

# Surface Electromagnetic Waves at a Single Interface of Superconductor and Left-Handed Materials

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#### **ABSTRACT**

The wave propagation characteristics along the single interface of superconductor and left-handed materials are investigated theoretically. An expression for the complex permittivity of a superconductor is derived in the approximation of two-component plasma containing "normal" and "superconducting" electrons. Basic relations are obtained in the general case at temperatures  $T \leq T_c$  where  $T_c$  is the critical temperature. The frequency, the structure, and the temperature dependences of surface electromagnetic waves propagating along a single interface of a superconductor-left-handed material interface are computed, analyzed and discussed.

#### **Keywords**

Surface wave; left-handed materials; superconductor; single interface.



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#### 1. INTRODUCTION

In the last decades, tremendos interest has been paid to study the propagation of electromagnetic surface waves propagating at interface between two media or multilayer of different materials theoretically and experimentally [1-5]. The existence of linear surface waves can be achieved under a special condition where the permittivity of one of the media has negative sign, so a surface electromagnetic waves can propagate along a planar interface in such a way that amplitude of the electric and magnetic fields are exponentially decrease away from the interface[2-5]. The field of the surface electromagnetic wave is localized at a boundary layer whose sizes on each side of the interface is a fraction of the operating wavelength.

The surface electromagnetic wave has also been solved for different types of materials, for example dielectrics, semiconductors, Left-handed material, metals, [2-5] and non-linear media [6-9]. It is also found that the surface electromagnetic wave can exist under certain conditions at the surface of uniaxial crystals where the permittivity tensor has a positive component [7], so the condition of the permittivity of one of the adjacent media has negative sign is not necessary. Since the developed of LHM Veselago [6] four dcads ago, the study of the electromagnetic waves in a composite structures containing Left-handed materials has gained more attentions because its refractive index has a negative value in a finite range of frequency. Recently, it has shown, theoretically that if one of the semi-infinite media is a left-handed medium (LHM) then the boundary conditions for the propagation of the surface waves can exist. Until now, the study of the LHM has been focused on their properties at microwave frequencies. However, considering potential technological applications are leading to fabricate LHM at at infrared and optical frequencies [10-15].

For metals, semiconductors, and also LHM, the permittivity is a complex quantity with a negative real part over a wide range of frequency due the contribute of the free charge electrons. This result can create conditions for the generation of surface electromagnetic waves and also introduce specific features into their polarization- and energy-related characteristics [2-5]. A similar situation can be found in superconductors where the temperature of the subsystem of free electrons below the critical point can be described into two-component plasma model as an ensemble of "normal" and "superconducting" electrons [16-17]. In our previous work, novel characteristics of surface waves in a layerd of antiferromagnetic- superconductor waveguide structures[18], and semiconductor-superconductor waveguide[9] have been studied. In this paper, we investigate the characteristics of a surface electromagnetic wave along a superconductor-LHM interface. The formula of the dispersion relation are presented, and the numerical results are given for typical parameters.

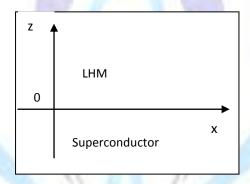


Fig. 1 Diagram of the surface waveguide composed of Superconductor- Left-Handed Material Structure.

#### 2. BASIC CONCEPTS

In Fig.1, we assume that a region z > 0 is occupied by a semifinite LHM with a frequency-dependent permittivity  $\varepsilon_1$  and permeability  $\mu_1$ , while a region z < 0 is filled by a superconductor substrate with a  $\varepsilon_2$ . In the two-fluid model, the total conduction electron in a superconductor is the sum of the super-electron in the lowest state and the normal-electron of the excited superconducting medium characterized by a frequency-dependent (in the general case) permittivity state with densities  $n_s$  and  $n_n$ , respectively. The densities of normal and superconducting electrons are functions of the temperature and according to the two liquid model as  $n = n_n + n_s$  at an arbitrary temperature. Taking into account that the total current in the superconducting medium is defined as  $J = J_n + J_s$  and the amplitude of the electric and magnetic fields of the propagating wave vary with time according to the exponential low (E,H)  $\sim$  exp(i $\omega$ t) we introduce the permittivity of the model from the relations [19]

 $n_n=n\theta^4$ , 9, where  $\theta=T/T_c$  is the normalized temperature and  $T_c$  is the temperature of the phase transition to the superconducting state. Then, at  $T\to 0$ , the density of superconducting electrons is approximately equal to the density of conduction electrons; i.e,  $n_s=n$ . At  $T=T_c$ , the density of normal electrons is equal to the density of conduction



electron  $n = n_n$  which satisfies the condition of the superconducting medium . By using the Maxwell equation, one can get the formula of the permttivity of the superconductor as follows:

$$\vec{\nabla} \times \vec{H} = \frac{i\,\omega}{c} \varepsilon_p E + \frac{4\pi}{c} (J_n + J_s) = ik_o \varepsilon_2 E, \tag{1}$$

Where  $k_o = \omega/c$ ,  $\omega$  is the angular frequency, c is the velocity of light in vacuum, and  $\varepsilon_p$  is the lattice contribution to the total permittivity of the superconductor.  $J_o$  can be derived from the equation of motion, as:

$$\frac{dj_n}{dt} = \frac{n_n e^2}{m} E - \frac{j_n}{\tau} \tag{2}$$

The solution of this equation is:

$$j_n = \frac{n_n e^2 \tau}{m} \frac{E}{1 + i \omega \tau} = \frac{n_n e^2 \tau}{m} \frac{\theta^4 E}{1 + i \omega \tau}$$
(3)

Where  $\tau^{-1} = \nu$  is the frequency of collisions of normal electrons as in Eqs. (2) and (3) and assuming that the frequency of collisions of superconducting electrons is equal to zero ( $\nu = 0$ ), the density of the electric current provided by the superconducting electrons can be written as:

$$j_s = \frac{n_s e^2}{i\omega m} E = \frac{ne^2}{i\omega m} (1 - \theta^4) E \tag{4}$$

Upon substituting relations (3) and (4) into expression (1), we obtain the the complex permittivity of the superconducting medium in the two liquid approximation as[19]:

$$\varepsilon_2 = \varepsilon_p - \frac{\omega_{os}^2}{\omega^2} \left( 1 + \frac{i \, v \theta^4}{\omega - i \, v} \right) \tag{5}$$

Where  $\omega_{os}=(4\pi ne^2/m)^{1/2}$  is the plasma frequency. The lattice contribution at frequencies  $\omega>\omega_{ps}$  where  $\omega_{ps}$  is the frequency of the resonance transition which lies in the optical range and governed by the static permittivity  $\varepsilon_p\approx\omega_o^2/\omega_{ps}^2$ .

It is Known that throuh the frequency range, the permittivity of the superconductor has negative value in the absence of dissipation, making the solutions of the problems in such structure to be nonexistent for bulk waves and existent for surface waves. Below, we will analyze the wave propagation conditions determined by the dispersion relation for waves of the surface type at a superconductor LHM interface taking into account expression derived for the permittivity of the superconductor shown in eq.(5).

At infrared and optical frequencies, the effective magnetic permeability  $\mu_1$ , and the effective permittivity  $\varepsilon_1$  of the LHM are given by the expression[20]:

$$\mu_1 = 1 - \frac{F\omega^2}{\omega^2 - \omega_o^2 + i\Gamma\omega}$$
, and  $\varepsilon_1 = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$  (6)

Where F is the fit factor,  $\Gamma$  is the resonance width,  $\omega_o$  is the resonance frequency,  $\omega_p$  and  $\gamma$  are the plasma and damping frequencies, respectively.

#### 3. DISPERSION RELATION

In this paper, we investigate the characteristics of transverse electromagnetic waves of the TM type propagating along a superconductor LHM interface in direction parallel to the x axis. The components of the wave field as a function of the coordinate x are normally proportional to the factor  $\exp(-ikx)$  where k is the propagation constant of the surface electromagnetic wave. In this case, the system of equations for the components of the magnetic and electric fields in the absence of bulk and surface charges and electric currents in each of the media has the form:



$$d^{2}H_{y}/dz^{2} + (k_{o}^{2}\varepsilon_{j}\mu_{j} - k^{2})H_{y} = 0,$$

$$E_{x} = \frac{i}{k_{o}\varepsilon_{j}\mu_{j}}\frac{dH_{y}}{dz}, E_{z} - \frac{k}{k_{o}\varepsilon_{j}\mu_{j}}H_{y}.$$
(7)

The solution to the first equation of system (7) determines the tangential component of the magnetic field of the surface electromagnetic wave in each of the media and its form can be written as a function of the coordinates and time in the following form:

$$H_{vi} = H_{0i} \exp[i(\omega t - kx) + (-1)^{j} \chi_{i} z],$$
(8)

Where the indices j=1 and 2 refer to the regions z > 0 and z < 0, respectively, and for the case of superconductor  $\mu_2 = 1$ . The transverse components of the wave vector in each medium are related to the propagation constant of the surface electromagnetic wave through the expressions:

$$\chi_1^2 = k^2 - k_o^2 \varepsilon_1 \mu_1$$
, and  $\chi_2^2 = k^2 - k_o^2 \varepsilon_2$ , (9)

In order to drive the dispersion relation, we use the boundary conditions for the tangential components of the electric and magnetic fields  $E_{x1} = E_{x2}$  and  $H_{y1} = H_{y2} = H_0$  at z = 0, respectively, the dispersion relation is then easily obtained:

$$k^{2} = \frac{\omega^{2}}{c^{2}} \frac{\varepsilon_{1} \varepsilon_{2} \left(\mu_{1} \varepsilon_{2} - \varepsilon_{1}\right)}{\varepsilon_{2}^{2} - \varepsilon_{1}^{2}} \tag{10}$$

#### 4. ANALYSIS OF THE DISPERSION RELATION

In our calculation, we consider the permittivity of the superconductor  $\varepsilon_2$ , and  $\varepsilon_1$ ,  $\mu_1$  of LHM are complex quantity leading the propagation constant of the surface electromagnetic wave k and the parameters  $\chi_j$  to be also complex quantity: k = k' - ik'', and  $\chi_i = \chi_i' - i\chi_i''$ .

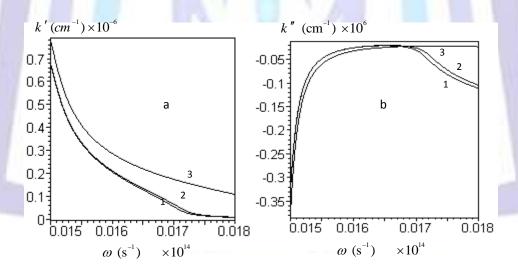


Fig.2 Frequency dependence of (a) the real part and (b) the imaginary part of the constant of propagation of the surface wave at normalized temperature  $\theta$ : (1) 0.3, (2)0.6, (3)

The dispersion relation (10) has been solved to get the propagation constant k. Fig.2 shows the frequency dependence of the real and imaginary parts of the propagation constant k of the surface electromagnetic wave for a waveguide structure with the following parameters:

$$\begin{split} &\mathcal{E}_p = 1.5 \,,\, \omega_p = 7.5 \times 10^{15} s^{-1} \,,\, \omega_{os} = 1.8 \times 10^{16} s^{-1} \,,\, \nu = 6 \times 10^{15} s^{-1} \,,\, \gamma = 2 \times 10^{13} s^{-1} \,,\, F = 0.2 \,,\\ &\Gamma = 2.4 \times 10^{12} s^{-1} \,,\, \omega_o = 1.489 \times 10^{14} s^{-1} \,,\, \text{and} \,\, \theta = 0.3, 0.6,\,\, \text{and} \,\, 0.9 \,\, \text{(curves} \,\, \text{1-3)} \,. \end{split}$$



In (Fig.2.a), it has been shown that throuh the range of the operatinf angular frequency in LHM  $1.5\times10^{14}\,\omega < 1.75\times10^{14}$ , the dispersion curves (1-2) were cutoff of k and the surface waves is no longer propagating, and by increasing the normalization temperature  $\theta$  making the cutoff vanishes. In (Fig.2.b) the imaginary part of the propagation constant k determines the propagation length of the surface wave l=1/k", and we can see that in the higher frequency and  $\theta$ , l are small. It is quite sufficient to detect a surface wave at superconductor-LHM interface. This behavior is totally different in the case of the superconductor-vacuum interface [13].

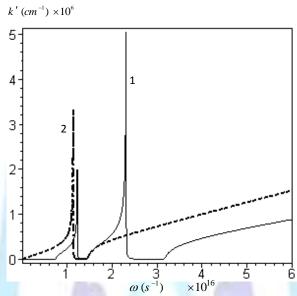


Fig.3 Frequency dependence of the real part of the constant of propagation of the surface wave at: (1)  $\varepsilon_1$  =1(vacuum), (2)  $\varepsilon_L$ ,  $\mu_L$  (LHM).

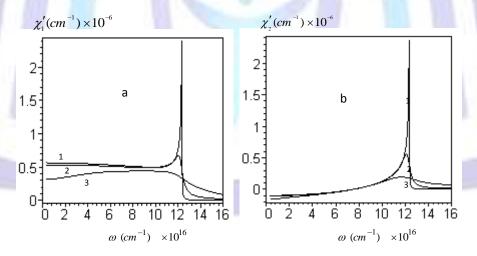


Fig.4 Frequency dependence of the real parts of the transverse components of the surface wave in (a) a superconductor and (b) a LHM at normalized temperature  $\theta$ : (1) 0.3, (2)0.6, (3)0.9.

In Fig.3 where  $\omega > 1.75 \times 10^{14} s^{-1}$  the LHM behaves as a dielectric and there exists of two maximum (curve 1) which is differ from the case where  $\varepsilon = 1$  in the curve 2, because in this case both the superconductor and LHM have the resonance frequencies  $\omega_{as}$  and  $\omega_{a}$  respectively.

The transverse components of the wave vector in each medium can be obtained from the dispersion relation (10) and the equation (9). The real part of transverse compound in Fig.4 determines the degree of localization of the field of the surface electromagnetic wave in the superconducting medium which is greater than in the LHM. It can also be seen that in the



frequency range  $\omega < 1.75 \times 10^{14} s^{-1}$ ,  $\chi_1$  have a negative values, so the medium behaves as a LHM, and if the nonlinear hasa positive value, the medium behaves as a dielectric, one.

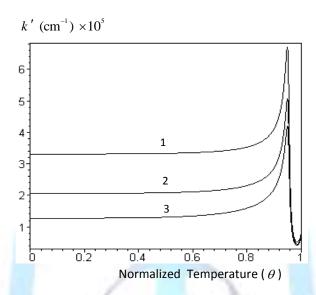


Fig.5 Frequency dependence of the real part of the constant of propagation of the surface wave at different values of the

angular frequency 
$$\omega$$
: (1)  $1.55 \times 10^{14}$ , (2)  $1.6 \times 10^{14}$ , (3)  $1.65 \times 10^{14}$  ( $s^{-1}$ ).

We also see that the degree of localization of the field is strongly dependent on the normalized temperature  $\theta$  in the superconductor medium than its behavior the LHM.

Fig.5 shows the temperature depends of the real part of the propagation constant k. It can be seen that k' is nearly constant with small  $\theta$  and becomes strongly depends on high values of  $\theta$ . We also see that k' has maxima with  $\theta$  and this maxima is shifted to the right with the high operating frequency, and showing some type of bistability.

#### 5. CONCLUSION

This work shows that the surface electromagnetic waves propagating along a superconducting-LHM interface substantially depend not only on the operating frequency and the physical parameters of the structure but also on the temperature of the superconductor and the physical properties of LHM. This results could lead to design a new optoelectronic devices.

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