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Stress fields in granular material and implications for performance of robot locomotion over granular media

Diandong Ren[1], Lance M. Leslie[2], Congbin Fu[3]
Department of Imaging and Applied Physics, Curtin University, Perth, Western Australia
School of Meteorology, University of Oklahoma, Norman, Oklahoma, USA
School of Atmospheric Sciences, Nanjing University Diandong Ren
Curtin University of Technology, Perth Western Australia
E-mail: Diandong.Ren@curtin.edu.au; rendianyun@gmail.com

Abstract

Legged locomotion of robots has advantages in reducing payload in contexts such as travel over deserts or in planet surfaces. A recent study (Li et al. 2013) partially addresses this issue by examining legged locomotion over granular media (GM). However, they miss one extremely significant fact. When the robot's wheels (legs) run over GM, the granules are set into motion. Hence, unlike the study of Li et al. (2013), the viscosity of the GM must be included to simulate the kinematic energy loss in striking and passing through the GM. Here the locomotion in their experiments is re-examined using an advanced Navier-Stokes framework with a parameterized granular viscosity. It is found that the performance efficiency of a robot, measured by the maximum speed attainable, follows a six-parameter sigmoid curve when plotted against rotating frequency. A correct scaling for the turning point of the sigmoid curve involves the footprint size, rotation frequency and weight of the robot. Our proposed granular response to a load, or the 'influencing domain' concept points out that there is no hydrostatic balance within granular material. The balance is a synergic action of multi-body solids. A solid (of whatever density) may stay in equilibrium at an arbitrary depth inside the GM. It is shown that there exists only a minimum set-in depth and there is no maximum or optimal depth. The set-in depth of a moving robot is a combination of its weight, footprint, thrusting/stroking frequency, surface property of the legs against GM with which it has direct contact, and internal mechanical properties of the GM. If the vehicle's working environment is known, the wheel-granular interaction and the granular mechanical properties can be grouped together. The unitless combination of the other three can form invariants to scale the performance of various designs of wheels/legs. Wider wheel/leg widths increase the maximum achievable speed if all other parameters are unchanged.

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1.Multibody problem

Granular material (GM) is a multibody, loose assemblage of solid granules of the same chemical and physical properties as their non-fractured source material. Because of the increased degree of freedom, the inter-particle interactions, through collisions as well as interfacial actions, can manifest macroscopically as having fluidity while remaining solids individually. The mean free path (measured by average granule dimension), analogous to the Prandtl mixing length in fluid dynamics, is much smaller than for fluid molecules. Moreover, exchanging locations with neighboring particles requires energy expense to overcome friction. This explains how shaking and adding interstitial fluids increase GM fluidity. This limited fluidity explains why a solid body placed on top of a GM bed finds an equilibrium position at any depth. There is no hydrostatic parallel inside GM's internal stress fields. Horizontal pressure may not even be a unanimous function of depth. The pressure field inside the GM is highly anisotropic (Fig. 1). Relationships between stresses in different principal directions are loosely confined by yielding criteria[2]. Although GMs are not true fluids, the resistance forces in separating the granules arise primarily from the gravity of the involved GM, whereas cohesive bonding is weak or non-existent. There are many applications of GM rheology, including landslides research[3] and designing vehicles for desert travel[1].

2.Range of influence

This is an extension of a recent, incomplete model[1]. All numerical experiments use a 3D full Navier Stokes framework[4] with parameterized granular viscosity[5]. We follow Ref. 1 in assuming no cohesion between granular particles: "friction-dominated forces are proportional to the hydrostatic-like pressure in granular media"[1]. Particles are thus assumed to have point-to-point, rather than area-to-area interaction as in reality.

Figure 1 shows how a load/intruder on a bed of an idealized granular substance perturbs stress fields. Because of complex internal stress fields within the GM, resistive forces to the load cannot use the hydrostatic relationship. If the dots, where two vertically adjacent iso-lines (white lines) become parallel and no longer unevenly affected by the load, are connected, an oval surface forms. The space inside this half oval is the "influence/ mobilization domain" (ID). Within the ID, granules experience different degrees of elastic compression. Suppose the largest extra strain rate due to the load is ε , granules that experience strain rate greater than the e-fold value of ε forms a more qualitative definition of ID. In Fig. 1a, due to the concave configuration of the compressed granules, the vertical component of the compression is a source of lift. As the GM weight within the ID cannot translate directly to lift balancing the load weight, equilibrium is provided by shear stress (friction) and normal stress (granule compression). Perturbations from the load cannot be localized. Instead, all granules within the ID are mobilized and work in concert to resist the load. Another lift source is lateral frictional forces exerted on the load by surrounding granules, and may dominate when the ID domain intercepts the boundary. For compacted GM (Fig. 1c), bridging effects may annihilate the vertical pressure and further invalidate the hydrostatic assumption of Ref. 1. However, the ID is a characteristic quantity, and explains the energetics of the load's motion inside the granular media (Fig. 1d). Because the ID envelope follows the intruder if it moves around, ID size is an appropriate invariant for evaluating the performance of robot locomotion with wheels/legs partially set in GM.

Although the load can maintain the force-balance at arbitrary depths exceeding the minimum depth, motion within the GM involves energy loss through irreversible heat dissipation. The rate of energy loss is nonlinearly proportional to ID volume, motion speed, and granular viscosity. The seemingly simple process of slowly placing a load on GM actually involves the gradual increasing of the ID domain, transforming potential energy into heat, and generating lift (Fig. 1a). Consequently, the depth at which the moving robot's wheels/legs stay depends on its weight, wheel/leg footprint and rotation/striking frequency. All are factors affecting the ID size and shape. When the ID does not intercept the boundaries, the eccentricity of the ID oval indicates the ratio of lateral lift force to bottom (head-on) lift force (Fig. 1d). Reducing the footprint decreases the portion of bottom lift in the total lift, explaining why lizards change their leg stances in the pushing and retracting phases of a step on sand (Fig. 1 of Ref. 1).

3. Granular viscosity should be factored in to address robot-granular material interactions

The static properties of dry repose angle and particle size are insufficient to describe the interaction of wheels/legs with GM. The granular viscosity must also be parameterized. Granular viscosity is not mentioned at all in previous recent work[1]. As the frequency increases, the repelled GM simply has no time to response rheologically before the wheels/legs move ahead. Using the parameters, our model reproduces the observed maximum attainable velocity vs. frequency relationship for the 0 to 5*Hz* range. Observational data were unavailable for higher frequencies. From Fig. 2, previous experiments[1] cover only part of the full range of sigmoidal curves. The thick black curve indicates the ideal case of a robot on a rigid, elastic and non-slip track. Because a significant amount of the kinematic energy is transformed into heat through internal friction, as the wheels pass through viscous GM, no cases are close to the ideal case. Counter-intuitively, at higher striking frequencies, the repelled granules have no time to response and act much more rigidly than at lower striking frequencies. Figure 2 also indicates that turning points are sensitive to wheel/leg-width. Greater wheel/leg width increases performance efficiency, and the transition point comes at higher striking frequency. Wheels/legs bearing different track depths encounter granular media of different viscosities because granular viscosity, unlike in true fluids, are also normal pressure dependent. Without granular viscosity parameterization, the portion of kinetic energy lost as heat dissipation is difficult to estimate.

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4. A more suitable scaling invariant

From the ID concept, a more suitable scaling is an invariant based on a dimensionless combination of the weight of the robot locomotor (*G*), the footprint (*A*), and wheel/leg rotation frequency (*f*). For the turning points of the maximum attainable steady speed (V_{max}), we propose $A^2 f\rho / GV_{\max}$ as a unit-less scaling invariant that constrains the wheel/leg rotation frequency and maximum attainable steady velocity at lower frequency range. Another dimensionless combination, $Gf/(\rho AV)$, is suitable for the higher frequency range where dynamic granular viscosity, not the dry repose angle, becomes the dominant factor for motion resistance. The turning points of the sigmoidal curves signify the regime transition. An earlier model[1] applies only to the repose angle dominated regime, corresponding to the low rotation frequency of the legs. The two dimensionless quantities vary only with the GM's mechanical properties. Critical frequencies (Fig. 2, red arrows) are estimated from the continuity requirement at the critical frequencies. For the same robot on the same granular media, the weight *G* and density ρ are fixed. The footprint area is linearly proportional to leg width (*w*). Here we present only the results of the C-leg configuration discussed in an earlier study[1]. Other wheel/leg configurations are qualitatively similar. The same reason lizards walking on sand adjust their stance in the pushing and retracting stages of a step also implies the close relationship between the wheel/leg width in the maximal attainable speed of the robot. For a fair comparision, different wheels/legs should, at standstill, have the same footprint size, but from their Fig. 4c, it is unlikely. The full C-leg case has an advantage because it has the largest footprint. The counter-C legs dig deeper, expending more energy in pulling out and are the least efficient leg shapes.

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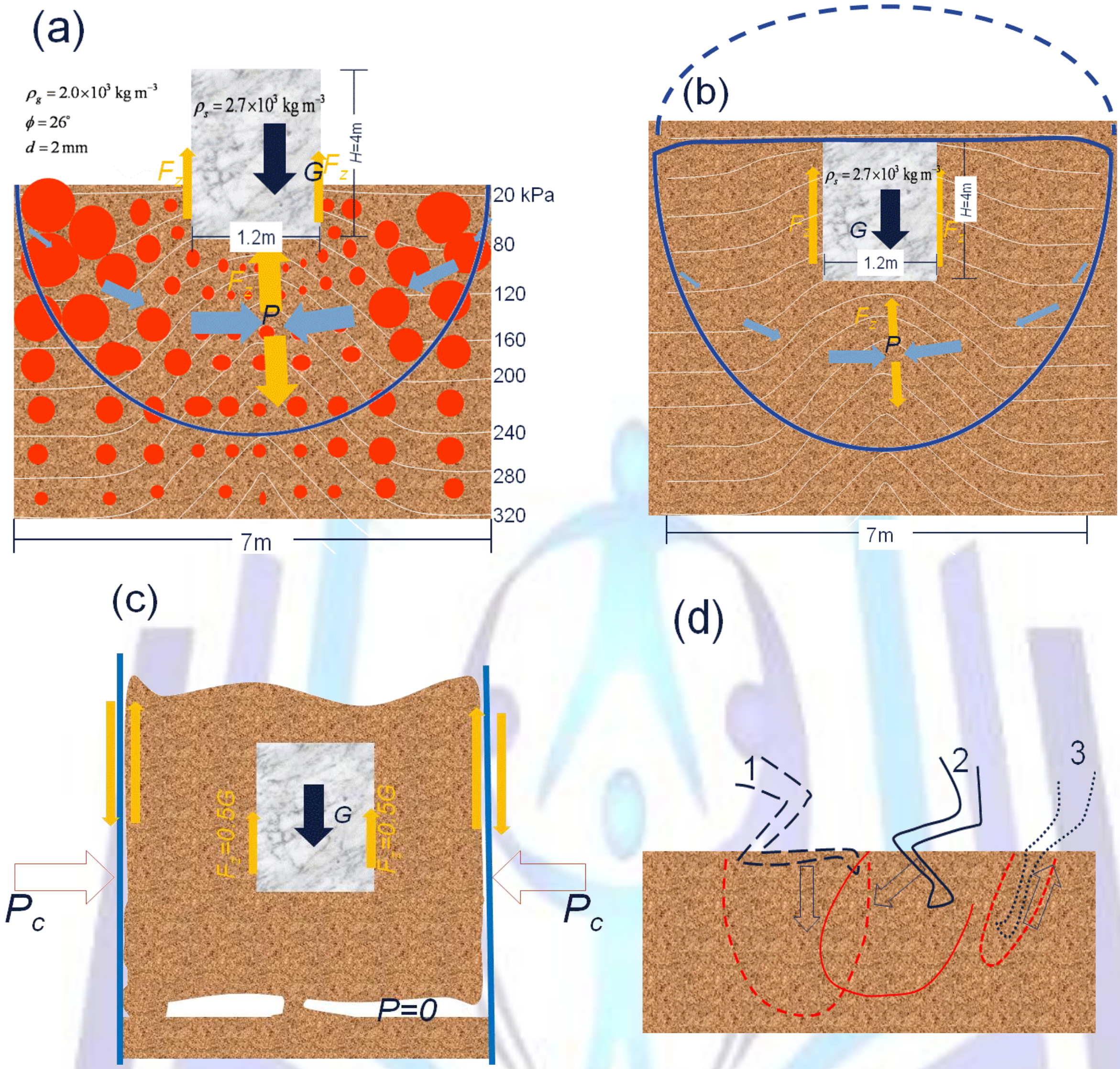

Figure 1. Granular responses to loading, showing 'lift' generation, minimum intruder depth, and domain of influence (ID, the region confined by the bold blue curve). Panel (a) is a cross-section of a load placed on GM. Contour lines are vertical components of compressive stress within the GM. Red blobs indicate elastic granules of identical size when not compressed. Bold arrows are forces; black arrows are gravity, yellow arrows are lifts, and cyan arrows are confining stresses within the ID. Red bubbles of different sizes show degrees of compression (exaggerated). Other GM assumptions follow an earlier study[1]. Panel (b) is identical to (a) except it shows the force balance and range of ID after submersion of the load. The ID of a cylindrical load resembles a downward-pointing half-oval of eccentricity (*e*), a function of weight and footprint area of the load, the dry repose angle of the GM (ϕ) and the fractional coefficients between load and GM. *e* also is an indicator of the ratio of lateral to bottom lift. When the load is moving inside the GM, its ID follows, as explained in the text. Internal granular stress is highly anisotropic with no 'hydrostatic' relationship. Doubling the weight of the load does not double the minimum set-in depth. Vertical pressure is not necessarily greater than the horizontal component, especially for large ϕ or if the ID intercepts the lateral container boundaries. Pressure components in different directions are only weakly constrained by yielding criteria[6]. Panels (a) and (b) are cases with no, or distant horizontal boundaries. For very high confinement pressure (P_c>G $tg\phi$, where G is the total gravity of the involved GM), 'bridging' effects (panel (c)) can cause voids/fissures/crevasses of zero vertical pressure, as lateral friction completely balances the load weight. Panel (d) is the hind foot of a zebra-tailed lizard. Dashed, solid and dotted tracings are leg positions at early-, mid-, and late- stance. Hollow arrows indicate local movement directions of the leg elements. Red lines (of the corresponding styles) indicate the ID at that stance.

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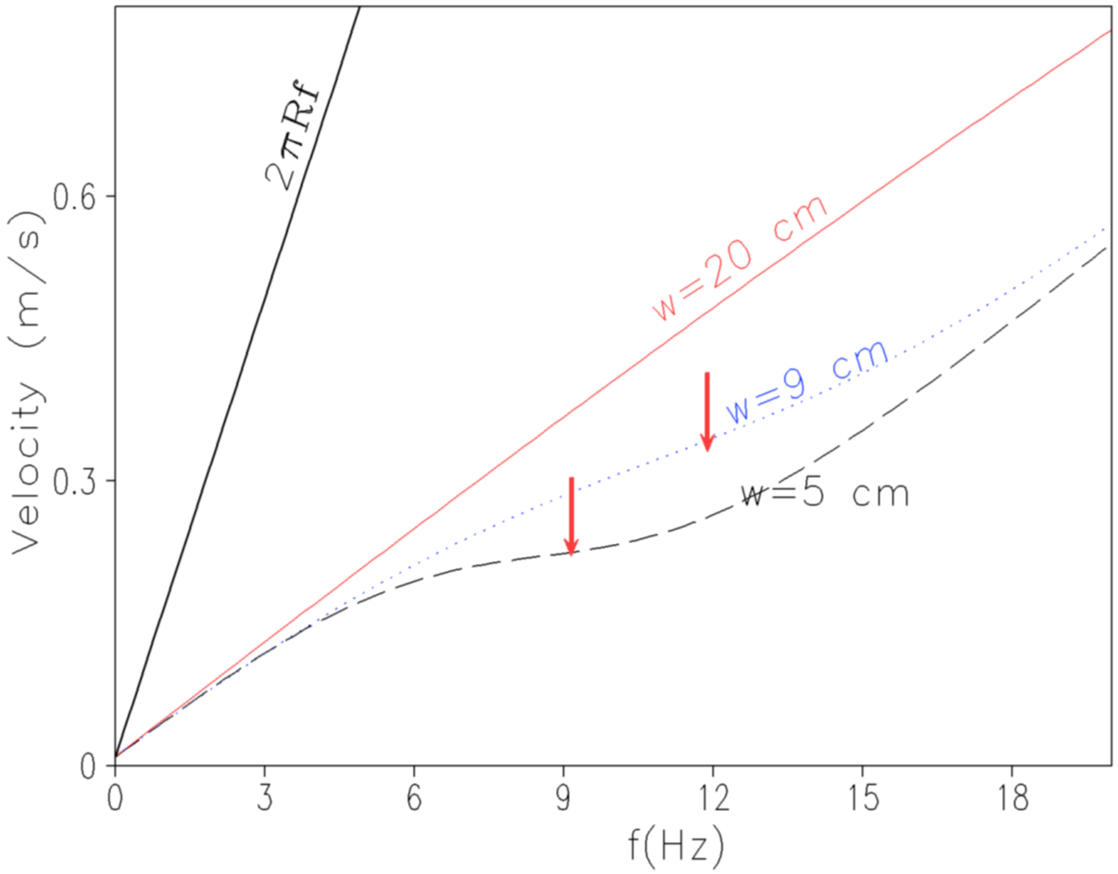

Fig. 2. Robot vehicle performance illustrated using maximum attainable translational speed as a function of wheel/leg striding frequencies. Performance curves for all experiments are sigmoidal but with varying transition turning points (red arrows), satisfying the constraint: $f_c = C_1/(C_2/w-1)$, where C_1 (Hz) and C_2 (m) are constants for the same robot intruder moving inside the same GM. All else the same, transitions occur at higher frequencies, as wheel/leg width increases. The turning point separates two flow regimes: a frictional angle dominated regime at low rotation frequency, and a viscosity dominated regime at high frequency. Parameters are: *G*=0.15 Kg; at *w*=1 cm, foot print $A=3\times10^{-4}$ m^2 (not shown in the figure for clarity); leg length *R'*=2.05 cm; dry repose angle for loosely packed situation $\phi = 36\,\text{degrees}$ Granules are assumed to be in non-cohesive point-to-point contact, and individual granules are not further crushed into smaller particles.