



## On the theory of the passive impurity distribution in the turbulent air flow

<sup>1</sup>George Jandieri, <sup>2</sup>Natalia Zhukova

1. Georgian Technical University, 77 Kostava Str., Tbilisi 0175, Georgia

E-mail: jandieri@access.sanet.ge

2. M. Nodia Institute of Geophysics, 1 Akexidze Str., Tbilisi 0193, Georgia

E-mail: natalia27cpp@hotmail.com

### ABSTRACT

The statistical model of passive impurity transfer in surface boundary layers of the turbulent atmosphere in the presence of wind is offered. Analytical expressions of the normalized concentration of impurity for arbitrary correlation tensor of the second rank of velocity pulsation when the emission source is located at a certain height over the Earth's surface are obtained. The effective coefficient of turbulent diffusion contains coefficient of molecular diffusion, longitudinal and transverse turbulent diffusion coefficients. Numerical calculations were carried out using experimental data of ground(-based) observations. The isolines describing distribution of the passive impurities at calm case are depicted at different values of a wind speed and at certain distances from a source. Dynamics of globules formation with various concentration of impurity transferred by wind is constructed. They have specific characteristic spatial scales and lifetimes.

### INTRODUCTION

Statistical theory of waves propagation in a randomly inhomogeneous media have been rather well studied [1,2]. Fluctuation phenomena in different physical systems have various nature. Statistical characteristics of scattered electromagnetic waves in the upper atmosphere and the ionosphere have been considered in [3-6].

In the lower atmosphere turbulent mixing and diffusion in the streams is especially important. It is known that process of the turbulent diffusion is defined by turbulent characteristics of the velocity field. For calculation of the passive impurity concentration it is necessary knowledge of the parameters of the lower atmospheric layers as coefficients of the turbulent diffusion, and also winds in the considered layer. Pollution control of the atmosphere first of all is connected with the impurity distribution (gases, aerosols, small solid particles) in the atmosphere erupting by a source. The most widespread factor leading to the negative consequences is the pollution of natural environment connecting with the transfer of various harmful substances on long distances owing to diffusion. At the same time the following parameters are considered: characteristics of the emit sources, the features of erupted harmful impurity, meteorological parameters of the atmosphere. The concentration of impurity in the air depends on the power of the source and on the disseminating ability of the atmosphere. The important characteristic of emissions scattering is stability of the atmosphere. A certain type of stability is a result of the influence of a complex of meteorological parameters, and the speed of wind is one of dominating.

The impurity erupting in the atmosphere from a source scatters and transferred in air by different scales turbulent vortices permanently existing in the atmosphere. Intensity of the atmospheric turbulence and, therefore, intensity of the impurity diffusion at different weather conditions are various and determined mainly by two factors: a vector of the wind speed and a vertical temperature gradient depending on the properties of the underlying surface, thermal balance on the Earth's surface, and also dynamic and temperature characteristics of the air participating in scattering.

The important feature of the turbulent motion, transfer and scattering of impurities in the atmosphere is its randomness makes impossible to describe turbulent transfer in details as function of time and space. However it is possible to define average values of various parameters: speeds of wind, pressure, temperature, etc. It is known that the evolution of concentration distribution caused by the temporal average characteristics of the velocity field in the turbulent atmosphere depends on averaging interval length. Entity of this phenomenon is defined by specific influence of different scale turbulent vortices on the scattering process of impurity in its different stages, revealing mainly on the horizontal diffusion, as the scales of vortices realizing vertical diffusion are restricted by the Earth's surface. Small-scale vortices cause instant pulsations of the wind. Their total influence determines impurity content in air during the time interval in several minutes. Large-scale vortices cause substantial variations of the mean wind for a long observed time interval.

Many papers are devoted to the estimation methods of the atmosphere pollution by different emission sources. Frequently the theory of turbulent diffusion is based on the parabolic type partial differential equation for the mean concentration of diffuse impurity (see for example [7]).

### THE NORMALIZED CONCENTRATION DISTRIBUTION AND TURBULENT DIFFUSION COEFFICIENTS

We consider model of the passive impurities propagation and distribution in the atmosphere using the modify mean field method [8]. Concentration  $N(\mathbf{r}, t)$  and the velocity  $V(\mathbf{r}, t)$  of a flow satisfy stochastic Fokker-Planck equation:

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x_\alpha} (NV_\alpha) = (D_0 \delta_{\alpha\beta} + D_{\alpha\beta}) \frac{\partial^2 N}{\partial x_\alpha \partial x_\beta} + S(\mathbf{r}, t), \quad (1)$$



where:  $D_0$  is the coefficient of the molecular diffusion,  $D_{\alpha\beta}$  is the second rank tensor of the turbulent diffusion with nonzero diagonal elements:  $D_{\parallel}$  and  $D_{\perp}$  are longitudinal and transversal turbulent diffusion coefficients, respectively;  $S(\mathbf{r}, t)$  is the deterministic function of the pollutant source. Submit these functions as sum of the mean and fluctuating terms:  $N(\mathbf{r}, t) = N_0(\mathbf{r}, t) + n(\mathbf{r}, t)$ ,  $V(\mathbf{r}, t) = \mathbf{V}_0 + \mathbf{u}(\mathbf{r}, t)$ ,  $\mathbf{V}_0 = const$ , which are random functions of the spatial coordinates and time. The equation (1) yields two integro-differential equations for the mean concentration of the passive impurity and fluctuating one:

$$\frac{\partial N_0(\mathbf{k}, t)}{\partial t} + a(k)N_0(\mathbf{k}, t) = -\langle u^2 \rangle k_{\alpha} \int_{-\infty}^{\infty} d\mathbf{k}' k'_{\beta} \int_0^t d\tau W_{\alpha\beta}(\mathbf{k} - \mathbf{k}', \tau) \cdot \exp[-a(k')\tau] N_0(\mathbf{k}, t - \tau) + S(\mathbf{k}, t), \quad (2)$$

$$n(\mathbf{k}, t) = ik_{\beta} \exp[-a(k)t] \int_0^t dt' \exp[-a(k)t'] \int_{-\infty}^{\infty} d\mathbf{k}' u_{\beta}(\mathbf{k} - \mathbf{k}', t') \cdot N_0(\mathbf{k}', t'), \quad (3)$$

where:  $a(k) = ik_{\alpha} V_{0\alpha} + D_0 k^2 + D_{\alpha\beta} k_{\alpha} k_{\beta}$ ,  $\alpha, \beta = x, y, z$ ;  $\langle u^2 \rangle^{1/2}$  is the root-mean-square pulsation of the wind velocity, angular brackets denote the ensemble average. Correlation tensor  $W_{\alpha\beta}(\mathbf{p}, \tau)$  of the velocity pulsations describes the homogeneous and stationary stochastic process. The mean concentration distribution of the passive impurity for the arbitrary pollutant source  $S(\mathbf{r}, t)$  and the solenoidal vector field ( $k_{\alpha} W_{\alpha\beta}(\mathbf{k}, t) = k_{\beta} W_{\alpha\beta}(\mathbf{k}, t) = 0$ ) satisfying the initial condition  $N(\mathbf{k}, 0) = 0$  may be written as:

$$N_0(\mathbf{r}, t) = \int_{-\infty}^{\infty} d\mathbf{k} \exp[i\mathbf{k}\mathbf{r} - b(k)t] \int_0^t dt' S(\mathbf{k}, t') \exp[b(k)t'], \quad (4)$$

where:  $b(k) = ik_{\alpha} V_{0\alpha} + k^2 D_{eff}(\mathbf{k})$ ,

$$D_{eff}(\mathbf{k}) = D_0 + D_{\alpha\beta} \frac{k_{\alpha} k_{\beta}}{k^2} + \frac{\langle u^2 \rangle}{(2\pi)^3} \cdot \frac{k_{\alpha} k_{\beta}}{k^2} \int_{-\infty}^{\infty} d\mathbf{k}' \int_{-\infty}^{\infty} d\mathbf{p} \int_0^{\infty} d\tau W_{\alpha\beta}(\mathbf{p}, \tau) \cdot \exp[-i(\mathbf{k} - \mathbf{k}')\mathbf{p} - a(k')\tau]. \quad (5)$$

Consider the following model. Let a point source having power  $M$  is located at a height  $H$  over the origin of coordinates

$$S(\mathbf{r}, t) = q(\mathbf{r}) \cdot Q(t) = QM \delta(x - H) \cdot \delta(y) \delta(z). \quad (6)$$

Fourier transform is [7]:  $S(\mathbf{k}) = 2QM \cos(k_x H)$ . If vector of the turbulent flow  $\mathbf{V}_0$  is directed along the Z-axis and X-axis perpendicular upward, equation (4) can be rewritten as:

$$N_0(\mathbf{r}, t) = 2QM \int_{-\infty}^{\infty} d\mathbf{k} \frac{\cos(k_x H)}{b(k)} [1 - \exp(-b(k)t)] \cdot \exp(i\mathbf{k}\mathbf{r}). \quad (7)$$

$$D_{eff}(\mathbf{k}) = D_0 + \frac{1}{8\pi^{3/2}} \frac{\langle u^2 \rangle}{D_{\perp} D_{\parallel}^{1/2}} \frac{k_{\alpha} k_{\beta}}{k^2} \int_{-\infty}^{\infty} d\mathbf{p} \int_0^{\infty} d\tau \frac{1}{\tau^{3/2}} W_{\alpha\beta}(\mathbf{p}, \tau) \cdot \exp\left[-i\mathbf{k}\mathbf{p} - \frac{\rho_x^2 + \rho_y^2}{4D_{\perp}\tau} - \frac{(\rho_z - V_0\tau)^2}{4D_{\parallel}\tau}\right], \quad (8)$$

$$W_{\alpha\beta}(\mathbf{p}, \tau) = G_{\alpha\beta}(\mathbf{p}) \cdot \exp\left(-\frac{\tau}{T}\right) = \left[ \left(1 - \frac{\rho^2}{l^2}\right) \delta_{\alpha\beta} + \frac{\rho^2}{l^2} n_{\alpha} n_{\beta} \right] \exp\left(-\frac{\rho^2}{l^2} - \frac{\tau}{T}\right), \quad n_{\alpha} \text{ is the unit vectors of } \mathbf{p};$$

anisotropic velocity pulsations have different characteristic spatial scales  $l_{\parallel}$  and  $l_{\perp}$  along and perpendicular to the turbulent stream, respectively;  $\rho^2/l^2 = \rho_{\perp}^2/l_{\perp}^2 + \rho_{\parallel}^2/l_{\parallel}^2$ ,  $\rho_{\perp}^2 = \rho_x^2 + \rho_y^2$ ;  $T$  is an integral Lagrangian temporal scale [9] characterizing life time of energetic turbulent inhomogeneities having spatial scale  $l = \langle u^2 \rangle T$ . Integrating over  $\mathbf{p}$  passing to the polar coordinate system, using Jacobi-Anger formulas [10]

$$\exp(iz \sin \varphi) = \sum_{n=-\infty}^{\infty} \exp(in\varphi) J_n(z), \quad \exp(iz \cos \varphi) = \sum_{n=-\infty}^{\infty} i^n \exp(in\varphi) J_n(z),$$



Integrating equation (8) over polar angle  $\varphi$  taking into account orthogonality condition [4]

$$\frac{1}{2\pi} \int_0^{2\pi} d\varphi \exp[-i(m-n)\varphi] = \begin{cases} 0 & \text{at } m \neq n \\ 1 & \text{at } m = n \end{cases}$$

and the recurrence condition  $J_{\nu-1}(z) - J_{\nu+1}(z) = 2J'_\nu(z)$  of the  $J_n(z)$   $n$ -th order Bessel function, we obtain:

$$D_{eff}(\mathbf{k}) = D_0 + \frac{1}{2} \frac{\langle u^2 \rangle}{D_\perp D_\square^{1/2} k^2} \sum_{n=-\infty}^{\infty} i^n \int_{-\infty}^{\infty} d\rho_z \int_0^{\infty} d\rho_\perp \rho_\perp \left( \frac{\rho_\perp^2}{D_\perp} + \frac{\rho_z^2}{D_\square} \right)^{-1/2} \left\{ \left[ \left( 1 - \frac{\rho_\perp^2}{l_\perp^2} - \frac{\rho_z^2}{l_\square^2} \right) k^2 - \frac{\rho_z^2}{l_\square^2} k_z^2 \right] \cdot J_n(k_x \rho_\perp) J_n(k_y \rho_\perp) + \frac{1}{4} \frac{\rho_\perp^2}{l_\perp^2} k_x^2 J_n(k_y \rho_\perp) [J_{n-2}(k_x \rho_\perp) - 2J_n(k_x \rho_\perp) + J_{n+2}(k_x \rho_\perp)] + \frac{1}{4} \frac{\rho_\perp^2}{l_\perp^2} k_y^2 J_n(k_x \rho_\perp) \cdot [J_{n-2}(k_x \rho_\perp) - 2J_n(k_x \rho_\perp) + J_{n+2}(k_x \rho_\perp)] \right\} \exp \left\{ -\frac{\rho_\perp^2}{l_\perp^2} - \frac{\rho_z^2}{l_\square^2} - \left[ \frac{1}{T} \left( 1 + \frac{TV_0^2}{4D_\square} \right) \left( \frac{\rho_\perp^2}{D_\perp} + \frac{\rho_z^2}{D_\square} \right) \right]^{1/2} + \rho_z \left( \frac{V_0}{2D_\square} - i k_z \right) \right\}. \tag{9}$$

Using the saddle-point method at integration over  $\rho_z$  [11] we obtain:

$$D_{eff}(\mathbf{k}) = D_0 + \frac{\sqrt{\pi} \langle u^2 \rangle}{4 (k l_\perp)^2} \cdot \left( \frac{D_\square}{V_0^2 D_\perp^3} \right)^{1/4} \int_0^{\infty} d\rho_\perp \rho_\perp^{1/2} \left\{ J_0(\alpha) J_0(\beta) [2(k l_\perp)^2 - (3\alpha^2 + 3\beta^2 + \gamma^2)] + \alpha^2 J_2(\alpha) J_0(\beta) + \beta^2 J_0(\alpha) J_2(\beta) + 2 \sum_{n=1}^{\infty} (-1)^n J_{2n}(\alpha) J_{2n}(\beta) [2(k l_\perp)^2 - (3\alpha^2 + 3\beta^2 + \gamma^2)] + \alpha^2 \sum_{n=1}^{\infty} (-1)^n J_{2n}(\beta) [J_{2n-2}(\alpha) + J_{2n+2}(\alpha)] + \beta^2 \sum_{n=1}^{\infty} (-1)^n J_{2n}(\alpha) [J_{2n-2}(\beta) + J_{2n+2}(\beta)] \right\} \cdot \exp \left( -\frac{\rho_\perp^2}{l_\perp^2} - \frac{V_0}{\sqrt{D_\square D_\perp}} \rho_\perp \right), \tag{10}$$

where:  $\alpha = k_x \rho_\perp$ ,  $\beta = k_y \rho_\perp$ ,  $\gamma = k_z \rho_\perp$ . Exponential function fast decreases inversely proportional to  $\rho_\perp$  and therefore the important integrated area is in the interval (0,1). Depending on the parameters  $\alpha$  and  $\beta$  we can either carry out series expansion or use asymptotic expansion [11].

a) If  $\alpha \ll 1$ ,  $\beta \ll 1$ , introducing new parameter  $\zeta = (V_0 l_\perp)^2 / 8 D_\square D_\perp$ , the expression (10) can be rewritten as:

$$D_{eff} = D_0 + \frac{\sqrt{2\pi}}{8} \langle u^2 \rangle \frac{l_\perp^2}{D_\perp} \zeta^{1/2} \exp(\zeta) [K_{3/4}(\zeta) - K_{1/4}(\zeta)], \tag{11}$$

where  $K_\nu(x)$  is the Macdonald function. We consider limiting cases of big and small values of the parameter  $\zeta$  using the asymptotic formulae for the Macdonald function [10].

At  $\zeta < 1$  Equation (11) yields

$$D_{eff} = D_0 + \frac{2^{1/4} \pi^{1/2}}{8} \Gamma\left(\frac{3}{4}\right) \frac{\langle u^2 \rangle l_\perp^2}{D_\perp \zeta^{1/4}} = D_0 + D_{turb}, \tag{12}$$



at  $\zeta > 1$

$$D_{eff} = D_0. \quad (13)$$

From the expression (12) follows that the velocity pulsations increase  $D_{eff}$ . Coefficient of the turbulent diffusion depends on the transversal spatial scale of the velocity fluctuations ( $\sim l_{\perp}^{3/2}$ ), horizontal ( $D_{\square}$ ) and vertical ( $D_{\perp}$ ) components of the diffusion coefficients. Velocity  $V_0$  increases inversely proportion to  $D_{urb}$ . If the velocity of the turbulent flow substantially increases, velocity pulsations do not give the contribution.

b) If  $\alpha \gg 1$ ,  $\beta \gg 1$ . At big values  $x$  using the asymptotic Bessel functions

$$J_{\nu}(x) = \left(\frac{2}{\pi x}\right)^{1/2} \cos\left(x - \frac{\nu\pi}{2} - \frac{\pi}{4}\right),$$

from equation (10) we obtain

$$D_{eff}(\mathbf{k}) = D_0 + \frac{1}{2\sqrt{\pi}} \frac{\langle u^2 \rangle}{(k l_{\perp})^2} \left(\frac{D_{\square}}{V_0^2 D_{\perp}^3}\right)^{1/4} \frac{1}{(k_x k_y)^{1/2}} \int_0^{\infty} d\rho_{\perp} \rho_{\perp}^{-1/2} \left[2(k l_{\perp})^2 - \rho_{\perp}^2(4k_x^2 + 4k_y^2 + k_z^2)\right] \cdot [\cos(k_{-}\rho_{\perp}) + \sin(k_{+}\rho_{\perp})] \exp\left(-\frac{\rho_{\perp}^2}{l_{\perp}^2} - \frac{V_0}{\sqrt{D_{\square} D_{\perp}}} \rho_{\perp}\right), \quad (14)$$

where  $k_{\pm} = k_x \pm k_y$ . Integrating, at  $\zeta > 1$  we obtain

$$D_{eff}(\mathbf{k}) = D_0 + \frac{\langle u^2 \rangle}{V_0} \left(\frac{D_{\square}}{D_{\perp}}\right)^{1/2} \frac{1}{(k_x k_y)^{1/2}}. \quad (15)$$

Hence, the effective turbulent diffusion coefficient at small scale velocity pulsations is determined only parameters characterizing turbulent flow; while at large scale pulsations it depends also on the wavelength, namely it is inversely proportional to the transverse wave number, i.e.  $k_{\perp}^{-1}$ .

Knowledge of the turbulent diffusion coefficient  $D_{eff}$  allows us to calculate integral (4). In small scale fluctuations parameter  $D_{eff}$  not depends on  $k$ . Substituting into equation (4) we can calculate first integral having simple pole  $k_z = i D k_{\perp}^2 / V_0$  in the upper half plane applying the residue theory.

For small spatial scale irregularities ( $k_{\perp} l_{\perp} \ll 1$ ) and big Peclet's number ( $V_0 l / D_{\square}$ ) we have

$$D_* = \frac{\sqrt{2\pi}}{8} \langle u^2 \rangle \frac{l_{\perp}^2}{D_{\perp}} \zeta^{1/2} \cdot \exp(\zeta) [K_{3/4}(\zeta) - K_{1/4}(\zeta)], \quad (16)$$

where:  $D_* = D_{eff}(\mathbf{k}) - D_0$ ,  $\zeta = (V_0 l_{\perp})^2 / 8 D_{\square} D_{\perp}$ ,  $K_{\nu}(x)$  is the Macdonald function. Consider different cases of the parameter  $\zeta$ . Using the asymptotic formulas of the Macdonald function from equation (10) follows, that: at  $\zeta < 1$  velocity pulsations lead to the increase of the turbulent diffusion coefficient  $D_*$  depending on as the characteristic transverse spatial scale of the velocity pulsation ( $l_{\perp}^{3/2}$ ), as well as the horizontal ( $D_{\square}$ ) and transversal ( $D_{\perp}$ ) diffusion coefficients. Increasing the wind velocity  $V_0$  parameter  $D_*$  decreases. If  $V_0$  substantially increases, velocity pulsations not give the contribution. At  $\zeta > 1$  turbulent diffusion coefficient is determined mainly by the velocity of a turbulent stream.

Особый интерес представляет изучение распространения примеси в атмосфере при аномальных метеорологических условиях, к которому относится, в частности, штиль. В этом случае отсутствует перенос примеси ветром, и около источника могут наблюдаться очень высокие концентрации. В областях с резко



континентальным климатом бывают штили с вертикальной протяженностью до нескольких сотен метров и более [7]. Turbulent diffusion coefficient of the passive impurities in the calm case ( $V_0 = 0$ ) is:

$$D_*^0 = 2\sqrt{\pi} \langle u^2 \rangle l_{\perp} \left( \frac{T}{D} \right)^{1/2} \zeta_1 \cdot \exp(\zeta_1) [K_{3/4}(\zeta_1) - K_{1/4}(\zeta_1)], \quad (17)$$

here:  $\zeta_1 = l_{\perp}^2 / 8TD_{\perp}$ .

Substituting (16) and (17) into equation (7) we obtain the evaluation of the pollutant mean concentration distribution:

$$\begin{aligned} \frac{N_0(\mathbf{r}, t)}{QM} = & \frac{2\pi^2}{D_* z} \exp\left\{-\frac{1}{4} \frac{V_0}{D_* z} [(x+H)^2 + y^2]\right\} \cdot \left\{1 + \frac{1}{4z^2} [(x+H)^2 + y^2]\right\} - \\ & - \frac{\sqrt{\pi}}{2} \frac{\alpha}{\sqrt{\beta}} \left[ \exp(2\sqrt{\beta\gamma}) \cdot \operatorname{erf}\left(\sqrt{\frac{\beta}{t}} + \sqrt{\gamma t}\right) + \exp(2\sqrt{\beta\gamma}) \cdot \operatorname{erf}\left(\sqrt{\frac{\beta}{t}} - \sqrt{\gamma t}\right) - \right. \\ & \left. - \exp(2\sqrt{\beta\gamma}) + \exp(-2\sqrt{\beta\gamma}) \right] + (H \rightarrow -H), \end{aligned} \quad (18)$$

$$\frac{N_0^0(\mathbf{r}, t)}{QM} = \frac{2\pi^2}{D_*^0} [(x+H)^2 + y^2 + z^2]^{-1/2} \left\{ 1 - \operatorname{erf}\left[\left(D_*^0 t\right)^{-1/2} ((x+H)^2 + y^2 + z^2)\right] \right\} + (H \rightarrow -H), \quad (19)$$

where:  $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x dt \exp(-t^2)$  is the Gaussian error function,  $\alpha = \frac{\pi^{3/2}}{2} \frac{1}{D_*^{3/2}} \exp\left(\frac{zV_0}{2D_*}\right)$ ,

$$\beta = \frac{1}{4D_*} [(x+H)^2 + y^2 + z^2], \quad \gamma = \frac{V_0^2}{4D_*}.$$

Equation (12) satisfies the initial condition

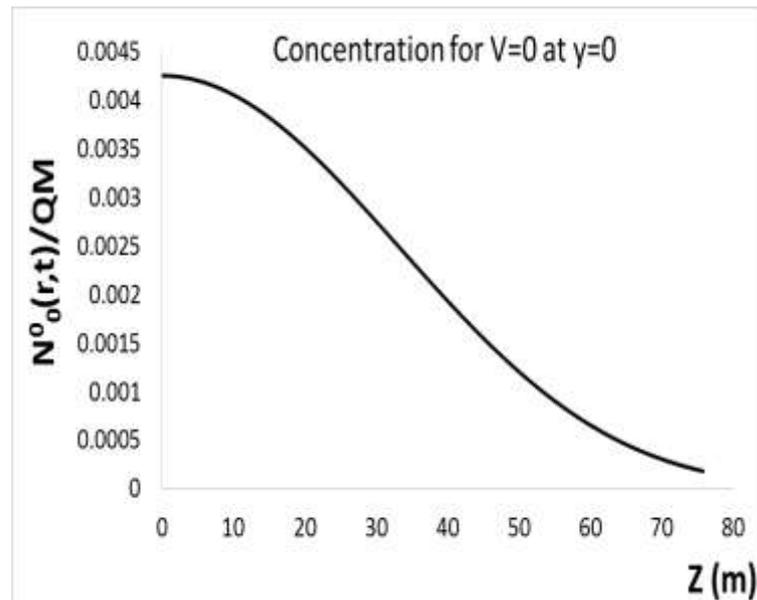
$$[(x+H)^2 + y^2] \leq z^2 \quad (20)$$

imposing the restriction on a distance  $z$ . The condition  $N_0^0(\mathbf{r}, t) = 0$  is satisfied for arbitrary distances  $z$  in the calm case. The first terms in (18) and (19) represent stationary concentration distribution. Actually, if  $t \rightarrow \infty$  taking into account  $\operatorname{erf}(0) = 0$ ,  $\operatorname{erf}(\infty) = 1$  and the inequality (20), the second term of equation (19) vanishes; the equation (18) passes into the well-known result [7] for passive pollutant concentration distribution with constant exchange coefficients.

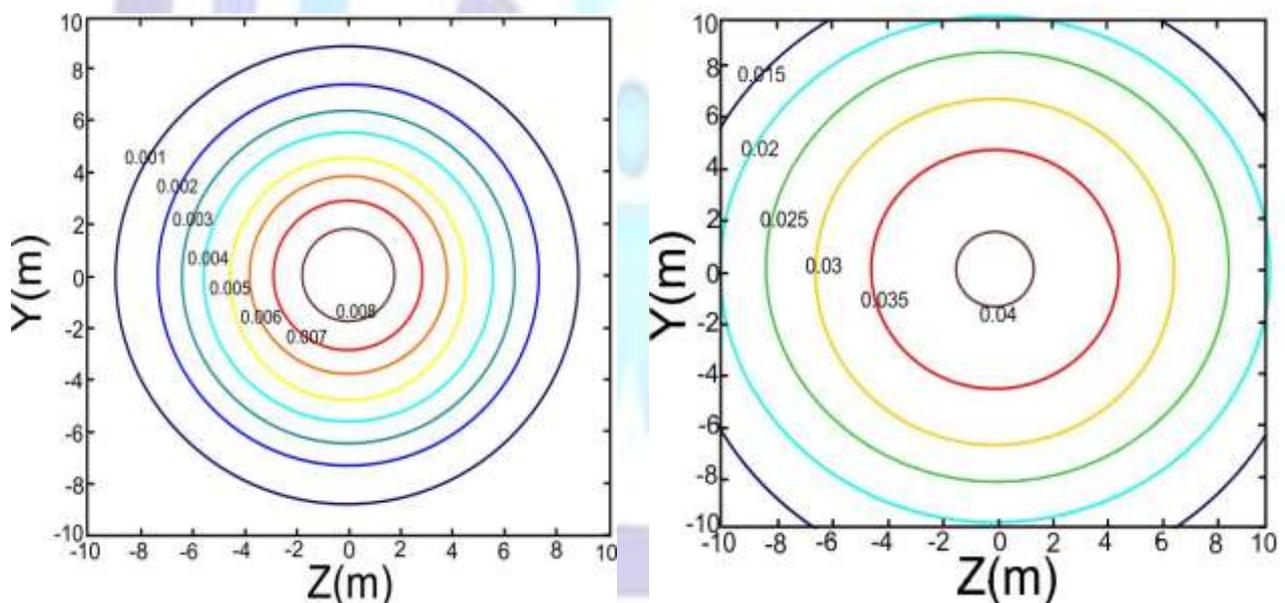
## Numerical calculations

Numerical calculations were carried using equations (10)-(13) and experimental data [9,12]. Characteristic spatial scale  $l_{\perp} = \langle u^2 \rangle^{1/2} T$  of the velocity pulsations, transversal coefficient of the turbulent diffusion  $D_{\perp} = \langle u^2 \rangle T$  and characteristic time  $t_0 = 2T = \langle u^2 \rangle / C_1 \varepsilon$ , where  $C_1$  is nondimensional constant of the order of one,  $\varepsilon$  is the velocity of the turbulent energy dissipation in the inertial subrange taking from the experimental data on a smoke jet at an altitude 100m [9]. According to [12]:  $D_{\perp} = nD_{\perp}$ ,  $n = 15; 30$ .

Coefficients of the turbulent diffusion calculating using the equations (10) and (11) are listed in the table 1 for the velocity  $V_0 = 1-3 \text{ m} \cdot \text{s}^{-1}$  and  $V_0 = 0$ . Analyses show that in the calm case  $D_* = D$  (turb) varies in the interval  $13 \text{ m}^2 \cdot \text{s}^{-1}$  and  $125 \text{ m}^2 \cdot \text{s}^{-1}$ . Increasing the wind velocity the value of  $D_*$  decreases to the interval from  $1 \text{ m}^2 \cdot \text{s}^{-1}$  to  $7 \text{ m}^2 \cdot \text{s}^{-1}$  at  $V_0 = 3 \text{ m} \cdot \text{s}^{-1}$ .



**Fig.1: Normalized concentration distribution of the passive impurities as a function of a distance  $Z$  from a pollutant source eruption in the calm case.**



**Fig. 2: Normalized concentration distribution of the passive impurities as a function of a distance  $Z$  at:  $H = 180$  m,  $n = 15$ ,  $D_{\perp} = 22$  m<sup>2</sup> · s<sup>-1</sup>,  $T = 110$  sec. Stationary case (left figure), nonstationary case  $t = 10$  sec (right figure).**

The curves of the passive pollutant concentration distribution normalizing on a power of the source are illustrated on Figures 1-3 as a function of the distance in the calm case. The dependence on the distance  $z$  of the normalized concentration  $N_0^0(\mathbf{r}, t) / QM$  in the stationary case using the data of  $D_*^0$  and equation (13) is plotted on Figure 1 for an altitude  $H = 100$  m,  $V_0 = 0$ ,  $D_{\perp} = 11$  m<sup>2</sup> · s<sup>-1</sup>,  $n = 15$ ,  $T = 15$  sec,  $l = 150$  m. At first it slowly decreases and since 80 m from the pollutant source it does not vary. The isolines of the constant concentration in the YOZ plane are shown in Figure 2 for both stationary and nonstationary cases using (13).

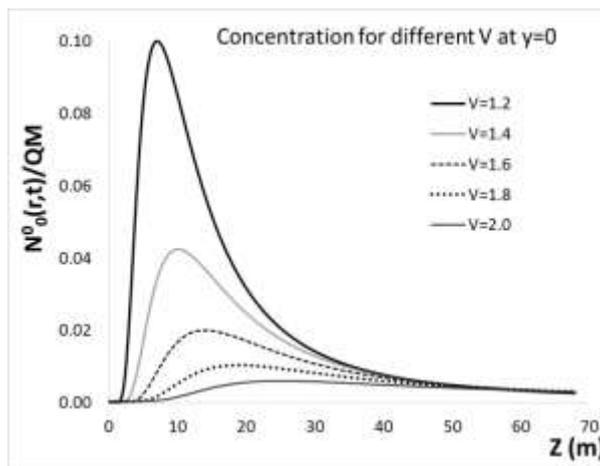


Fig. 3: Normalized concentration distribution of the passive impurities as a function of distance Z for different velocities of the turbulent flow.

Table 1. To the Figure 3

Wind velocity $V_0$ (m/sec)	$N_{0max}^0$	$Z_{max}$	$Z_{min}$
1.2	0.1	6.45	70
1.4	0.042	10.65	100
1.6	0.02	12.85	200
1.8	0.01	17.65	180
2.0	0.006	26.45	130

Table 2. Parameters of the lower atmosphere

N	$l, m$	T, sec	$D_{\perp}, m^2 \cdot s^{-1}$	$D$ (turb), $m^2 \cdot s^{-1}$							
				$V_0 = 0, m \cdot s^{-1}$		$V_0 = 1 m \cdot s^{-1}$		$V_0 = 2 m \cdot s^{-1}$		$V_0 = 3 m \cdot s^{-1}$	
				n=15	n=30	n=15	n=30	n=15	n=30	n=15	n=30
1	150	415	27	34,7	14,1	21,0	5,3	8,90	2,7	4,8	
2	100	470	11	39,1	7,90	12,7	2,5	4,60	1,2	2,3	
3	100	220	25	18,1	9,90	14,0	4,2	6,60	2,3	3,8	
4	160	225	58	18,7	15,8	21,5	7,5	11,2	4,4	7,0	
5	150	240	46	20,1	14,9	20,6	6,7	10,3	3,8	6,2	
6	140	290	36	24,0	13,8	19,6	5,9	9,30	3,2	5,4	
7	160	340	36	28,6	15,6	22,5	6,4	10,3	3,4	5,8	
8	180	630	25	52,8	15,6	24,3	5,4	9,50	2,7	4,9	
9	93	160	27	13,4	9,20	12,9	4,6	6,30	2,3	3,8	
10	193	740	25	61,9	16,3	25,7	5,5	9,80	2,7	5,0	
11	180	1500	11	124,9	9,90	17,6	7,8	5,30	1,3	2,5	
12	190	350	9	31,5	8,6	15,8	2,3	4,5	1,04	2,1	
13	200	500	15	30,0	12,7	22	3,7	7	1,7	3,3	

The curves of the pollutant concentration distribution in stationary case and at different wind velocities in the interval  $V_0 = 1.2 - 2 \text{ m} \cdot \text{s}^{-1}$ ; at  $H = 180 \text{ m}$ ,  $t = 42 \text{ hours}$ ,  $D_{\perp} = 30 \text{ m}^2 \cdot \text{s}^{-1}$ ,  $n = 15$ ,  $T = 180 \text{ sec}$ ,  $l = 150 \text{ m}$  are depicted in Figure 3 using the equation (12). At first concentration increases near the source reaching maximum  $N_{0\text{max}}$  at any height  $Z_{\text{max}}$  and then decreases with the distance  $Z$ . Symmetry of the curves is violated in proportion to the wind velocity. The behavior of these curves is close to the pollutant distribution obtained in [13] in the atmosphere in the presence and absent of the mist on the altitude 100 m.

Investigation of nonstationary distribution of the pollutant concentration at great distances for a source is of particular interest. Appearance, evaluation of isolines corresponding to the normalized concentration of the passive impurities  $N_0^0(\mathbf{r}, t) / QM$  and formation of globules in nonstationary case are illustrated on Figures 4-11 at 35 sec after pollutant emission at the wind velocity  $V_0 = 1 \text{ m} \cdot \text{s}^{-1}$  using the equations (10) and (12) demonstrating wavy character of the nonstationary stream and clarity picture of separate globules formation. Numerical calculations were carried out for the parameters:  $H = 180 \text{ m}$ ,  $D_{\perp} = 9 \text{ m}^2 \cdot \text{s}^{-1}$ ,  $n = 15$ ,  $T = 50 \text{ sec}$ ,  $l = 100 \text{ m}$ .

After eruption from a source in the interval  $t = 35 - 200 \text{ sec}$  isolines # 0.001 - # 0.006 have half ellipse forms. These isolines are stretched along the  $Z$  direction i.e. in the direction of the wind.

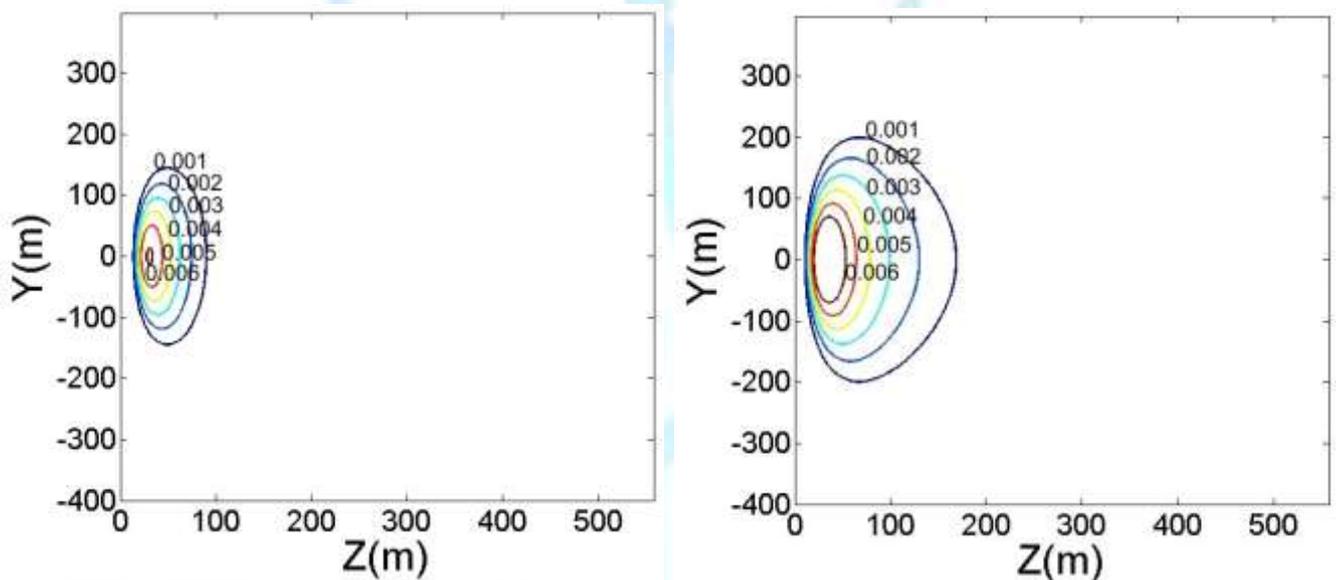


Fig. 4: Passive impurities concentration distribution in the time interval  $t = 35 - 200 \text{ sec}$ .

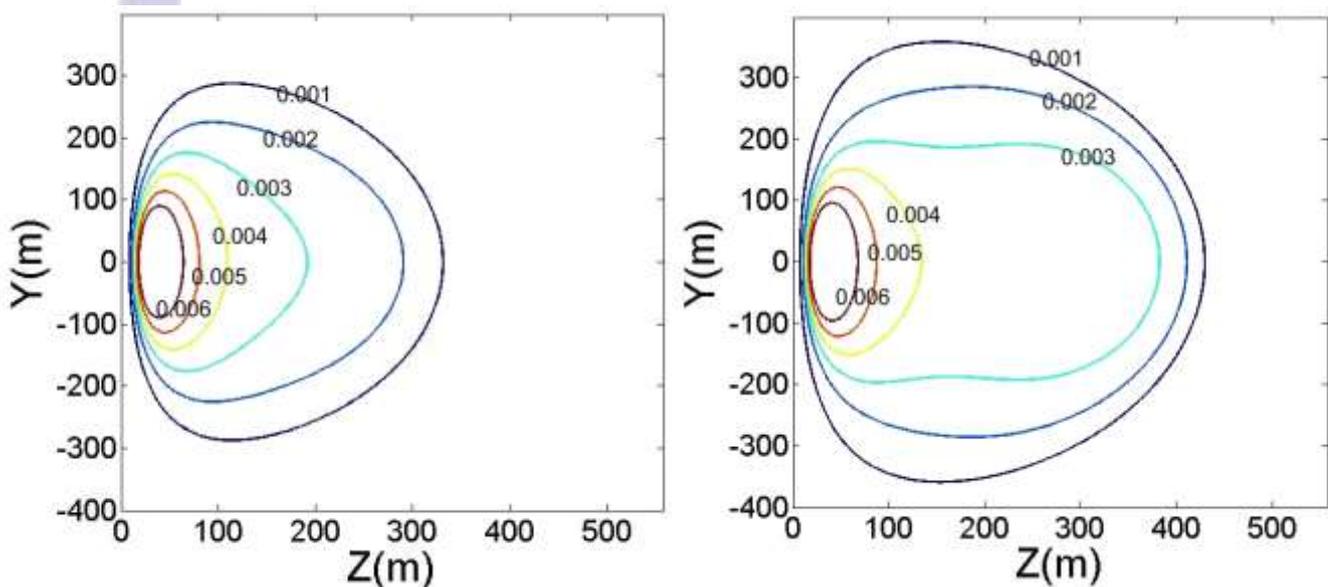


Fig. 5: Evaluation of the concentration isolines in the time interval  $t = 380 - 470 \text{ sec}$ .

The curves corresponding to the passive impurities concentration continue stretching along the wing direction. Isoline # 0.001 covers a distance of 300 m at  $t = 329$  sec moment. Isolines # 0.002 and # 0.003 move slowly following # 0.001. At  $t = 420$  sec moment these curves are extended approximately equally occupying big are of the Earth surface. At  $t = 475$  sec these isolines generate globule (#0.004) of the passive impurity at the distance 300 m from a source having characteristic linear scale 15 m.

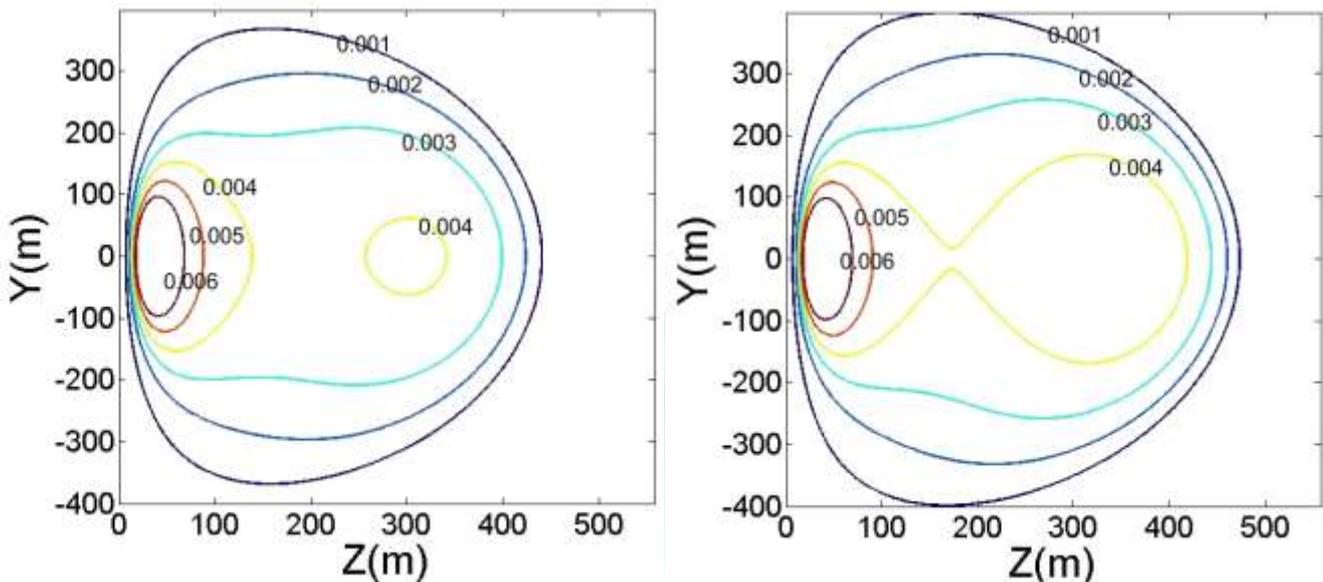


Fig. 6. Evaluation of the concentration isolines in the time interval  $t = 475 - 510$  sec.

This globule continues spreading up to 110 m during 35 sec and then disappears interflow with the wind (life time of this globule is 35 sec) (Figure 6).

New separate globule #0.005 originates at time  $t = 517$  sec and at the distance 360 m from a source (Figure 7).

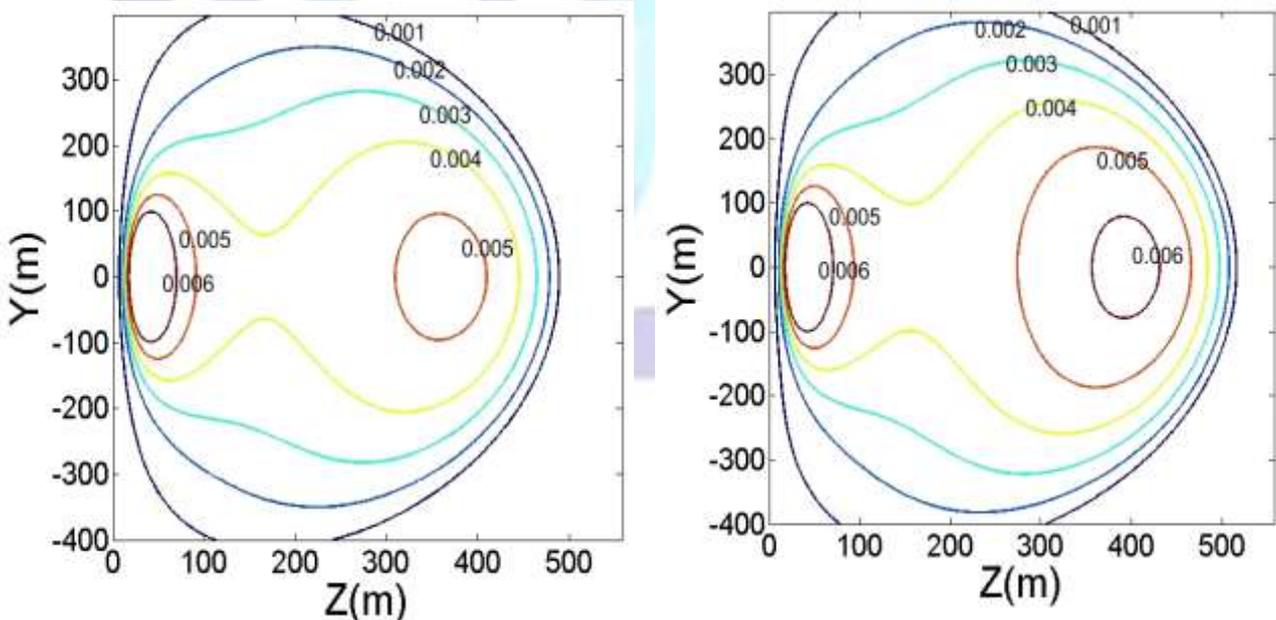


Fig. 7: Evaluation of the concentration isolines in the time interval  $t = 517 - 550$  sec.

Characteristic linear scale is about 40 m, lifetime is 505 sec. This globule extends occupying are 210 m and then disappears. Separate globule #0.006 arises at time  $t = 545$  sec having linear scale 20 m and life time 600 sec (Figure 7). This globule appears at the distance 390 m from a source, extends in 180 m throw area and disappears.

**Table3. Globules parameters**

Concentration N/QM	Time of creation sec	Time of disappearance sec	Lifetime sec	Initial coordinate of the globule formation, m	Diameter of the globules m
0,006	545	1145	600	390	20
0,005	517	1022	505	360	40
0,004	475	510	35	300	15

## CONCLUSION

The statistical model of passive impurity transfer in surface boundary layers of the turbulent atmosphere in the presence of wind is offered. Analytical expressions of the normalized concentration of impurity for arbitrary correlation tensor of the second rank of velocity pulsation when the emission source is located at a certain height over the Earth's surface are obtained. The effective coefficient of turbulent diffusion contains coefficient of molecular diffusion, longitudinal and transverse turbulent diffusion coefficients. Analytical calculations were carried out for the Gaussian correlation function of the velocity pulsations. Numerical calculations were carried out using experimental data of ground(-based) observations. The isolines describing distribution of the passive impurities at calm case (in stationary and non-stationary cases) are depicted at different values of a wind speed and at certain distances from a source. They approximately have a circle shapes. Dynamics of globules formation with various concentration of impurity transferred by wind is constructed. Globules pull away from the main stream and move separately along the wind. They have specific characteristic spatial scales and lifetimes. The offered model can find application at transfer of ashes after volcano eruption and other natural phenomena.

During development and gradual implementation of new technological processes limited, strictly controlled emission of passive impurities in the atmosphere determining by the special techniques, considering scattering of harmful substances, their accumulation and migration can be allowed.

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