# ON PARTICLE WAVES SOLUTIONS OF THE EQUATION OF ETHER WAVES <br> DAVID ZARESKI <br> IAI, Israel Aerospace Industries, Yehud, Israël. Former from Université Paul Sabatier, Toulouse France. <br> E-Mail: zareski@inter.net.il 


#### Abstract

In previous publications, we demonstrated that the monochromatic waves $\xi$ associated to particles, (including photons), submitted to forces, (gravitational regarding the photons), are the solutions of the ether elastic wave equation for sufficiently large pulsation $\omega$. In these publications we did not considered the fact that it may exist waves $\xi$ that are the solutions of this wave equation even for small values of $\omega$, like for example waves associated to the free photons. In the present paper, we show cases where waves $\xi$ associated to particles submitted to forces are solutions of this ether wave equation for any values of $\omega$, i.e., even for its small values.

\section*{RESUME}

Dans des publications antérieures, nous démontrâmes que les ondes monochromatiques $\xi$ associées aux particules, (photons compris), soumises a des forces, (gravitationnelles en ce concerne les photons ) sont les solutions de l'équation des ondes d'éther élastiques lorsque la pulsation $\omega$ est suffisamment élevée. Dans ces publication nous n'avons pas considéré le fait qu'il pourrait exister de telles ondes $\xi$ qui soient des solutions de cette équation des ondes même pour de faibles valeurs de $\omega$ comme par exemple les ondes associées aux photons libres. Dans ce présent article, nous montrons des cas ou les $\xi$ associées a des particules soumises a des forces sont des solutions de cette équation des ondes pour n'importe quelles valeurs de $\omega$, i.e., même pour ses faibles valeurs.


## KEYWORDS

waves associated to particle; equation of the ether elastic waves; small pulsation.

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## 1. RECALL OF SOME RESULTS

As in previous publications, we denote by $\xi$ the waves associated to particles that may have a rest mass $m$ and an electrical charge e, and be submitted to a gravitational field $g_{\mu \nu}$ and to an electromagnetic field $A_{\mu}$. We also denoted by $E_{T}$ the particle total energy, and by $V_{P}$ the phase velocity of $\xi$. As shown, e.g., in Ref. [1], $V_{P}$ ensues from the Lagrange-Einstein function $\mathrm{L}_{\mathrm{G}}$ defined by

$$
\begin{equation*}
\mathrm{L}_{\mathrm{G}} \equiv-\mathrm{mc} \dot{\mathrm{~s}}+\mathrm{eA}_{\mu} \dot{\mathrm{x}}^{\mu} / \mathrm{c} \tag{1}
\end{equation*}
$$

and the general expressions for $\mathrm{V}_{\mathrm{P}}$, is the following:

$$
\begin{equation*}
\mathrm{V}_{\mathrm{p}}=\frac{\mathrm{cE}_{\mathrm{T}} \mathrm{~g}_{44}}{\mathrm{eA}_{\mathrm{j}} \mathrm{u}^{\mathrm{j}} \mathrm{~g}_{44}+\left(\mathrm{E}_{\mathrm{T}}+\mathrm{eA}_{4}\right)\left(\mathrm{GB}-\mathrm{g}_{\mathrm{j} 4} \mathrm{u}^{\mathrm{j}}\right)} \tag{2}
\end{equation*}
$$

where $\mathrm{B}, \mathrm{G}, \mathrm{u}^{\mathrm{j}}$, are defined in Sec. III of Ref. [1] and here below for the cases dealt here. In the particular case where $\mathrm{e}=0$ and where the gravitational field is of Schwarzschild, then (2) becomes

$$
\begin{equation*}
\mathrm{V}_{\mathrm{P}}=\mathrm{c} \gamma /\left(\gamma_{\mathrm{a}} \mathrm{~B}\right) \tag{3}
\end{equation*}
$$

where $\gamma, \gamma_{\mathrm{a}}$, and B are defined in the three following equations:

$$
\begin{gather*}
\gamma^{2} \equiv 1-\alpha / \mathrm{r}  \tag{4}\\
\gamma_{\mathrm{a}}^{2} \equiv 1+\alpha\left(\cos ^{2} \mathrm{a}\right) /\left(\mathrm{r} \gamma^{2}\right),  \tag{5}\\
\mathrm{B} \equiv \sqrt{1-\left(\gamma \mathrm{mc}^{2} / \mathrm{E}_{\mathrm{T}}\right)^{2}}, \tag{6}
\end{gather*}
$$

see Sec. V of Ref. [2]. One reminds that in Eq. (5), "a" denotes the angle made by the direction of the trajectory at the current point of it, and by the radius vector issued from the source O of this field and joining this point.

When furthermore $m=0$, i.e., when the particle is a photon, then one has,

$$
\begin{equation*}
\mathrm{V}=\mathrm{V}_{\mathrm{P}}=\mathrm{c} \gamma / \gamma_{\mathrm{a}} \tag{7}
\end{equation*}
$$

where V denotes the photon velocity. The fact that the phase velocity of the photon wave, i.e., of the electromagnetic wave, equals the velocity of the photon is a general property, (ibid).

Now, the general expression for $\xi$ is given by (38) of Ref. [1] namely, by

$$
\begin{equation*}
\xi=\xi_{0} \exp (\mathrm{i} \phi) \tag{8}
\end{equation*}
$$

where $\xi_{0}$ is a vector that depends uniquely upon the spatial coordinates and where the phase $\phi$ is related to the Lagrange-Einstein function $L_{G}$ by the relation

$$
\begin{equation*}
\mathrm{d} \phi=\mathrm{L}_{\mathrm{G}} \mathrm{dt} / \hbar \tag{9}
\end{equation*}
$$

Cf. (13) of Ref. [1]. In the general case, the expression for $\phi$ is, Cf. (14), (ibid),

$$
\begin{equation*}
\left.\phi \equiv(1 / \hbar) \mid-\int \mathrm{E}_{\mathrm{T}} \mathrm{dt}+\int\left(\mathrm{E}_{\mathrm{T}} / \mathrm{V}_{\mathrm{P}}\right) \mathrm{d} \ell\right\rfloor \tag{10}
\end{equation*}
$$

where the spatial integral is taken along the particle trajectory. When the fields to which the particle is submitted are constant, then (10) becomes

$$
\begin{equation*}
\phi \equiv \omega(-t+S) \tag{11}
\end{equation*}
$$

where $\omega$ denotes the constant pulsation defined by

$$
\begin{equation*}
\omega \equiv\left(\mathrm{E}_{\mathrm{T}} / \hbar\right) \tag{12}
\end{equation*}
$$

which, in this case, is constant since $\mathrm{E}_{\mathrm{T}}$ is constant, and S denotes the eikonal defined by

$$
\begin{equation*}
\mathrm{S} \equiv \int \mathrm{~d} \ell / \mathrm{V}_{\mathrm{P}} \tag{13}
\end{equation*}
$$

this integral being taken also along the particle trajectory. Therefore, in this case where the fields to which the particle is submitted are constant, the explicit expression for (8) is

$$
\begin{equation*}
\xi=\xi_{0} \exp [i \omega(-\mathrm{t}+\mathrm{S})] \tag{14}
\end{equation*}
$$

Yet in, e.g., Ref. [1], we demonstrated that the ether is a specific elastic medium in which, in particular, propagate monochromatic waves $\boldsymbol{\Theta}$. In a region void of their generators, these $\boldsymbol{\Theta}$ are the solutions the following Navier-Stokes-Durand wave equation

$$
\begin{equation*}
\operatorname{curl}\left(\mathrm{V}_{\mathrm{P}}^{2} \operatorname{curl} \boldsymbol{\Theta}\right)=\omega^{2} \boldsymbol{\Theta} \tag{15}
\end{equation*}
$$

Then we demonstrated, (ibid), that
i. a wave $\xi$ associated to free photons, i.e., to free masseless particles, is a solution of (15) for any value of the pulsation $\omega$, i.e., even for small values of $\omega$;
ii. a wave $\xi$ associated to particles submitted to forces, including photons submitted to a gravitational field, is a solution of (15) for sufficiently large $\omega$.

## Remark

In Ref. [1], we demonstrated the result ii without trying to search waves $\xi$ that could be solutions of Eq. (15) even for small values of $\omega$. It is here below that we show that there exists waves $\xi$ associated to particles submitted to fields of forces, that are solutions of Eq. (15) for any value of $\omega$,i.e., even for small $\omega$.

## 2. CASES WHERE WAVES ASSOCIATED TO PARTICLES SUBMITTED TO FORCES ARE SOLUTIONS OF THE ETHER WAVE EQUATION EVEN FOR SMALL FREQUENCIES

## 2. 1 GENERALITIES

From here on, $\xi$ will denote monochromatic waves of the form given in (14). We present here below, two cases where $\xi$ are solutions of Eq. (15) for any value of the pulsation $\omega$, i.e., even for small $\omega$. These two cases are the following:
a) $\quad \xi$ is associated to particles submitted to a Schwarzschild field and describe a rectilinear trajectory issued from the source O of this field (this case includes the one where the particle is free);
b) $\quad \xi$ is associated to particles submitted to also a Schwarzschild field but describe a circle centered at $O$.

## 2. 2 CASE WHERE THE TRAJECTORY OF THE PARTICLE SUBMITTED TO A SCHWARZSCHILD FIELD OR FREE, IS RECTILINEAR

When the particle is submitted to a Schwarzschild field, and describes, e.g., the x-axe issued from O , we know that this trajectory is a possible one, then considering Eqs. (3)-(6), and the fact that $\mathbf{a}=0$, Eq. (3) becomes

$$
\begin{equation*}
\mathrm{V}_{\mathrm{P}}=\frac{\mathrm{c} \gamma^{2}}{\sqrt{1-\left(\gamma \mathrm{mc}^{2} / \mathrm{E}_{\mathrm{T}}\right)^{2}}} \tag{16}
\end{equation*}
$$

where furthermore $\gamma^{2}$ and $S$ becomes

$$
\begin{align*}
& \gamma^{2} \equiv 1-\alpha / x  \tag{17}\\
& S=\int d x / V_{P} \tag{18}
\end{align*}
$$

Therefore in this case, $x$ is the sole coordinate upon which $V_{P}$ and $S$ depend. It follows that, after having substituted in (15), $\boldsymbol{\Theta}$ by $\boldsymbol{\xi}$ defined in (14), this Eq. (15) depends also only upon the coordinate x and after simplification of the term $\exp (-i \omega t)$, it becomes

$$
\begin{equation*}
\operatorname{curl}\left[\mathrm{V}_{\mathrm{P}}^{2} \operatorname{curl}\left(\xi_{0} \mathrm{e}^{\mathrm{i} \omega \mathrm{~S}}\right)\right]=\omega^{2} \xi_{0} \mathrm{e}^{\mathrm{i} \omega \mathrm{~S}} \tag{19}
\end{equation*}
$$

Now one has

$$
\mathrm{V}_{\mathrm{P}}^{2} \operatorname{curl}\left(\xi_{0} \mathrm{e}^{\mathrm{i} \omega \mathrm{~S}}\right)=\mathrm{V}_{\mathrm{P}}^{2}\left|\begin{array}{l}
\partial_{\mathrm{x}}  \tag{20}\\
0 \\
0
\end{array} \wedge\left(\mathrm{e}^{\mathrm{i} \omega \mathrm{~S}} \left\lvert\, \begin{array}{l}
\xi_{01} \\
\xi_{02} \\
\xi_{03}
\end{array}\right.\right)=\mathrm{e}^{\mathrm{i} \omega \mathrm{~S}}\right| \begin{aligned}
& 0 \\
& -\left(\mathrm{V}_{\mathrm{P}}^{2} \partial_{\mathrm{x}} \xi_{03}+\mathrm{i} \omega \mathrm{~V}_{\mathrm{P}} \xi_{03}\right) \\
& +\left(\mathrm{V}_{\mathrm{P}}^{2} \partial_{\mathrm{x}} \xi_{02}+\mathrm{i} \omega \mathrm{~V}_{\mathrm{P}} \xi_{02}\right)
\end{aligned}
$$

and

$$
\operatorname{curl}\left[\mathrm{V}_{\mathrm{P}}{ }^{2} \operatorname{curl}\left(\xi_{0} \mathrm{e}^{\mathrm{i} \omega \mathrm{~S}}\right)\right]=\left\lvert\, \begin{aligned}
& \partial_{\mathrm{x}} \\
& 0 \\
& 0
\end{aligned} \wedge\left(\begin{array}{l}
\left.\mathrm{e}^{\mathrm{i} \omega \mathrm{~S}} \left\lvert\, \begin{array}{l}
0 \\
-\left(\mathrm{V}_{\mathrm{P}}^{2} \partial_{\mathrm{x}} \xi_{03}+\mathrm{i} \omega \mathrm{~V}_{\mathrm{P}} \xi_{03}\right) \\
+\left(\mathrm{V}_{\mathrm{P}}^{2} \partial_{\mathrm{x}} \xi_{02}+\mathrm{i} \omega \mathrm{~V}_{\mathrm{P}} \xi_{02}\right)
\end{array}\right.\right)
\end{array}\right)\right.
$$

i.e.,

$$
\operatorname{curl}\left[V_{P}^{2} \operatorname{curl}\left(\xi_{0} e^{i \omega S}\right)\right]=\left\{\begin{array}{l}
0  \tag{21}\\
-\partial_{x}\left[e^{i \omega S}\left(V_{P}^{2} \partial_{x} \xi_{02}+i \omega V_{P} \xi_{02}\right)\right]=\omega^{2} \xi_{0} e^{i \omega S} \\
-\partial_{x}\left[e^{i \omega S}\left(V_{P}^{2} \partial_{x} \xi_{03}+i \omega V_{P} \xi_{03}\right)\right]
\end{array}\right.
$$

It follows the three following equations

$$
\begin{gather*}
\xi_{01}=0  \tag{22}\\
-\partial_{\mathrm{x}}\left[\mathrm{e}^{\mathrm{i} \omega \mathrm{~S}}\left(\mathrm{~V}_{\mathrm{P}}^{2} \partial_{\mathrm{x}} \xi_{02}+\mathrm{i} \omega \mathrm{~V}_{\mathrm{P}} \xi_{02}\right)\right]=\omega^{2} \xi_{02} \mathrm{e}^{\mathrm{i} \omega \mathrm{~S}}  \tag{23}\\
-\partial_{\mathrm{x}}\left[\mathrm{e}^{\mathrm{i} \omega \mathrm{~S}}\left(\mathrm{~V}_{\mathrm{P}}^{2} \partial_{\mathrm{x}} \xi_{03}+\mathrm{i} \omega \mathrm{~V}_{\mathrm{P}} \xi_{03}\right)\right]=\omega^{2} \xi_{03} \mathrm{e}^{\mathrm{i} \omega \mathrm{~S}} \tag{24}
\end{gather*}
$$

The explicit expression for, e.g., Eq. (23), is

$$
\begin{equation*}
i \omega\left(2 \mathrm{~V}_{\mathrm{P}} \partial_{\mathrm{x}} \xi_{02}+\partial_{\mathrm{x}} \mathrm{~V}_{\mathrm{P}} \xi_{02}\right)+\left(\partial_{\mathrm{x}} \mathrm{~V}_{\mathrm{P}}^{2} \partial_{\mathrm{x}} \xi_{02}+\mathrm{V}_{\mathrm{P}}^{2} \partial_{\mathrm{xx}} \xi_{02}\right)=0 \tag{25}
\end{equation*}
$$

that is, since $\xi_{02}$ and $\xi_{03}$ are real, one has the two equations

$$
\begin{gather*}
2 \mathrm{~V}_{\mathrm{P}} \partial_{\mathrm{x}} \xi_{02}+\partial_{\mathrm{x}} \mathrm{~V}_{\mathrm{P}} \xi_{02}=0  \tag{26}\\
2 \mathrm{~V}_{\mathrm{P}} \partial_{\mathrm{x}} \mathrm{~V}_{\mathrm{P}} \partial_{\mathrm{x}} \xi_{02}+\mathrm{V}_{\mathrm{P}}^{2} \partial_{\mathrm{xx}} \xi_{02}=0 \tag{27}
\end{gather*}
$$

Now Eq. (25) implies furthermore that these two equations (26) and (27) must be simultaneously null. Therefore, considering that Eq. (26) can be written also

$$
\begin{equation*}
2 \mathrm{~V}_{\mathrm{P}} \partial_{\mathrm{x}} \mathrm{~V}_{\mathrm{P}} \partial_{\mathrm{x}} \xi_{02}+\left(\partial_{\mathrm{x}} \mathrm{~V}_{\mathrm{P}}\right)^{2} \xi_{02}=0 \tag{28}
\end{equation*}
$$

(27) and (28) have to be simultaneously null. It follows by eliminating the term $2 \mathrm{~V}_{\mathrm{P}} \partial_{\mathrm{x}} \mathrm{V}_{\mathrm{P}} \partial_{\mathrm{x}} \xi_{02}$, between them, that one has the relation

$$
\begin{equation*}
\mathrm{V}_{\mathrm{P}}^{2} \partial_{\mathrm{xx}} \xi_{02}=\left(\partial_{\mathrm{x}} \mathrm{~V}_{\mathrm{P}}\right)^{2} \xi_{02} \tag{29}
\end{equation*}
$$

Equation (29) permits to determine $\xi_{02}$ since $\mathrm{V}_{\mathrm{P}}$ and of course $\partial_{\mathrm{x}} \mathrm{V}_{\mathrm{P}}$ are known from (16) and (17). Obviously one has also the same equations for $\xi_{03}$. Equation (22) shows that $\xi$ is perpendicular to the trajectory, i.e., to the x-axis since $\xi_{01}=0$, but as it ensues from Eq. (29), $\xi_{02}$ and $\xi_{03}$ are not constant on this trajectory.

For the free particle, i.e., where $\mathrm{V}_{\mathrm{P}}$ is constant, (39) shows that in this case $\partial_{\mathrm{xx}} \xi_{02}=\partial_{\mathrm{xx}} \xi_{03}=0$,
this implies that $\xi_{02}$ and $\xi_{03}$ are constant since they cannot increase indefinitely with x .

## 2. 3 CASE WHERE THE TRAJECTORY OF THE PARTICLE SUBMITTED TO A SCHWARZSCHILD FIELD IS A CIRCLE

One considers the case where the particle trajectory denoted $\Sigma$ is a circumference centered at the source O of the field and is contained in the plane $z=0$, we know that this trajectory is also a possible one. In this case one uses the cylindrical coordinates $\mathrm{r}, \varphi, \mathrm{z}$. Now, on $\Sigma$, r is constant, the sole variable is the angle $\varphi$, and the angle "a" in Eq.
(5) takes the value $\mathrm{a}=\pi / 2$, the expression for $\mathrm{V}_{\mathrm{P}}$ is then

$$
\begin{equation*}
\mathrm{V}_{\mathrm{P}}=\mathrm{c} \gamma / \sqrt{1-\left(\gamma \mathrm{mc}^{2} / \mathrm{E}_{\mathrm{T}}\right)^{2}} \tag{30}
\end{equation*}
$$

$V_{P}$ is constant since $r$ is also constant on $\Sigma$, and $S$ defined in (13) becomes simply

$$
\begin{equation*}
\mathrm{S}=\mathrm{r} \varphi / \mathrm{V}_{\mathrm{P}} \tag{31}
\end{equation*}
$$

It follows, denoting

$$
\begin{equation*}
\boldsymbol{\psi} \equiv \xi_{0} \exp \left(\mathrm{i} \omega \mathrm{r} \varphi / \mathrm{V}_{\mathrm{P}}\right) \tag{32}
\end{equation*}
$$

that, in this case, (19) becomes

$$
\begin{equation*}
\operatorname{curl} \operatorname{curl} \psi=\left(\omega / V_{\mathrm{P}}\right)^{2} \psi \tag{33}
\end{equation*}
$$

where, using the cylindrical basis $\mathbf{e}_{\mathbf{r}}, \mathbf{e}_{\varphi}, \mathbf{e}_{\mathrm{z}}$, the components $\xi_{0 r}, \xi_{0 \varphi}, \xi_{0 \mathrm{z}}$ of the vector $\xi_{0}$ defined by

$$
\begin{equation*}
\xi_{0}=\xi_{0 \mathbf{r}} \mathbf{e}_{\mathbf{r}}+\xi_{0 \varphi} \mathbf{e}_{\varphi}+\xi_{0 \mathrm{z}} \mathbf{e}_{\mathrm{z}} \tag{34}
\end{equation*}
$$

are constant by symmetry on $\Sigma$. Now in general, one has in cylindrical coordinates

$$
\operatorname{curl} \mathbf{M}=\left(\frac{1}{r} \partial_{\varphi} M_{z}-\partial_{z} M_{\varphi}\right) \mathbf{e}_{r}+\left(\partial_{z} M_{z}-\partial_{r} M_{z}\right) \mathbf{e}_{\varphi}+\frac{1}{r}\left[\partial_{r}\left(r M_{\varphi}\right)-\partial_{\varphi} M_{r}\right] \mathbf{e}_{z}
$$

Since in this case, the sole variable is the angle $\varphi$, it follows that

$$
\begin{equation*}
\operatorname{curl} \Psi=\frac{1}{\mathrm{r}}\left(\mathbf{e}_{\mathrm{r}} \partial_{\varphi} \Psi_{\mathrm{z}}-\mathbf{e}_{\mathrm{z}} \partial_{\varphi} \Psi_{\mathrm{r}}\right) \tag{35}
\end{equation*}
$$

and also, since the $\xi_{0 \mathrm{r}}, \xi_{0 \varphi}, \xi_{0 \mathrm{z}}$ are constant

$$
\begin{equation*}
\operatorname{curl} \boldsymbol{\Psi}=\mathrm{i}\left(\omega / \mathrm{V}_{\mathrm{P}}\right) \exp \left(\mathrm{i} \omega \mathrm{r} \varphi / \mathrm{V}_{\mathrm{P}}\right)\left(\mathbf{e}_{\mathrm{r}} \xi_{0 \mathrm{z}}-\mathbf{e}_{\mathrm{z}} \xi_{0 \mathrm{r}}\right) \tag{36}
\end{equation*}
$$

and

$$
\begin{equation*}
\text { curl curl } \Psi=\left(\omega / V_{\mathrm{p}}\right)^{2} \exp \left(\mathrm{i} \omega \operatorname{} \varphi / \mathrm{V}_{\mathrm{P}}\right)\left(\xi_{0 \mathrm{r}} \mathbf{e}_{\mathbf{r}}+\xi_{0 \mathrm{z}} \mathbf{e}_{\mathrm{z}}\right) \tag{37}
\end{equation*}
$$

Comparing (37) with (33) taking (32) into account, one sees that $\xi_{0 \varphi}=0$, that is, the vector $\xi_{0}$ is perpendicular to the trajectory i.e., to $\Sigma$.

## REMARK

We have shown in Refs.[ 3], [4] and [5], that the waves $\xi$ associated to these particles that describe a closed loop, interfere with themselves and only are not destructed and even are amplified by this interference waves of specific frequencies, they are the quantum states defined in quantum mechanics, that exists when $r$ and $h \nu$ are defined in, e.g., Eqs, (47) \& (48), of Ref. [3].

## 3. CONCLUSION

In previous publications we have shown that all the waves $\xi$ associated to massive or masseless particles are the solutions of the equation of the ether waves for sufficiently large frequencies, but here we have shown that it exists waves $\xi$ associated to massive or masseless particle, that are solutions of this equation even for small frequencies,.

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