\nu}{}^{\nu)} + 2\partial_\nu z_{(\mu}{}^{\nu)} = \gamma_{(IJ)} G^I_\mu G^{\nu J}_\nu + 2\gamma_{(IJ)} G^I_\nu G^{\nu J}_\mu. \tag{7.4}$$

The local Noether equations associated to the SU($N$) symmetry are

$$\partial_\mu J_N^{\mu a}(G) = 0, \tag{7.5}$$

$$\frac{i}{g_I}\partial_\mu \frac{\delta \mathscr{L}}{\delta(\partial_\mu G^a_{\nu I})} = J_N^{\mu a}, \tag{7.6}$$

$$\frac{i}{g_I}\frac{\delta \mathscr{L}}{\delta(\partial_\mu G^a_{\nu I})}\partial^\mu \partial^\nu \omega^a = 0 \tag{7.7}$$

where

$$J_N^{\mu a} = \Big[\frac{\delta \mathscr{L}}{\delta(\partial_\mu G^a_{\nu I})}, G^I_\nu\Big]^a. \tag{7.8}$$

Eq. (7.6) is understood as the symmetry equation involving a set of fields.

The above classical equations are showing that from SU($N$) symmetry it is possible to generate a physics with interlaced Yang-Mills families. Something that move us to understand that QCD is not the only one theory related to SU$(3)_c$.

## 8. Conclusion

A fields set physics is introduced and a symmetry of difference is obtained. A non abelian whole quantum is developed. Yang-Mills symmetry is preserved and new associative properties are obtained. A non abelian constructivist lagrangian includes different families transforming under a common symmetry group $SU(N)$. Each field transforms like $A_{\mu I}{}' = UA_{\mu I}U^{-1} + \frac{i}{g_I}\partial_\mu U.U^{-1}$ where $U = e^{i\omega_a t_a}$[27-30]. Antireductionist properties are obtained from Yang-Mills symmetry. They are a physics containing set, quanta diversity, interdependence, nonlinearity, chance.

The objective of this work was to study the non abelian constructivist lagrangian. Consider the whole classical equations. The set action minimal principle, $\delta S = 0$, and corresponding $N$-equations of motion. Three Noether equations derived from gauge symmetry plus $N + 3$ Bianchi identities. They will define a whole structure where a new Yang-Mills whole quantum is defined.

Yang-Mills theory is extended. Preserving its own gauge symmetry a field $A^a_\mu$ is inserted in a fields set $\{A^a_{\mu I}\} = \{A^a_\mu, B^a_\mu, \ldots N^a_\mu\}$. A grouping physics appears. A physics studied by an antireductionist Yang-Mills symmetry gauge theory takes form. A set action is derived beyond the usual Yang-Mills. The part is inserted in the whole and new properties are derived.

Massive gluons are introduced under the usual non abelian gauge symmetry. The enlarged lagrangian is

$$\begin{aligned} L = &\Big\{(Z_{\mu\nu} + z_{\mu\nu})(Z^{\mu\nu} + z^{\mu\nu})\Big\} + \eta\Big\{(Z_{\mu\nu} + z_{\mu\nu})(\widetilde{Z}^{\mu\nu} + \widetilde{z}^{\mu\nu})\Big\} \\ &+ m_{II}G_{\mu I}G^{\mu I} + \overline{\Psi}D_\mu\Psi \end{aligned} \tag{8.1}$$

where the covariant derivative preserves the usual Yang-Mills case and includes a massive gluon

$$D_\mu = \partial_\mu + igA_{\mu 1} + ig' A_{\mu 2} \tag{8.2}$$

where $A^a_{\mu 1}$ and $A^a_{\mu 2}$ are a massless and a massive gluon respectively.

Massive gluons may be helpful to solve QCD infrared problems at high energy [31-32]. Although asymptotic freedom is an achievement for quarks behaviour at deep inelastic scattering [33-34], QCD contains an incompleteness due to infrared problems at quarks and gluons scattering at high energies. Massive gluons under asymptotic freedom expected.

## A. Constructor basis $\{D, X_i\}$

For a more immediate comprehension about the properties that such extended gauge model carries, it should be written in terms of a composition where just one field transforms inhomogeneously. This is because it works as the boundary conditions for the usual models. Thus the field $D_\mu$ works as the usual gauge field and the fields $X^i_\mu$ as a kind of vector-matter fields transforming in the adjoint representation. It gives

$$D_\mu \to D_\mu{}' = UD_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1},$$
$$X^i_\mu \to X^i_\mu{}' = UX^i_\mu U^{-1}.$$

Geometrically the potential fields $X^i_\mu$ from the torsion tensor of the higher- dimensional manifold that spontaneously compactify to $M^4 \times B^4$, where $M^4$ is the Minkowski space-time and $B^k$ some $k$-dimensional internal space. Thus, the origin of the potential fields can be treated back to the *vielbein*, spin-connection and potential fields of higher-dimensional gravity-matter coupled theory spontaneously compactified for an internal space with torsion [ ].

## A. Granular and collective field strength

We consider here Lagrangian terms constructed from a family of fields.

### A.1.1 $D_{\mu\nu}$

Let us consider, in the construction base:

$$D_{\mu\nu} = \partial_\mu D_\nu - \partial_\nu D_\mu - ig[D_\mu, D_\nu]. \tag{A.1}$$

Then, remembering from (**??**) that $D_\mu = \Omega_{0,I}\, G^I_\mu$, we have:

$$D_{\mu\nu} = \partial_\mu(\Omega_{0,I} G^I_\nu) - \partial_\nu(\Omega_{0,I} G^I_\mu) - ig[\Omega_{0,I} G^I_\mu, \Omega_{0,J} G^J_\nu].$$

Then,

$$D_{\mu\nu} = \Omega_{0,I}(\partial_\mu G^I_\nu - \partial_\nu G^I_\mu - ig\Omega_{0,J}[G^I_\mu, G^J_\nu]), \tag{A.2}$$

where $I, J \in \{0, 1, \ldots, N-1\}$. It is verified that $D_{\mu\nu} = -D_{\nu\mu}$.

### A.1.2 $X^i_{[\mu\nu]}$

Now let us consider:

$$X^i_{[\mu\nu]} = \partial_\mu X^i_\nu - \partial_\nu X^i_\mu - ig([D_\mu, X^i_\nu] - [D_\nu, X^i_\mu]. \tag{A.3}$$

Remembering that $X^i_\mu = \Omega_{i,I} G^I_\mu$, we have:

$$\begin{aligned} X^i_{[\mu\nu]} &= \partial_\mu(\Omega_{i,I} G^I_\nu) - \partial_\nu(\Omega_{i,I} G^I_\mu) - ig([\Omega_{0,J} G^J_\mu, \Omega_{i,I} G^I_\nu] - [\Omega_{0,J} G^J_\nu, \Omega_{i,I} G^I_\mu]) \\ &= \Omega_{i,I}(\partial_\mu G^I_\nu - \partial_\nu G^I_\mu) - ig(\Omega_{i,I}\Omega_{0,J}[G^J_\mu, G^I_\nu] - \Omega_{i,I}\Omega_{0,J}[G^J_\nu, G^I_\mu]). \end{aligned}$$

Then, simplifying:

$$X^i_{[\mu\nu]} = \Omega_{i,I}(\partial_\mu G^I_\nu - \partial_\nu G^I_\mu) - ig(\Omega_{0,I}\Omega_{i,J} + \Omega_{0,J}\Omega_{i,I})[G^I_\mu, G^J_\nu], \tag{A.4}$$

where $I, J \in \{0, 1, \ldots, N-1\}$, $i \in \{1, \ldots, N-1\}$. Note that $X^i_{[\mu\nu]} = -X^i_{[\nu\mu]}$.

### A.1.3 Antisymmetric granular sector $Z_{[\mu\nu]}$

The antisymmetric granular tensor is defined in terms of the tensors $D_{\mu\nu}$ and $X^i_{[\mu\nu]}$ according to:

$$Z_{[\mu\nu]} = dD_{\mu\nu} + \alpha_i X^i_{[\mu\nu]} \tag{A.5}$$

where $d$ and $\alpha_i$ are constants.

Then, using the equations (A.2) and (A.4) we have:

$$\begin{aligned}
Z_{[\mu\nu]} &= d\Omega_{0,I}(\partial_\mu G^I_\nu - \partial_\nu G^I_\mu) - igd\Omega_{0,I}\Omega_{0,J}[G^I_\mu, G^J_\nu] \\
&\quad + \alpha_i\Omega_{i,I}(\partial_\mu G^I_\nu - \partial_\nu G^I_\mu) - ig\alpha_i(\Omega_{0,I}\Omega_{i,J} + \Omega_{0,J}\Omega_{i,I})[G^I_\mu, G^J_\nu] \\
&= (d\Omega_{0,I} + \alpha_i\Omega_{i,I})(\partial_\mu G^I_\nu - \partial_\nu G^I_\mu) \\
&\quad - ig(d\Omega_{0,I}\Omega_{0,J} + \alpha_i\Omega_{0,I}\Omega_{i,J} + \alpha_i\Omega_{0,J}\Omega_{i,I})[G^I_\mu, G^J_\nu].
\end{aligned}$$

Then,

$$Z_{[\mu\nu]} = a_I(\partial_\mu G^I_\nu - \partial_\nu G^I_\mu) - ige_{(IJ)}[G^I_\mu, G^J_\nu] \tag{A.6}$$

where:

$$\begin{aligned}
a_I &\equiv d\Omega_{0,I} + \alpha_i\Omega_{i,I} \\
e_{(IJ)} &\equiv d\Omega_{0,I}\Omega_{0,J} + \alpha_i\Omega_{0,I}\Omega_{i,J} + \alpha_i\Omega_{0,J}\Omega_{i,I},
\end{aligned}$$

and where $I, J \in \{0, 1, \ldots, N-1\}$, $i \in \{1, \ldots, N-1\}$.

### A.1.4 Antisymmetric collective sector $z_{[\mu\nu]}$

The antisymmetric collective tensor is defined in terms of the tensor $X^i_{\mu\nu}$ according to:

$$z_{[\mu\nu]} = a_{(ij)}[X^i_\mu, X^j_\nu] + b_{[ij]}\{X^i_\mu, X^j_\nu\} + \gamma_{[ij]}\, X^i_\mu X^j_\nu. \tag{A.7}$$

Using the relation (**??**) corresponding to the field $X^i_\mu$ in the above equation, we have:

$$\begin{aligned}
z_{[\mu\nu]} &= a_{(ij)}[\Omega_{i,I}G^I_\mu, \Omega_{j,J}G^J_\nu] + b_{[ij]}\{\Omega_{i,I}G^I_\mu, \Omega_{j,J}G^J_\nu\} + \gamma_{[ij]}\Omega_{i,I}G^I_\mu\Omega_{j,J}G^J_\nu \\
&= a_{(ij)}\Omega_{i,I}\Omega_{j,J}[G^I_\mu, G^J_\nu] + b_{[ij]}\{\Omega_{i,I}\Omega_{j,J}\{G^I_\mu, G^J_\nu\} + \gamma_{[ij]}\Omega_{i,I}\Omega_{j,J}G^I_\mu G^J_\nu.
\end{aligned}$$

Then

$$z_{[\mu\nu]} = a_{(IJ)}[G^I_\mu, G^J_\nu] + b_{[IJ]}\{G^I_\mu, G^J_\nu\} + \gamma_{[IJ]}G^I_\mu G^J_\nu \tag{A.8}$$

where:

$$\begin{aligned}
a_{(IJ)} &\equiv a_{(ij)}\Omega_{i,I}\Omega_{j,J} \\
b_{[IJ]} &\equiv b_{[ij]}\Omega_{i,I}\Omega_{j,J} \\
\gamma_{[IJ]} &\equiv \gamma_{[ij]}\Omega_{i,I}\Omega_{j,J},
\end{aligned}$$

and where $I, J \in \{0, 1, \ldots, N-1\}$, $i \in \{1, \ldots, N-1\}$.

#### A.1.5 Symmetric granular sector $Z_{(\mu\nu)}$

The symmetric granular tensor is defined in terms of the tensor $X^i_{\mu\nu}$ according to:

$$Z_{(\mu\nu)} = \beta_i X^i_{(\mu\nu)} + \delta_i g_{\mu\nu}\, X^{\alpha)i}_{(\alpha} \tag{A.9}$$

where

$$X^i_{(\mu\nu)} = \partial_\mu X^i_\nu + \partial_\nu X^i_\mu - ig([D_\mu, X^i_\nu] + [D_\nu, X^i_\mu]). \tag{A.10}$$

Now, replacing in the above definitions the expression (**??**) for the fields $X^i_\mu$, we have:

$$\begin{aligned} X^i_{(\mu\nu)} &= \partial_\mu(\Omega_{i,I} G^I_\nu) + \partial_\nu(\Omega_{i,I} G^I_\mu) - ig([\Omega_{0,I} G^I_\mu, \Omega_{i,J} G^J_\mu] + [\Omega_{0,I} G^I_\nu, \Omega_{i,J} G^J_\nu]) \\ &= \Omega_{i,I}(\partial_\mu G^I_\nu + \partial^I_\nu G^I_\mu) - \Omega_{i,I}\Omega_{0,J} ig([G^J_\mu, G^I_\nu] + [G^J_\nu, G^I_\mu]). \end{aligned}$$

Then

$$X^i_{(\mu\nu)} = \Omega_{i,I}\big\{\partial_\mu G_\nu + \partial_\nu G^I_\mu - ig\Omega_{0,J}([G^I_\mu, G^J_\nu] + [G^I_\nu, G^J_\mu])\big\}.$$

But, $[G^J_\mu, G^I_\nu] = -[G^I_\nu, G^J_\mu]$, $[G^J_\nu, G^I_\mu] = -[G^I_\mu, G^J_\nu]$, then:

$$X^i_{(\mu\nu)} = \Omega_{i,I} G^I_{(\mu\nu)} \tag{A.11}$$

where:

$$G^I_{(\mu\nu)} \equiv \partial_\mu G^I_\nu + \partial_\nu G^I_\mu + ig\Omega_{0,J}([G^I_\mu, G^J_\nu] + [G^I_\nu, G^J_\mu]). \tag{A.12}$$

Also, we have:

$$X^{\alpha)i}_{(\alpha} = \Omega_{i,I} G^{\alpha)I}_{(\alpha}. \tag{A.13}$$

Then

$$Z_{(\mu\nu)} = \beta_i \Omega_{i,I} G^I_{(\mu\nu)} + \delta_i \Omega_{i,I} g_{\mu\nu}\, G^{\alpha)I}_{(\alpha}, \tag{A.14}$$

which can be written as:

$$Z_{(\mu\nu)} = \beta_I G^I_{(\mu\nu)} + \delta_I g_{\mu\nu} G^{\alpha)I}_{(\alpha} \tag{A.15}$$

where:

$$\beta_I \equiv \beta_i \Omega_{i,I} \quad \text{and} \quad \delta_I \equiv \delta_i \Omega_{i,I},$$

and where $I \in \{0, 1, \ldots, N-1\}$, $i \in \{1, \ldots, N-1\}$.

#### A.1.6 Symmetric collective sector $z_{(\mu\nu)}$

The symmetric granular tensor is defined in terms of the tensor $X^i_{\mu\nu}$ according to:

$$z_{(\mu\nu)} = b_{[ij]}[X^i_\mu, X^j_\nu] + c_{(ij)}\{X^i_\mu, X^j_\nu\} + u_{[ij]} g_{\mu\nu}[X^i_\alpha, X^{\alpha j}] + v_{(ij)} g_{\mu\nu}\{X^i_\alpha, X^{\alpha j}\}. \tag{A.16}$$

Placing the $X^i_\mu$ fields in terms of the $G^I_\mu$ fields, in the above equation, we have:

$$\begin{aligned} z_{(\mu\nu)} = {} & b_{[ij]}[\Omega_{i,I} G^I_\mu, \Omega_{j,J} G^J_\nu] + c_{(ij)}\{\Omega_{i,I} G^I_\mu, \Omega_{j,J} G^J_\nu\} \\ & + u_{[ij]} g_{\mu\nu}[\Omega_{i,I} G^I_\alpha, \Omega_{j,J} G^{\alpha J}] + v_{(ij)} g_{\mu\nu}\{\Omega_{i,I} G^I_\alpha, \Omega_{j,J} G^{\alpha J}\} \end{aligned}$$

So, rearranging:

$$z_{(\mu\nu)} = b_{[IJ]}[G^I_\mu, G^J_\nu] + c_{(IJ)}\{G^I_\mu, G^J_\nu\} + u_{[IJ]}g_{\mu\nu}[G^I_\alpha, G^{\alpha J}] + v_{(IJ)}g_{\mu\nu}\{G^I_\alpha, G^{\alpha J}\} \tag{A.17}$$

where:

$$b_{[IJ]} \equiv b_{[ij]}\Omega_{i,I}\Omega_{j,J}, \quad c_{(IJ)} \equiv c_{(ij)}\Omega_{i,I}\Omega_{j,J},$$
$$u_{[IJ]} \equiv u_{[ij]}\Omega_{i,I}\Omega_{j,J}, \quad v_{(IJ)} \equiv v_{(ij)}\Omega_{i,I}\Omega_{j,J},$$

and where $I \in \{0, 1, \ldots, N-1\}$, $i \in \{1, \ldots, N-1\}$.

### A. General Lagrangian

The most general expression for the Lagrangian is

$$\begin{aligned}\mathscr{L} &= \mathrm{tr}(Z_{\mu\nu}\ Z^{\mu\nu}) + \mathrm{tr}(z_{\mu\nu}\ z^{\mu\nu}) + \mathrm{tr}(Z_{\mu\nu}\ z^{\mu\nu})\\ &+ \eta\mathrm{tr}(Z_{\mu\nu}\ \bar{Z}^{\mu\nu}) + \eta\mathrm{tr}(z_{\mu\nu}\ \bar{z}^{\mu\nu}) - \frac{1}{2}m^2_{ij}X^i_\mu X^{\mu j}\end{aligned}$$

where $Z_{\mu\nu}$ is the most general covariant field strength with an unique dependence on fields,

$$Z_{\mu\nu} \to Z_{\mu\nu}{}' = UZ_{\mu\nu}U^{-1}$$

which can be decomposed as

$$Z_{\mu\nu} = Z_{[\mu\nu]} + Z_{(\mu\nu)}.$$

The anti-symmetric field strength is

$$Z_{[\mu\nu]} = dD_{\mu\nu} + \alpha_i X^{\ i}_{[\mu\nu]}$$

with

$$D_{\mu\nu} = \partial_\mu D_\nu - \partial_\nu D_\mu + ig[D_\mu, D_\nu],$$
$$X^{\ i}_{[\mu\nu]} = \partial_\mu X^i_\nu - \partial_\nu X^i_\mu + ig([D_\mu, X^i_\nu] - [D_\nu, X^i_\mu])$$

and the symmetric is

$$Z_{(\mu\nu)} = \beta_i X^{\ i}_{(\mu\nu)} + \rho_i g_{\mu\nu} X_\alpha{}^{\alpha i}$$

where

$$X^{\ i}_{(\mu\nu)} = \partial_\mu X^i_\nu + \partial_\nu X^i_\mu + ig([D_\mu, X^i_\nu] + [D_\nu, X^i_\mu]).$$

Another type of field strength is $z_{\mu\nu}$. It is a collective field which does not depend on derivatives

$$z_{\mu\nu} = z_{[\mu\nu]} + z_{(\mu\nu)}$$

where

$$z_{[\mu\nu]} = a_{(ij)}[X^i_\mu, X^j_\nu] + b_{[ij]}\{X^i_\mu, X^j_\nu\} + \gamma_{[ij]}X^i_\mu X^j_\nu$$

and

$$\begin{aligned}z_{(\mu\nu)} &= a_{[ij]}[X^i_\mu, X^j_\nu] + u_{[ij]}g_{\mu\nu}[X^i_\alpha, X^{\alpha j}]\\ &+ b_{(ij)}\{X^i_\mu, X^j_\nu\} + v_{(ij)}g_{\mu\nu}\{X^i_\alpha, X^{\alpha j}\}.\end{aligned}$$

It is understood the notation $A_\mu = A^a_\mu t_a$, where $t_a$ are the matrices which satisfy the Lie algebra for SU(N). Observe that $Z_{\mu\nu}$ is not Lie algebra valued as it is $F_{\mu\nu}$ in the usual QCD. However in order to explore the abundance of gauge scalars that such extended model offers one should also consider the non- irreducible sector contribution.

Decomposing the fields strength in terms of group terms $t_a$, $t_a t_b$, $[t_a, t_b]$, $\{t_a, t_b\}$, one gets an expansion where each coefficient transforms covariantly. Working out the total antisymmetric field tensor, one gets

$$z_{[\mu\nu]} = A^a_{\mu\nu} t_a$$

$$z_{[\mu\nu]} = B_{\mu\nu} + A^a_{\mu\nu}{}' t_a + C^{ab}_{\mu\nu} t_a t_b$$

where

$$A^a_{\mu\nu} = dD^a_{\mu\nu} + \alpha_i X^{ia}_{[\mu\nu]}$$

$$A^a_{\mu\nu}{}' = C^{abc}_{(ij)} X^{ic}_\mu X^j_{\nu b}$$

$$B_{\mu\nu} = \frac{1}{N} b_{[ij]} X^i_{\mu a} X^{ja}_\nu$$

$$C^{ab}_{(ij)} = \gamma_{[ij]} X^{ia}_\mu X^{jb}_\nu$$

$$C^{abc}_{(ij)} = -i a_{(ij)} f^{abc} + b_{[ij]} d^{cba}$$

Similarly one expands the symmetric field strength

$$\begin{aligned} Z_{(\mu\nu)} + z_{(\mu\nu)} = {} & (\beta X^{ia}_{(\mu\nu)} + \rho_i g_{\mu\nu} X^{ia\alpha}_\alpha) t_a + \\ & + (a_{[ij]} [X^{ia}_\nu, X^{jb}_\nu] + u_{[ij]} g_{\mu\nu} [X^{i\alpha}_\alpha, X^{ib\alpha}] + \\ & + b_{(ij)} \{X^i_\mu, X^j_\nu\}) [t_a, t_b] + \\ & + v_{(ij)} g_{\mu\nu} \{X^i_\mu, X^j_\nu\} \{t_a, t_b\} + \gamma_{(ij)} X^i_\mu X^j_\nu \, t_a t_b. \end{aligned}$$

We should now split the Lagrangian in transverse and longitudinal parts. From group theory arguments one knows that a four-vector carries information about different spin states. Thus, although Lorentz covariance turns out to be organised explicitly, the potential field Lagrangian plays with different quanta. Nevertheless as gauge invariance acts differently one the vector and scalar sectors, one expects that it will work as a source for rendering explicit a different dynamics for each one of those parts.

Therefore the Lagrangian should be written as

$$\mathscr{L}(D, X_i) = \mathscr{L}_A + \mathscr{L}_S - \frac{1}{2} m^2_{ij} X^i_\mu X^{\mu j},$$

$$\begin{aligned} \mathscr{L} = {} & \lambda_1 Z_{[\mu\nu]} Z^{[\mu\nu]} + \lambda_2 z_{[\mu\nu]} z^{[\mu\nu]} + \lambda_3 Z_{[\mu\nu]} z^{[\mu\nu]} + \\ & + \xi_1 Z_{(\mu\nu)} Z^{(\mu\nu)} + \xi_2 z_{(\mu\nu)} z^{(\mu\nu)} + \xi_3 Z_{(\mu\nu)} z^{(\mu\nu)}. \end{aligned}$$

## A. Equations of motion

### A.3.1 Equations of motion for the $D_\mu$ field

$$\frac{\partial \mathscr{L}}{\partial D^a_\mu} - \partial_\nu \frac{\partial \mathscr{L}}{\partial(\partial_\nu D^a_\mu)} = 0$$

$$
\begin{aligned}
&4\lambda_1\Big(d\partial_\nu\, Z^{[\mu\nu]}t_a + ig(dD^b_\nu + \alpha_i X^{ib}_\nu)\, Z^{[\mu\nu]}[t_a,t_b]\Big)\\
&+ 4\xi_1 ig\Big(\beta_i X^{ib}_\nu Z^{(\mu\nu)} + \rho_i X^{\mu ib} Z^{\nu)}_{(\nu}\Big)[t_a,t_b]\\
&+ 2\lambda_3\Big(d\partial_\nu\, z^{[\mu\nu]}t_a + i\frac{g}{N}(dD^b_\nu + \alpha_i X^{ib}_\nu)\, z^{[\mu\nu]}[t_a,t_b]\Big)\\
&+ 2\xi_3 ig\Big(\beta_i X^{ib}_\nu\; z^{(\mu\nu)} + \rho_i X^{\mu ib}\; z^{\nu)}_{(\nu}\Big)[t_a,t_b] = 0.
\end{aligned}
\tag{A.18}
$$

Taking the trace in above equation, one gets

$$
\begin{aligned}
&2\lambda_1\Big(d\partial_\nu\, Z^{[\mu\nu]a} - gf_{abc}(dD^b_\nu + \alpha_i X^{ib}_\nu)\, Z^{[\mu\nu]c}\Big)\\
&- 2\xi_1 g f_{abc}\Big(\beta_i X^{ib}_\nu Z^{(\mu\nu)c} + \rho_i X^{\mu ib} Z^{\nu)c}_{(\nu}\Big)\\
&+ \lambda_3\Big(d\partial_\nu\; z^{[\mu\nu]a} - gf_{abc}\frac{g}{N}(dD^b_\nu + \alpha_i X^{ib}_\nu)\; z^{[\mu\nu]c}\Big)\\
&- \xi_3 g f_{abc}\Big(\beta_i X^{ib}_\nu\; z^{(\mu\nu)c} + \rho_i X^{\mu ib}\; z^{\nu)c}_{(\nu}\Big) = 0.
\end{aligned}
\tag{A.19}
$$

Multiplying the equation of motion by $t_k$ and taking again the corresponding trace, we have

$$
\begin{aligned}
&\lambda_1\begin{pmatrix} d(d_{aek} - if_{aek})\partial_\nu\, Z^{[\mu\nu]e} + \\ +g(dD^b_\nu + \alpha_i X^{ib}_\nu)(if_{abc}f_{cek} - f_{abc}d_{cek})Z^{[\mu\nu]e}\end{pmatrix} +\\
&\xi_1\begin{pmatrix} g\rho_i(if_{abc}f_{cek} - f_{abc}d_{cek})X^{\mu ib}Z^{\nu)e}_{(\nu} + \\ g\beta_i(if_{abc}f_{cek} - f_{abc}d_{cek})X^{ib}_\nu Z^{(\mu\nu)e}\end{pmatrix} +\\
&\frac{\lambda_3}{2}\begin{pmatrix} d(d_{aek} - if_{aek})\partial_\nu\, z^{[\mu\nu]a} + \\ +g(dD^b_\nu + \alpha_i X^{ib}_\nu)(if_{abc}f_{cek} - f_{abc}d_{cek})z^{[\mu\nu]e}\end{pmatrix} +\\
&\frac{\xi_3}{2}\begin{pmatrix} g\rho_i(if_{abc}f_{cek} - f_{abc}d_{cek})X^{\mu ib}z^{\nu)e}_{(\nu} + \\ +g\beta_i(if_{abc}f_{cek} - f_{abc}d_{cek})X^{ib}_\nu z^{(\mu\nu)e}\end{pmatrix} = 0.
\end{aligned}
\tag{A.20}
$$

### A.3.2 Equations of motion for the $X^i_\mu$ field

$$
\begin{aligned}
&4\lambda_1\alpha_i(\partial_\nu\, Z^{[\mu\nu]}t_a + igD^b_\nu Z^{[\mu\nu]}[t_a,t_b]) +\\
&- 4\xi_1\begin{pmatrix} \beta_i\partial_\nu\, Z^{(\mu\nu)}t_a + \rho_i\partial^\mu\, Z^{\nu)}_{(\nu}t_a + \\ +ig\Big(\beta_i D^b_\nu\, Z^{(\mu\nu)} + \rho_i D^{\mu b} Z^{\nu)}_{(\nu}\Big)[t_a,t_b]\end{pmatrix} +\\
&+ 2\lambda_2\Big(2a_{(ij)}\, X^{jb}_\nu z^{[\mu\nu]}[t_a,t_b] + (2b_{[ij]} + \gamma_{[ij]})\, X^{jb}_\nu z^{[\mu\nu]}\{t_a,t_b\}\Big) +\\
&+ 4\xi_2\begin{pmatrix} \Big(a_{[ij]}\, X^{jb}_\nu z^{(\mu\nu)} + u_{[ij]}\, X^{\mu jb} z^{\nu)}_{(\nu}\Big)[t_a,t_b] + \\ +(b_{(ij)}\, X^{jb}_\nu z^{(\mu\nu)} + v_{(ij)}\, X^{\mu jb} z^{\nu)}_{(\nu})\{t_a,t_b\}\end{pmatrix} +\\
&+ \lambda_3\begin{pmatrix} 2\alpha_i\Big(\partial_\nu\, z^{[\mu\nu]}t_a + igD^b_\nu\, z^{[\mu\nu]}[t_a,t_b]\Big) + \\ +2a_{(ij)}\, X^{jb}_\nu Z^{[\mu\nu]}[t_a,t_b] + \Big(2b_{[ij]} + \gamma_{[ij]}\Big)\, X^{jb}_\nu Z^{[\mu\nu]}\{t_a,t_b\}\end{pmatrix} +\\
&+ 2\xi_3\begin{pmatrix} -\beta_i\partial_\nu\, z^{(\mu\nu)}t_a - \rho_i\partial^\mu\, z^{\nu)}_{(\mu}t_a + \\ -ig\Big(\beta_i D^b_\nu\, z^{(\mu\nu)} + \rho_i D^{\mu b}\, z^{\nu)}_{(\nu}\Big)[t_a,t_b] + \\ +\Big(a_{[ij]}\, X^{jb}_\nu Z^{(\mu\nu)} + u_{(ij)}\, X^{\mu jb} Z^{\nu)}_{(\nu}\Big)[t_a,t_b] + \\ +\Big(b_{(ij)}\, X^{jb}_\nu Z^{(\mu\nu)} + v_{(ij)}\, X^{\mu jb} Z^{\nu)}_{(\nu}\Big)\{t_a,t_b\}\end{pmatrix} = 0.
\end{aligned}
\tag{A.21}
$$

Taking the trace in above equation, one gets

$$
\begin{aligned}
&2\lambda_1\alpha_i(\partial_\nu Z^{[\mu\nu]a} - g f_{abc} D^b_\nu Z^{[\mu\nu]c}) + \\
&+ 2\xi_1 \begin{pmatrix} -\beta_i\partial_\nu Z^{(\mu\nu)a} - \rho_i\partial^\mu Z^{\nu)a}_{(\nu} + \\ +g f_{abc}\left(\beta_i D^b_\nu Z^{(\mu\nu)c} + \rho_i D^{\mu b} Z^{\nu)c}_{(\nu}\right) \end{pmatrix} + \\
&+ \lambda_2\left(2ia_{(ij)} f_{abc} + (2b_{[ij]} + \gamma_{[ij]}) d_{abc}\right) X^{jb}_\nu z^{[\mu\nu]c} + \\
&+ 2\xi_2 \begin{pmatrix} \left(ia_{[ij]} f_{abc} + b_{(ij)} d_{abc}\right) X^{jb}_\nu z^{(\mu\nu)c} + \\ +\left(iu_{[ij]} f_{abc} + v_{(ij)} d_{abc}\right) X^{\mu jb} z^{\nu)c}_{(\nu} \end{pmatrix} + \\
&+ \lambda_3 \begin{pmatrix} \alpha_i\left(\partial_\nu z^{[\mu\nu]a} - g f_{abc} D^b_\nu z^{[\mu\nu]c}\right) + \\ +\left(ia_{(ij)} f_{abc} + b_{[ij]} d_{abc} + \frac{1}{2}\gamma_{[ij]} d_{abc}\right) X^{jb}_\nu Z^{[\mu\nu]c} \end{pmatrix} + \\
&+ \xi_3 \begin{pmatrix} -\beta_i\partial_\nu z^{(\mu\nu)a} - \rho_i\partial^\mu z^{\nu)a}_{(\mu} + \\ +g f_{abc}\left(\beta_i D^b_\nu z^{(\mu\nu)c} + \rho_i D^{\mu b} z^{\nu)c}_{(\nu}\right) + \\ +\left(iu_{[ij]} f_{abc} + v_{(ij)} d_{abc}\right) X^{\mu jb} Z^{\nu)c}_{(\nu} + \\ +\left(ia_{[ij]} f_{abc} + b_{(ij)} d_{abc}\right) X^{jb}_\nu Z^{(\mu\nu)c} \end{pmatrix} = 0. \qquad \text{(A.22)}
\end{aligned}
$$

Multiplying the equation of motion by $t_k$ and taking again the corresponding trace, we have

$$
\begin{aligned}
&\lambda_1\begin{pmatrix} \alpha_i(d_{aek}-if_{aek})\partial_\nu\, Z^{[\mu\nu]e}+\\ +g\alpha_i(if_{abc}f_{cek}-f_{abc}d_{cek})D^b_\nu Z^{[\mu\nu]e}\end{pmatrix}+\\
&+\xi_1\begin{pmatrix} \beta_i(if_{aek}-d_{aek})\partial_\nu\, Z^{(\mu\nu)e}+\\ +g\beta_i(f_{abc}d_{cek}-if_{abc}f_{cek})D^b_\nu Z^{(\mu\nu)e}+\\ +\rho_i(if_{aek}-d_{aek})\partial^\mu\, Z^{\nu)e}_{(\nu}+\\ +g\rho_i(f_{abc}d_{cek}-if_{abc}f_{cek})D^{\mu b}Z^{\nu)e}_{(\nu}\end{pmatrix}+\\
&+\lambda_2\begin{pmatrix} \frac{1}{N}\left(2b_{[ij]}+\gamma_{[ij]}\right)X^{ja}_\nu\, z^{[\mu\nu]k}+\\ \begin{pmatrix} a_{(ij)}(f_{abc}f_{cek}+if_{abc}d_{cek})+\\ +\left(b_{[ij]}+\frac{1}{2}\gamma_{[ij]}\right)(d_{abc}d_{cek}-id_{abc}f_{cek})\end{pmatrix}X^{jb}_\nu\, z^{[\mu\nu]e}\end{pmatrix}+\\
&+\xi_2\begin{pmatrix} \frac{2}{N}b_{(ij)}X^{ja}_\nu\, z^{(\mu\nu)k}+\frac{2}{N}v_{(ij)}X^{\mu ja}\, z^{\nu)k}_{(\nu}+\\ +\begin{pmatrix} a_{[ij]}(f_{abc}f_{cek}+if_{abc}d_{cek})+\\ +b_{(ij)}(d_{abc}d_{cek}-id_{abc}f_{cek})\end{pmatrix}X^{jb}_\nu\, z^{(\mu\nu)e}+\\ +\begin{pmatrix} u_{[ij]}(f_{abc}f_{cek}+if_{abc}d_{cek})+\\ +v_{(ij)}(d_{abc}d_{cek}-id_{abc}f_{cek})\end{pmatrix}X^{\mu jb}\, z^{\nu)e}_{(\nu}\end{pmatrix}+\\
&+\frac{\lambda_3}{2}\begin{pmatrix} \alpha_i(d_{aek}-if_{aek})\partial_\nu\, z^{[\mu\nu]e}+\\ +g\alpha_i(if_{abc}f_{cek}-f_{abc}d_{cek})D^b_\nu\, z^{[\mu\nu]e}+\\ +\frac{1}{N}\left(b_{[ij]}+\frac{1}{2}\gamma_{[ij]}\right)X^{ja}_\nu Z^{[\mu\nu]k}+\\ +\begin{pmatrix} a_{(ij)}(if_{abc}d_{cek}+f_{abc}f_{cek})+\\ \left(b_{[ij]}+\frac{1}{2}\gamma_{[ij]}\right)(d_{abc}d_{cek}-id_{abc}f_{cek})\end{pmatrix}X^{jb}_\nu Z^{[\mu\nu]e}\end{pmatrix}+\\
&+\xi_3\begin{pmatrix} \frac{1}{2}\begin{pmatrix} \beta_i(if_{aek}-d_{aek})\partial_\nu\, z^{(\mu\nu)e}+\\ +g\beta_i(f_{abc}d_{cek}-if_{abc}f_{cek})D^b_\nu\, z^{(\mu\nu)e}+\\ +\rho_i(if_{aek}-d_{aek})\partial^\mu\, z^{\nu)e}_{(\nu}+\\ +g\rho_i(f_{abc}d_{cek}-if_{abc}f_{cek})D^{\mu b}\, z^{\nu)e}_{(\nu}\end{pmatrix}+\\ +\frac{1}{N}b_{(ij)}X^{ja}_\nu Z^{(\mu\nu)k}+\frac{1}{N}v_{(ij)}X^{\mu ja}Z^{\nu)k}_{(\nu}+\\ +\frac{1}{2}\begin{pmatrix} a_{[ij]}(f_{abc}f_{cek}+if_{abc}d_{cek})+\\ +b_{(ij)}(d_{abc}d_{cek}-id_{abc}f_{cek})\end{pmatrix}X^{jb}_\nu Z^{(\mu\nu)e}+\\ +\frac{1}{2}\begin{pmatrix} u_{[ij]}(f_{abc}f_{cek}+if_{abc}d_{cek})+\\ +v_{(ij)}(d_{abc}d_{cek}-id_{abc}f_{cek})\end{pmatrix}X^{\mu jb}Z^{\nu)e}_{(\nu}\end{pmatrix}=0.
\end{aligned}
\tag{A.23}
$$

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