Pentagram Geometry

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Abstract

The Natural Constant Phi is included in the geometrical figure of a Pentagram as many artists have found out. Michelangelo may be the most famous one, whose sketch was transported in the payload of the first successful mission to the moon. By deriving this natural constant from the geometrics of the Pentagram a new analytical expression is found for Phi.

Keywords

Geometry, Mathematics, Physics, Natural Constants, Phi, Phi Ratio, Golden Ratio

Introduction

The rules of trigonometry are used to find the distances and angles inside the Pentagram.

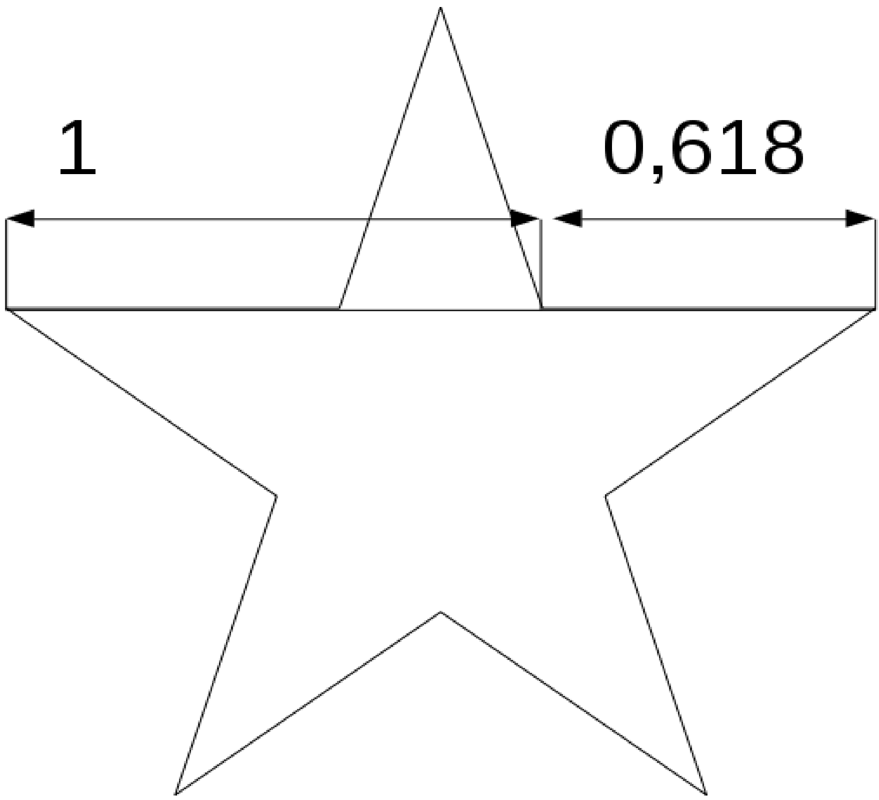

Fig 1: Phi inside the Pentagram

The section line in a circle is calculated as

$$s = 2r\sin\left(\frac{\alpha}{2}\right) \tag{1}$$

For the pentagram the angle alpha is given by

$$\alpha = 2\frac{2\pi}{5} \tag{2}$$

If we di~~e~~vide the section line into the two equal outer parts a and the middle part b we get

$$2a + b = 2r\sin\left(\frac{2\pi}{5}\right) \tag{3}$$

The pentagram contains only these two different distances a and b and only three different angles named alpha, beta and gamma. To calculate these five values we need a set of five equations, the first of which is equation (3). The others follow from the general laws for a triangle as equation (5),(6),(7),(8)

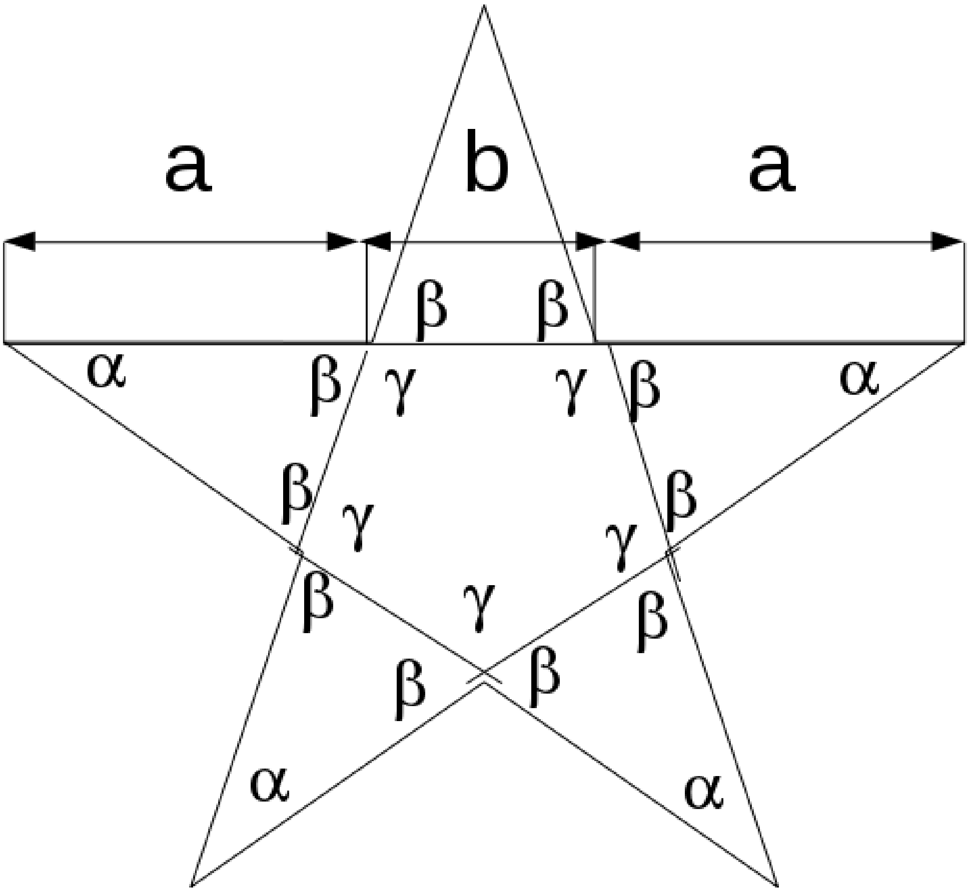

Fig 2: The three angles inside the Pentagram

From the law of the relationships of the angles to their opposite sides [1]

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} \tag{4}$$

we get

$$\frac{\sin\gamma}{2a+b}=\frac{\sin\alpha}{a+b} \tag{5}$$

The other three equations can be directly ob~~p~~tained from the drawing of the pentagram, because these angles add up to 180 degree.

$$2\alpha+\gamma=\pi \tag{6}$$

$$\alpha+2\beta=\pi \tag{7}$$

$$\beta+\gamma=\pi \tag{8}$$

So we got the five equations (3),(5),(6),(7),(8) and start calculating alpha, beta and gamma and then a and b. From (6) (7) (8) we get

$$\alpha=\frac{1}{5}\pi \tag{9}$$

$$\beta=\frac{2}{5}\pi \tag{10}$$

$$\gamma=\frac{3}{5}\pi \tag{11}$$

From (3) we get

$$b=2r\sin\left(\frac{2\pi}{5}\right)-2a \tag{12}$$

From (5) we get

$$b=\frac{a(2\sin\alpha-\sin\gamma)}{\sin\gamma-\sin\alpha} \tag{13}$$

From (12) and (13) we get a as

$$a=\frac{2r\sin\left(\frac{2}{5}\pi\right)}{2+\frac{2\sin\left(\frac{1}{5}\pi\right)-\sin\left(\frac{3}{5}\pi\right)}{\sin\left(\frac{3}{5}\pi\right)-\sin\left(\frac{1}{5}\pi\right)}} \tag{14}$$

So finally inserting (14) into (12) or (13) we can calculate b.
From Figure 1 we get Phi by

$$\frac{a}{a+b}=\frac{0.618...}{1} \tag{15}$$

or

$$\phi = 1 + \frac{a}{a+b} \tag{16}$$

Inserting (12) into (16) results in

$$\phi = 1 + \frac{a}{2r\sin(\frac{2\pi}{5}) - a} \tag{17}$$

Inserting (14) for a we get

$$\varphi = 1 + \frac{\dfrac{2r\sin(\frac{2}{5}\pi)}{2 + \dfrac{2\sin(\frac{1}{5}\pi) - \sin(\frac{3}{5}\pi)}{\sin(\frac{3}{5}\pi) - \sin(\frac{1}{5}\pi)}}}{2r\sin(\frac{2}{5}\pi) - \dfrac{2r\sin(\frac{2}{5}\pi)}{2 + \dfrac{2\sin(\frac{1}{5}\pi) - \sin(\frac{3}{5}\pi)}{\sin(\frac{3}{5}\pi) - \sin(\frac{1}{5}\pi)}}} \tag{18}$$

Here is some arithmetic necessary to simplify this equation. First the nominator can be eliminated to get

$$\varphi = 1 + \frac{\dfrac{1}{2 + \dfrac{2\sin(\frac{1}{5}\pi) - \sin(\frac{3}{5}\pi)}{\sin(\frac{3}{5}\pi) - \sin(\frac{1}{5}\pi)}}}{1 - \dfrac{1}{2 + \dfrac{2\sin(\frac{1}{5}\pi) - \sin(\frac{3}{5}\pi)}{\sin(\frac{3}{5}\pi) - \sin(\frac{1}{5}\pi)}}} \tag{19}$$

or by multiplying with the denominator

$$\varphi = 1 + \cfrac{1}{1 + \cfrac{2\sin(\frac{1}{5}\pi) - \sin(\frac{3}{5}\pi)}{\sin(\frac{3}{5}\pi) - \sin(\frac{1}{5}\pi)}} \tag{20}$$

which is simply

$$\varphi = \frac{\sin(\frac{3}{5}\pi)}{\sin(\frac{1}{5}\pi)} \tag{21}$$

Results and Discussion

A new analytical form for the famous Phi Ratio was found with formula (21). In literature Phi is given by [2] as

$$\varphi = \frac{1}{2}\left(1 + \sqrt{(5)}\right) \tag{22}$$

or by [3] as

$$\phi = 2\cos\left(\frac{\pi}{5}\right) \tag{23}$$

Even if the occurrence of the number 5 is quite obvious in all the three formulas and also in the geometric object leading to formula (21), the reason for this connection is still not known and may invite for further investigation.

Acknowledgments

My thanks to all the people I met on Hawai'i who have contributed in inspir~~e~~ing me to start these calculations in the first place and thus convinced me to get these calculations~~m~~ published. May this work inspire the young scientists all over the world to publish their own results and get them discussed and developed further ~~on~~ to lead to the next fundamental step of science we are all so desperately waiting for.

References

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3. Wolfram MathWorld, Golden Ratio, http://mathworld.wolfram.com/GoldenRatio.html, formula (9)

Dipl.-Phys. Ulrich Winter biography

2010 Presentation of "The Two Ways of Contact" at the N.A.S.A. Contact Conference, St. Jose, California, USA

2009 Presentation of "N.I.N.A.", the Neural Integrated Network Architecture, at the Maritim, Königswinter, Germany

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1962 born on the lovely countryside between the two rivers Rhine, Germany, and Maas, Netherlands