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# Introduction to Quantum Mechanics in Four-Dimensional Space Based on the Theory of Space

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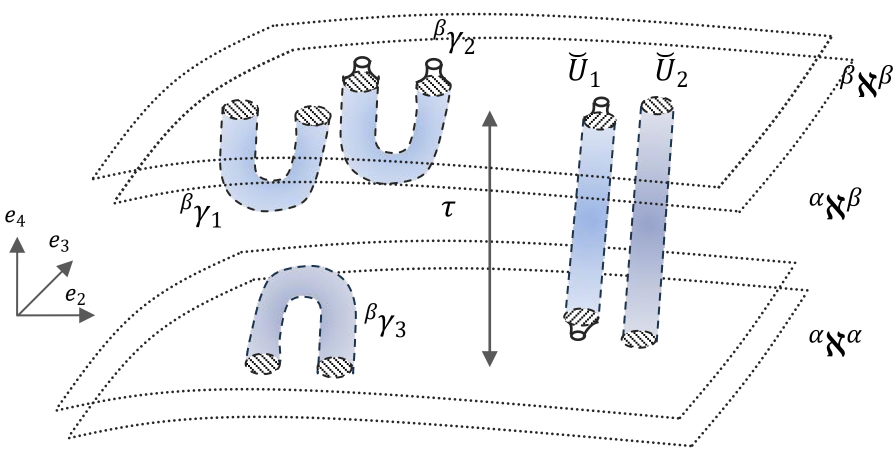
**Abstract**

The article, based on the structure of elementary particles in the form of spatial vortices revealed in the Theory of Space, the four-dimensional structure of space and the essence of electromagnetic interactions, formulates Hamilton and Hamiltonian-Jacobi functions describing the free motion of an elementary particle and the system of elementary particles in a hydrogen atom, approximating elementary particles with a single spatial vortex. It is shown that the complex nutation and precession of a spatial vortex identified with an elementary particle can be identified with matter waves described by transverse de Broglie plane waves of classical quantum mechanics on the boundary hypersurface $^{\beta}\aleph^{\beta}$, which also determines the limit of applicability of classical quantum mechanics, which reduces the complex trajectory of spatial vortices to the analysis of deformations on one of the boundary hypersurfaces.

**Keywords**: quantum mechanics, theory of space, modern quantum mechanics, Hamiltonian function, Hamiltonian-Jacobi function, @LecturesOnTheoreticalPhysics.

## Introduction

The structure of space and elementary particles revealed by the Theory of Space (Sobolewski D. S., Theory of Space, 2016) , (Sobolewski D. S., Theory of Space, 2017) (Sobolewski D. S., Theory of Space, 2024) allows quantum mechanics to be adapted to contemporary challenges posed to science such as controlled thermonuclear fusion, superconductivity, interplanetary travel technology, etc., therefore this article addresses such challenges by being an introduction to modern quantum mechanics describing interacting systems of elementary particles in the form of vortices in four-dimensional space[1]. The Figure 1. shows the structure of photons and quarks, which according to the cited Theory of Space are vortices in four-dimensional space with a layered structure.




[1] As an introduction to this article, it is worth watching the lecture entitled "Hidden knowledge in Physics" available on YouTube on the channel: @LecturesOnTheoreticalPhysics.

Figure 1. The structure of photons ${}^{\beta}\gamma_1, {}^{\beta}\gamma_2$, the Higgs boson ${}^{\beta}\gamma_3$ and quarks $\breve{U}_1, \breve{U}_2$ in four-dimensional space with layers ${}^{\alpha}\aleph^{\alpha}, {}^{\alpha}\aleph^{\beta}, {}^{\beta}\aleph^{\beta}$ (Sobolewski D. S., Theory of Space, 2016) (Sobolewski D. S., Theory of Space, 2017) (Sobolewski D. S., Theory of Space, 2024)

To sum up, in this article the subject of analysis will be vortices in four-dimensional space of type $\breve{U}_1$ and $\breve{U}_2$ shown in Figure 1, for which Hamilton and Hamiltonian-Jacobi functions will be presented in the case of the hydrogen atom and freely moving vortices.

**Materials and Methods**

The presented results are based on the theory entitled "Theory of Space" revealing the structure of space, elementary particles and the nature of all types of interactions (Sobolewski D. S., Theory of Space, 2016) (Sobolewski D. S., Theory of Space, 2017) (Sobolewski D. S., Theory of Space, 2024).

In summary, this publication uses the analytical method and logical structure based on the concepts, models and equations introduced by "Theory of Space", along with the theory entitled "New Generations of Rocket Engines" (Sobolewska, Sobolewska, Sobolewski, Sobolewski, & Sobolewski, New Generations of Rocket Engines, 2021) and „The Structure of Space and its Impact on Interplanetary Travel and Answers to Fundamental Cosmological Questions" (Sobolewska, Sobolewska, Sobolewski, Sobolewski i Sobolewski, The Structure of Space and its Impact on Interplanetary Travel and Answers to Fundamental Cosmological Questions, 2022).

**Results and Discussion**

***Hamiltonian of a single vortex connecting hypersurfaces ${}^{\alpha}\aleph^{\alpha}$ and ${}^{\beta}\aleph^{\beta}$ moving freely***

Based on the Lagrangian function $\mathcal{L}$ given in the Theory of Space (Sobolewski D. S., Theory of Space, 2016), (Sobolewski D. S., Theory of Space, 2017), (Sobolewski D. S., Theory of Space, 2024) and the Legendre'a transformation, an explicit form of the Hamiltonian $H$ of a space channel moving freely with velocity $v$ was given[2]:

$$H(\boldsymbol{p},\boldsymbol{q}) = \frac{{M_{\psi}}^2}{2\,I_{\breve{S}}} + \frac{{M_{\Theta_v}}^2}{2\,I_{\breve{W}}} + \frac{{M_{\phi_{4W_1}}}^2}{2\,I_{\breve{S}}} + \frac{{M_{\phi_{4W_2}}}^2}{2\,I_{\breve{S}}} + \frac{\left(M_{\Theta_{4W_1}} - \cos\Theta_v\, M_{\phi_{4W_1}}\right)^2}{2\,I_{\breve{W}}\,\sin^2\Theta_v} + \frac{\left(M_{\Theta_{4W_2}} - \cos\Theta_v\, M_{\phi_{4W_2}}\right)^2}{2\,I_{\breve{W}}\,\sin^2\Theta_v} + 4m_0c^2\sec\Theta_v\,\sin^4\frac{\Theta_v}{2} \tag{1}$$

, where:

- $\boldsymbol{q} \overset{\text{def}}{=} \left(\Theta_v, \psi, \Theta_{4W_1}, \Theta_{4W_2}, \phi_{4W_1}, \phi_{4W_2}\right)$ - denotes the generalized coordinates in the configuration space $SO(4)$, which are the Euler angles,
- $\boldsymbol{p} \overset{\text{def}}{=} \left(M_{\Theta_v}, M_{\psi}, M_{\Theta_{4W_1}}, M_{\Theta_{4W_2}}, M_{\phi_{4W_1}}, M_{\phi_{4W_2}}\right)$ - denotes the generalized momenta, which in the four-dimensional space are in fact angular momentum,
- $\cos\Theta_v = \sqrt{1-\frac{v^2}{c^2}}$ - the relationship between the orientation of a spatial vortex and its velocity.

**Hamiltonian-Jacobi function of a single vortex connecting hypersurfaces ${}^{\alpha}\aleph^{\alpha}$ and ${}^{\beta}\aleph^{\beta}$ moving freely**

[2] In the cited Theory of Space the meaning of angles $\phi_{4W_i}$ was swapped with angles $\Theta_{4W_i}$, which should not lead to misunderstandings.

Based on the Hamiltonian function from the equation (1), the Hamilton–Jacobi equation for the action function $S(\boldsymbol{q}, t)$ for a freely moving spatial vortex with velocity $v$ is given:

$$\frac{\partial S}{\partial t} = -\frac{1}{2\, I_{\breve{S}}}\left(\frac{\partial S}{\partial \psi}\right)^2 - \frac{1}{2\, I_{\breve{W}}}\left(\frac{\partial S}{\partial \Theta_v}\right)^2 - \frac{1}{2\, I_{\breve{S}}}\left(\frac{\partial S}{\partial \phi_{4W_1}}\right)^2 - \frac{1}{2\, I_{\breve{S}}}\left(\frac{\partial S}{\partial \phi_{4W_2}}\right)^2 - \frac{\left(\frac{\partial S}{\partial \Theta_{4W_1}} - \cos\Theta_v \frac{\partial S}{\partial \phi_{4W_1}}\right)^2}{2\, I_{\breve{W}}\ \sin^2\Theta_v} - \frac{\left(\frac{\partial S}{\partial \Theta_{4W_2}} - \cos\Theta_v \frac{\partial S}{\partial \phi_{4W_2}}\right)^2}{2\, I_{\breve{W}}\, \sin^2\Theta_v} - 4 m_0 c^2 \sec\Theta_v \sin^4\frac{\Theta_v}{2} \tag{2}$$

, where:

- $\Theta_v, \psi, \Theta_{4W_1}, \Theta_{4W_2}, \phi_{4W_1}, \phi_{4W_2}$ – Euler angles in four-dimensional space,
- $\cos\Theta_v = \sqrt{1 - \frac{v^2}{c^2}}$ - the relationship between the orientation of a spatial vortex and its velocity.

**Approximation of classical quantum mechanics**

The essence of the approximation used in classical quantum mechanics is explained, which, abstracting from the complex four-dimensional structure of elementary particles in four-dimensional space, approximates them with de Broglie'a plane waves in the form:

$$\psi(\boldsymbol{r}, t) = A\, exp[i\, (\boldsymbol{k}\, \boldsymbol{r} - \omega t)] \tag{3}$$

, that is, it reduces complex analyses of spatial vortex motion to the description of deformations of the $^{\beta}\aleph^{\beta}$ boundary hypersurface.

Wherein the angular wavenumber in the four dimensional space $\boldsymbol{k} = \sum_{i=1}^{4} k_i e_i = \sum_{i=1}^{4} \frac{2\pi}{\int_0^T v_i dt} e_i$ can be determined for one of the ends of the spatial vortex:

$$\boldsymbol{v}_E = \boldsymbol{v}_{E,S} + *^T\Big(= -\dot{\psi}_E e_1 \wedge e_4 - \left(\dot{\Theta}_{E,4W_1} \cos\Theta_{v_E} + \dot{\phi}_{E,4W_1}\right) e_2 \wedge e_4 - \dot{\Theta}_{E,4W_1} \sin\Theta_{v_E}\, e_2 \wedge e_1 - \dot{\Theta}_{E,4W_2} \sin\Theta_{v_E}\, e_3 \wedge e_1 - \left(\dot{\Theta}_{E,4W_2} \cos\Theta_{v_E} + \dot{\phi}_{E,4W_2}\right) e_3 \wedge e_4 + \dot{\Theta}_{v_E} e_2 \wedge e_3\Big) \wedge \left(e'_4 \frac{1}{2}\sqrt{\frac{1}{3}\frac{h\, \bar{s}}{m_{0,E} c}}\right) \tag{4}$$

, where the components of the velocity vector $\boldsymbol{v}_{E,S}$ are equal to:

$$v_{E,S_1} = r\cos(\Theta_{E,4W_1})\cos(\Theta_{E,4W_2})\dot{\Theta}_{E,4W_1} + \cos\left(\Theta_{E,4W_2}\right)\sin\left(\Theta_{E,4W_1}\right)\dot{r} - r\sin\left(\Theta_{E,4W_1}\right)\sin\left(\Theta_{E,4W_2}\right)\dot{\Theta}_{E,4W_2} \tag{5}$$

$$v_{E,S_2} = r\cos\left(\Theta_{E,4W_2}\right)\sin\left(\Theta_{E,4W_1}\right)\dot{\Theta}_{4W_2} + r\cos\left(\Theta_{E,4W_1}\right)\sin\left(\Theta_{E,4W_2}\right)\dot{\Theta}_{E,4W_1} + \sin\left(\Theta_{E,4W_1}\right)\sin\left(\Theta_{E,4W_2}\right)\dot{r} \tag{6}$$

$$v_{E,S_3} = \cos\left(\Theta_{E,4W_1}\right)\dot{r} - r\sin(\Theta_{E,4W_1})\dot{\Theta}_{E,4W_1} \tag{7}$$

$$v_{E,S_4} = 0 \tag{8}$$

***The Lagrangian function for an electron in the central field of an atom's nucleus***

The publication gives the Lagrangian function $\mathcal{L}_E$ for the electron in a hydrogen atom, which is as follows[3]:

$$\begin{aligned}\mathcal{L}_E = T - U = \frac{1}{2} I_{\breve{S}} \left(\dot{\psi}^2 + \left(\dot{\phi}_{4W_1} + \dot{\Theta}_{4W_1} \cos\Theta_v\right)^2 + \left(\dot{\phi}_{4W_2} + \dot{\Theta}_{4W_2} \cos\Theta_v\right)^2\right) + \\ + \frac{1}{2} I_{\breve{W}} \left(\dot{\Theta}_{4W_1}{}^2 \sin^2\Theta_v + \dot{\Theta}_{4W_2}{}^2 \sin^2\Theta_v + \dot{\Theta}_v{}^2\right) + \frac{1}{2} m_0 \left(\dot{r}^2 + r^2 \dot{\Theta}_{4W_1}{}^2 + r^2 \sin\Theta_{4W_1} \dot{\Theta}_{4W_2}{}^2\right) \\ + \frac{1}{4\,\pi\,\varepsilon\,\varepsilon_0} \frac{\breve{q}^2}{r} - m_0 c^2 (\cos^{-1}\Theta_v + \cos\Theta_v - 2)\end{aligned} \tag{9}$$

, where $\boldsymbol{q} \stackrel{\text{def}}{=} \left(\Theta_v, \psi, \Theta_{4W_1}, \Theta_{4W_2}, \phi_{4W_1}, \phi_{4W_2}, r\right)$.

***Hamilton's function for an electron in the central field of an atom's nucleus***

Based on the Lagrangian function $\mathcal{L}_E$ for the electron, the Hamiltonian function for the electron in the central field of the atomic nucleus is given:

$$\begin{aligned}H_E(\boldsymbol{p},\boldsymbol{q}) = \frac{M_{E,\Theta_v}{}^2}{2I_{E,\breve{W}}} + \frac{M_{E,\psi}{}^2}{2I_{E,\breve{S}}} + \frac{M_{E,\phi_{4W_1}}{}^2}{2I_{E,\breve{S}}} + \frac{p_{E,r}{}^2}{2m_{0,E}} + \frac{M_{E,\phi_{4W_2}}{}^2}{2I_{E,\breve{S}}} + \frac{\left(M_{E,\Theta_{4W_1}} - \cos\Theta_{v_E} M_{E,\phi_{4W_1}}\right)^2}{I_{E,\breve{W}} \sin^2\Theta_{v_E} + m_{0,E}\, r^2} \\ + \frac{\left(M_{E,\Theta_{4W_2}} - \cos\Theta_{v_E} M_{E,\phi_{4W_2}}\right)^2}{I_{E,\breve{W}} \sin^2\Theta_{v_E} + m_{0,E}\, r^2 \sin\Theta_{E,4W_1}} - \frac{1}{2} m_E\, r^2 \left(\frac{M_{E,\Theta_{4W_1}} - \cos\Theta_{v_E} M_{E,\phi_{4W_1}}}{I_{E,\breve{W}} \sin^2\Theta_{v_E} + m_{0,E}\, r^2}\right)^2 \\ - \frac{1}{2} m_{0,E}\, r^2 \sin\Theta_{4W_1} \left(\frac{M_{E,\Theta_{4W_2}} - \cos\Theta_{v_E} M_{E,\phi_{4W_2}}}{I_{E,\breve{W}} \sin^2\Theta_{v_E} + m_{0,E}\, r^2 \sin\Theta_{E,4W_1}}\right)^2 - \frac{1}{4\,\pi\,\varepsilon\,\varepsilon_0} \frac{\breve{q}^2}{r} \\ + 4 m_{0,E} c^2 \sec\Theta_{v_E} \sin^4\frac{\Theta_{v_E}}{2}\end{aligned} \tag{10}$$

**Hamilton's function for the hydrogen atom**

The publication gives an explicit form of the Hamilton function $H_H(\boldsymbol{p},\boldsymbol{q})$ for the hydrogen atom:

[3] We added an accent to the charge quantities notation $\breve{q}$ to distinguish it from generalized coordinates $q$.

$$
\begin{aligned}
H_H(\boldsymbol{p},\boldsymbol{q}) = {} & \frac{{M_{C,\psi}}^2}{2\, I_{C,\breve{S}}} + \frac{{M_{C,\Theta_{v_C}}}^2}{2\, I_{C,\breve{W}}} + \frac{{M_{C,\Theta_{4W_1}}}^2}{2\, I_{C,\breve{S}}} + \frac{{M_{C,\Theta_{4W_2}}}^2}{2\, I_{C,\breve{S}}} + \frac{\left(M_{C,\phi_{4W_1}} - cos\, \Theta_{v_C}\ M_{C,\Theta_{4W_1}}\right)^2}{2\, I_{C,\breve{W}}\ sin^2\, \Theta_{v_C}} \\
& + \frac{\left(M_{C,\phi_{4W_2}} - cos\, \Theta_{v_C}\ M_{C,\Theta_{4W_2}}\right)^2}{2\, I_{C,\breve{W}}\, sin^2\, \Theta_{v_C}} + 4m_{0,C}c^2\, sec\, \Theta_{v_C}\, sin^4 \frac{\Theta_{v_C}}{2} + \frac{{M_{E,\Theta_v}}^2}{2I_{E,\breve{W}}} + \frac{{M_{E,\psi}}^2}{2I_{E,\breve{S}}} \\
& + \frac{{M_{E,\phi_{4W_1}}}^2}{2I_{E,\breve{S}}} + \frac{{p_{E,r}}^2}{2m_{0,E}} + \frac{{M_{E,\phi_{4W_2}}}^2}{2I_{E,\breve{S}}} + \frac{\left(M_{E,\Theta_{4W_1}} - cos\, \Theta_{v_E}\, M_{E,\phi_{4W_1}}\right)^2}{I_{E,\breve{W}}\, sin^2\, \Theta_{v_E} + m_{0,E}\, r^2} \\
& + \frac{\left(M_{E,\Theta_{4W_2}} - cos\, \Theta_{v_E}\, M_{E,\phi_{4W_2}}\right)^2}{I_{E,\breve{W}}\, sin^2\, \Theta_{v_E} + m_{0,E}\, r^2 \sin \Theta_{E,4W_1}} - \frac{1}{2} m_{0,E}\, r^2 \left(\frac{M_{E,\Theta_{4W_1}} - cos\, \Theta_{v_E}\, M_{E,\phi_{4W_1}}}{I_{E,\breve{W}}\, sin^2\, \Theta_{v_E} + m_{0,E}\, r^2}\right)^2 \\
& - \frac{1}{2} m_{0,E}\, r^2 \sin \Theta_{E,4W_1} \left(\frac{M_{E,\Theta_{4W_2}} - cos\, \Theta_{v_E}\, M_{E,\phi_{4W_2}}}{I_{E,\breve{W}}\, sin^2\, \Theta_{v_E} + m_{0,E}\, r^2 \sin \Theta_{E,4W_1}}\right)^2 - \frac{1}{4\, \pi\, \varepsilon\, \varepsilon_0} \frac{\breve{q}^2}{r} \\
& + 4m_{0,E}c^2\, sec\, \Theta_{v_E}\, sin^4 \frac{\Theta_{v_E}}{2}
\end{aligned} \tag{11}
$$

## Hamiltonian-Jacobi equations of the hydrogen atom

The publication also gives an explicit form of the Hamiltonian-Jacobi equation for the action function $S_H(\boldsymbol{q}, t)$, which can be identified with the wave function in classical quantum mechanics[4]:

$$
\begin{aligned}
-\frac{\partial S}{\partial t} = {} & \frac{1}{2\, I_{C,\breve{S}}}\left(\frac{\partial S}{\partial \psi_C}\right)^2 + \frac{1}{2\, I_{C,\breve{W}}}\left(\frac{\partial S}{\partial \Theta_{v_C}}\right)^2 + \frac{1}{2\, I_{C,\breve{S}}}\left(\frac{\partial S}{\partial \Theta_{C,4W_1}}\right)^2 + \frac{1}{2\, I_{C,\breve{S}}}\left(\frac{\partial S}{\partial \Theta_{C,4W_2}}\right)^2 \\
& + \frac{\left(\frac{\partial S}{\partial \phi_{C,4W_1}} - cos\, \Theta_{v_C} \frac{\partial S}{\partial \Theta_{C,4W_1}}\right)^2}{2\, I_{C,\breve{W}}\ sin^2\, \Theta_{v_C}} + \frac{\left(\frac{\partial S}{\partial \phi_{C,4W_2}} - cos\, \Theta_{v_C} \frac{\partial S}{\partial \Theta_{C,4W_2}}\right)^2}{2\, I_{C,\breve{W}}\, sin^2\, \Theta_{v_C}} \\
& + 4m_{0,C}c^2\, sec\, \Theta_{v_C}\, sin^4 \frac{\Theta_{v_C}}{2} + \frac{1}{2I_{E,\breve{W}}}\left(\frac{\partial S}{\partial \Theta\ _{v_E}}\right)^2 + \frac{1}{2I_{E,\breve{S}}}\left(\frac{\partial S}{\partial \psi_E}\right)^2 \\
& + \frac{1}{2I_{E,\breve{S}}}\left(\frac{\partial S}{\partial \phi_{E,4W_1}}\right)^2 + \frac{1}{2m_{0,E}}\left(\frac{\partial S}{\partial r}\right)^2 + \frac{1}{2I_{E,\breve{S}}}\left(\frac{\partial S}{\partial \phi_{E,4W_2}}\right)^2 \\
& + \frac{\left(\frac{\partial S}{\partial \Theta_{E,4W_1}} - cos\, \Theta_{v_E} \frac{\partial S}{\partial \phi_{E,4W_1}}\right)^2}{I_{E,\breve{W}}\, sin^2\, \Theta_v + m_{0,E}\, r^2} + \frac{\left(\frac{\partial S}{\partial \Theta_{E,4W_2}} - cos\, \Theta_{v_E} \frac{\partial S}{\partial \phi_{E,4W_2}}\right)^2}{I_{E,\breve{W}}\, sin^2\, \Theta_{v_E} + m_{0,E}\, r^2 \sin \Theta_{E,4W_1}} \\
& - \frac{1}{2} m_{0,E}\, r^2 \left(\frac{\frac{\partial S}{\partial \Theta_{E,4W_1}} - cos\, \Theta_{v_E} \frac{\partial S}{\partial \phi_{E,4W_1}}}{I_{E,\breve{W}}\, sin^2\, \Theta_{v_E} + m_{0,E}\, r^2}\right)^2 \\
& - \frac{1}{2} m_{0,E}\, r^2 \sin \Theta_{E,4W_1} \left(\frac{\frac{\partial S}{\partial \Theta_{E,4W_2}} - cos\, \Theta_{v_E} \frac{\partial S}{\partial \phi_{E,4W_2}}}{I_{\breve{W}}\, sin^2\, \Theta_{v_E} + m_{0,E}\, r^2 \sin \Theta_{E,4W_1}}\right)^2 - \frac{1}{4\, \pi\, \varepsilon\, \varepsilon_0} \frac{\breve{q}^2}{r} \\
& + 4m_{0,E}c^2\, sec\, \Theta_{v_E}\, sin^4 \frac{\Theta_{v_E}}{2}
\end{aligned} \tag{12}
$$

An explicit form of the Hamiltonian-Jacobi equation for the action function $S'_H(\boldsymbol{q}, t)$ is also given, assuming that the atomic nucleus is stationary and replacing the electron mass $m_{0,E}$ by the reduced mass $m_r = \frac{m_{0,E}\, m_{0,C}}{m_{0,E}+m_{0,C}}$, but without the rest mass present in the component defining the strain energy of the boundary hypersurfaces:

[4] The publication reveals the essence of classical quantum mechanics by referring to the cited Theory of Space.

$$-\frac{\partial S'}{\partial t}=\frac{1}{2I_{E,\breve{W}}}\left(\frac{\partial S'}{\partial \Theta_{v_E}}\right)^2+\frac{1}{2I_{E,\breve{S}}}\left(\frac{\partial S'}{\partial \psi_E}\right)^2+\frac{1}{2I_{E,\breve{S}}}\left(\frac{\partial S'}{\partial \phi_{E,4W_1}}\right)^2+\frac{1}{2m_r}\left(\frac{\partial S'}{\partial r}\right)^2+\frac{1}{2I_{E,\breve{S}}}\left(\frac{\partial S'}{\partial \phi_{E,4W_2}}\right)^2$$
$$+\frac{\left(\frac{\partial S'}{\partial \Theta_{E,4W_1}}-\cos\Theta_{v_E}\frac{\partial S'}{\partial \phi_{E,4W_1}}\right)^2}{I_{E,\breve{W}}\sin^2\Theta_v+m_r r^2}+\frac{\left(\frac{\partial S'}{\partial \Theta_{E,4W_2}}-\cos\Theta_{v_E}\frac{\partial S'}{\partial \phi_{E,4W_2}}\right)^2}{I_{E,\breve{W}}\sin^2\Theta_{v_E}+m_r r^2\sin\Theta_{E,4W_1}}$$
$$-\frac{1}{2}m_r r^2\left(\frac{\frac{\partial S'}{\partial \Theta_{E,4W_1}}-\cos\Theta_{v_E}\frac{\partial S'}{\partial \phi_{E,4W_1}}}{I_{E,\breve{W}}\sin^2\Theta_{v_E}+m_r r^2}\right)^2 \tag{13}$$
$$-\frac{1}{2}m_r r^2\sin\Theta_{E,4W_1}\left(\frac{\frac{\partial S'}{\partial \Theta_{E,4W_2}}-\cos\Theta_{v_E}\frac{\partial S'}{\partial \phi_{E,4W_2}}}{I_{E,\breve{W}}\sin^2\Theta_{v_E}+m_r r^2\sin\Theta_{E,4W_1}}\right)^2-\frac{1}{4\pi\varepsilon\varepsilon_0}\frac{\breve{q}^2}{r}$$
$$+4m_{0,E}c^2\sec\Theta_{v_E}\sin^4\frac{\Theta_{v_E}}{2}$$

***Quantum numbers in the hydrogen atom***

It has been shown that the stationary states in the hydrogen atom are those in which the angular velocities in the configuration space $SO(4)\times SO(4)\times\mathbb{R}^+$ are equal to each other or are multiples of each other, so that after the electron and the nucleus of the atom rotate around the common center of mass after time $t-t_0=T$ the spatial vortices overlap in the phase $\boldsymbol{q}(t)=\boldsymbol{q}(t_0)$ and do not lead to self-interaction of the electron or self-interaction of the nucleus of the atom.

This resulted in the introduction of twelve quantum numbers: $k_{\Theta_{v_C}}\in\mathbb{N}$, $k_{\psi_C}\in\mathbb{N}$, $k_{\Theta_{C,4W_1}}\in\mathbb{N}$, $k_{\Theta_{C,4W_2}}\in\mathbb{N}$, $k_{\phi_{C,4W_1}}\in\mathbb{N}$, $k_{\phi_{C,4W_2}}\in\mathbb{N}$, $k_{\Theta_{v_E}}\in\mathbb{N}$, $k_{\psi_E}\in\mathbb{N}$, $k_{\Theta_{E,4W_1}}\in\mathbb{N}$, $k_{\Theta_{E,4W_2}}\in\mathbb{N}$, $k_{\phi_{E,4W_1}}\in\mathbb{N}$, $k_{\phi_{E,4W_2}}\in\mathbb{N}$.

***Hamiltonian of a single vortex connecting hypersurfaces*** $^{\alpha}\aleph^{\alpha}$ **and** $^{\beta}\aleph^{\beta}$ ***moving freely*** **(Main Text)**

The angular velocity of the vortices $\overset{2}{\omega}$ has the form (Sobolewski D. S., Theory of Space, 2016) (Sobolewski D. S., Theory of Space, 2017), (Sobolewski D. S., Theory of Space, 2024):

$$\overset{2}{\omega}=-\dot{\psi}e_1\wedge e_4-\left(\dot{\Theta}_{4W_1}\cos\Theta_v+\dot{\phi}_{4W_1}\right)e_2\wedge e_4-\dot{\Theta}_{4W_1}\sin\Theta_v\,e_2\wedge e_1-\dot{\Theta}_{4W_2}\sin\Theta_v\,e_3\wedge e_1$$
$$-\left(\dot{\Theta}_{4W_2}\cos\Theta_v+\dot{\phi}_{4W_2}\right)e_3\wedge e_4+\dot{\Theta}_v e_2\wedge e_3 \tag{14}$$

, where $\Theta_v,\psi,\Theta_{4W_1},\Theta_{4W_2},\phi_{4W_1},\phi_{4W_2}$ are the Euler angles in four-dimensional space.

The bivectors in equation (14) of the form $e_i\wedge e_k$ actually denote invariant hyperplanes of vortex rotations, creating not vortex lines but vortex hypersurfaces.

Therefore, for example, the $\dot{\psi}e_1\wedge e_4$ component means rotations around the $e_4$ axis but also around the $e_1$ axis, which is identified with the spin of elementary particles (Sobolewski D. S., Theory of Space, 2016) (Sobolewski D. S., Theory of Space, 2017) (Sobolewski D. S., Theory of Space, 2024).

This means that vortices with $e_1\wedge e_4, e_2\wedge e_4, e_3\wedge e_4$ invariant hypersurfaces can have different dynamics than vortices with $e_2\wedge e_1, e_3\wedge e_1, e_2\wedge e_3$ invariant hypersurfaces, which is denoted by different coefficients[5] in the kinetic energy $T$, namely, respectively, by $I_{\breve{S}}$ and $I_{\breve{W}}$:

[5] These coefficients correspond to the moments of inertia in solid state physics.

$$T = \frac{1}{2} I_{\breve{S}} \left( \dot{\psi}^2 + \left( \dot{\phi}_{4W_1} + \dot{\Theta}_{4W_1} \cos \Theta_v \right)^2 + \left( \dot{\phi}_{4W_2} + \dot{\Theta}_{4W_2} \cos \Theta_v \right)^2 \right) + \\ + \frac{1}{2} I_{\breve{W}} \left( \dot{\Theta}_{4W_1}{}^2 \sin^2 \Theta_v + \dot{\Theta}_{4W_2}{}^2 \sin^2 \Theta_v + \dot{\Theta}_v{}^2 \right) \tag{15}$$

The $\breve{U}_1, \breve{U}_2$ vortices moving freely with velocity $v$ corresponding to quarks connecting different boundary hypersurfaces deform the boundary hypersurfaces and thus oscillate around the equilibrium position $\Theta_{v_0}$ performing precession and nutation identified with matter waves, as shown in Figure 2.

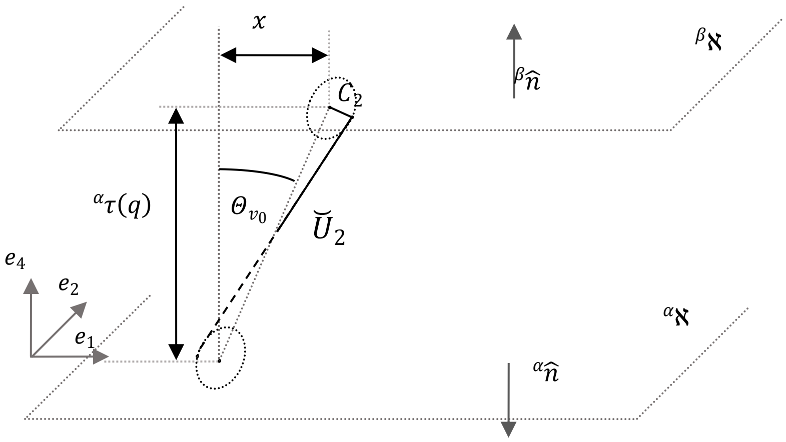



Figure 2. The $\breve{U}_2$ vortex moving freely with speed $\boldsymbol{v} = v\,\boldsymbol{e_1}$ oscillates around the equilibrium position $\Theta_{v_0}$ performing precession and nutation[6] *(Sobolewski D. S., Theory of Space, 2016)*, *(Sobolewski D. S., Theory of Space, 2017)*, *(Sobolewski D. S., Theory of Space, 2024)*.

The energy of deformed boundary hypersurfaces is equal to $E_s = m_0 c^2 (\cos^{-1} \Theta_v + \cos \Theta_v - 2)$ according to the formula (Sobolewski D. S., Theory of Space, 2016) (Sobolewski D. S., Theory of Space, 2017) (Sobolewski D. S., Theory of Space, 2024):

$$E = m_0 c^2 + E_{\Delta J} + E_s \tag{16}$$

, where $E_{\Delta J}$ is the kinetic energy associated with the change of angular momentum orientation and $E_s$ is the strain energy of the boundary hypersurfaces:

$$E_{\Delta J} = m_0 c^2 (1 - \cos \Theta_v) \tag{17}$$

$$E_s = 4 m_0 c^2 \sec \Theta_v \sin^4 \frac{\Theta_v}{2} \qquad \left\{ \Theta_v : 0 \le \Theta_v \le \frac{\pi}{2} \right\} \tag{18}$$

$$\cos \Theta_v = \sqrt{1 - \frac{v^2}{c^2}} \tag{19}$$

The essence of quantum mechanics, as revealed in the publication "Theory of Space" (Sobolewski D. S., Theory of Space, 2016) (Sobolewski D. S., Theory of Space, 2017) (Sobolewski D. S., Theory of Space, 2024), is the

[6] Not all oscillations are shown in the figure.

approximation of the oscillations of moving space channels, while approximating them by means of a wave function with wavelengths found using the de Broglie equation:

$$\lambda = \frac{h}{p} \tag{20}$$

, where $h = 6{,}626 * 10^{-34} Js$.

The Lagrangian function $\mathcal{L}$ determined for a single spatial channel is equal to:

$$\begin{aligned} \mathcal{L} = T - U = \frac{1}{2} I_{\breve{S}} \left(\dot{\psi}^2 + \left(\dot{\phi}_{4W_1} + \dot{\Theta}_{4W_1} \cos\Theta_v\right)^2 + \left(\dot{\phi}_{4W_2} + \dot{\Theta}_{4W_2} \cos\Theta_v\right)^2\right) + \\ + \frac{1}{2} I_{\breve{W}} \left(\dot{\Theta}_{4W_1}{}^2 \sin^2\Theta_v + \dot{\Theta}_{4W_2}{}^2 \sin^2\Theta_v + \dot{\Theta}_v{}^2\right) - m_0 c^2 \left(\cos^{-1}\Theta_v + \cos\Theta_v - 2\right) \end{aligned} \tag{21}$$

, where $\boldsymbol{q} \stackrel{\text{def}}{=} \left(\Theta_v, \psi, \Theta_{4W_1}, \Theta_{4W_2}, \phi_{4W_1}, \phi_{4W_2}\right)$ denote the Euler angles in four-dimensional space, of which the angles $\left(\psi, \Theta_{4W_1}, \Theta_{4W_2}, \phi_{4W_1}, \phi_{4W_2}\right)$ are cyclic coordinates (Sobolewski D. S., Theory of Space, 2016) (Sobolewski D. S., Theory of Space, 2017) (Sobolewski D. S., Theory of Space, 2024).

In order to obtain an explicit form of the Hamiltonian function from the Lagrangian function $\mathcal{L}$, we will apply the Legendre transformation with respect to $\dot{\boldsymbol{q}}$**:**

$$H(\boldsymbol{p}) = \boldsymbol{p}\,\dot{\boldsymbol{q}} - \mathcal{L}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t) \tag{22}$$

, where $\boldsymbol{p}$ is the generalized momentum $\boldsymbol{p} \stackrel{\text{def}}{=} \left(M_{\Theta_v}, M_{\psi}, M_{\Theta_{4W_1}}, M_{\Theta_{4W_2}}, M_{\phi_{4W_1}}, M_{\phi_{4W_2}}\right)$[7] is equal to:

$$\boldsymbol{p} = \frac{\partial \mathcal{L}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t)}{\partial \dot{\boldsymbol{q}}} \tag{23}$$

By taking partial derivatives from equation (23) we get:

$$M_{\Theta_v} = \frac{\partial \mathcal{L}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t)}{\partial \dot{\Theta}_v} = I_{\breve{W}}\, \dot{\Theta}_v \tag{24}$$

$$M_{\psi} = \frac{\partial \mathcal{L}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t)}{\partial \dot{\psi}} = I_{\breve{S}}\, \dot{\psi} \tag{25}$$

$$M_{\Theta_{4W_1}} = \frac{\partial \mathcal{L}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t)}{\partial \dot{\Theta}_{4W_1}} = I_{\breve{W}} \sin^2\Theta_v\, \dot{\Theta}_{4W_1} + \cos\Theta_v\, I_{\breve{S}}\left(\dot{\phi}_{4W_1} + \cos\Theta_v\, \dot{\Theta}_{4W_1}\right) \tag{26}$$

$$M_{\Theta_{4W_2}} = \frac{\partial \mathcal{L}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t)}{\partial \dot{\Theta}_{4W_2}} = I_{\breve{W}} \sin^2\Theta_v\, \dot{\Theta}_{4W_2} + \cos\Theta_v\, I_{\breve{S}}\left(\dot{\phi}_{4W_2} + \cos\Theta_v\, \dot{\Theta}_{4W_2}\right) \tag{27}$$

$$M_{\phi_{4W_1}} = \frac{\partial \mathcal{L}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t)}{\partial \dot{\phi}_{4W_1}} = I_{\breve{S}}\left(\dot{\phi}_{4W_1} + \cos\Theta_v\, \dot{\Theta}_{4W_1}\right) \tag{28}$$

$$M_{\phi_{4W_2}} = \frac{\partial \mathcal{L}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t)}{\partial \dot{\phi}_{4W_2}} = I_{\breve{S}}\left(\dot{\phi}_{4W_2} + \cos\Theta_v\, \dot{\Theta}_{4W_2}\right) \tag{29}$$

[7] In this case we are talking about angular momentum, so the notation M for angular momentum was introduced to clearly distinguish it from the Lagrangian function.

By determining $\dot{\boldsymbol{q}}$ from equations (24), (25), (26), (27), (28) and (29) we obtain (Sobolewski D. S., Theory of Space, 2016) (Sobolewski D. S., Theory of Space, 2017) (Sobolewski D. S., Theory of Space, 2024):

$$\dot{\Theta}_v = \frac{M_{\Theta_v}}{I_{\breve{W}}} \tag{30}$$

$$\dot{\psi} = \frac{M_{\psi}}{I_{\breve{S}}} \tag{31}$$

$$\dot{\Theta}_{4W_1} = \frac{M_{\Theta_{4W_1}} - \cos\Theta_v\, M_{\phi_{4W_1}}}{I_{\breve{W}} \sin^2\Theta_v} \tag{32}$$

$$\dot{\Theta}_{4W_2} = \frac{M_{\Theta_{4W_2}} - \cos\Theta_v\, M_{\phi_{4W_2}}}{I_{\breve{W}} \sin^2\Theta_v} \tag{33}$$

$$\dot{\phi}_{4W_1} = \frac{M_{\phi_{4W_1}}}{I_{\breve{S}}} - \cos\Theta_v\, \dot{\Theta}_{4W_1} \tag{34}$$

$$\dot{\phi}_{4W_2} = \frac{M_{\phi_{4W_2}}}{I_{\breve{S}}} - \cos\Theta_v\, \dot{\Theta}_{4W_2} \tag{35}$$

Substituting the right sides of equations (32) and (33) into equations respectively (34) and (35) , we obtain equations for $\dot{\boldsymbol{q}}$ expressed using the integrals of motion and the variable $\Theta_v$:

$$\dot{\phi}_{4W_1} = \frac{M_{\phi_{4W_1}}}{I_{\breve{S}}} - \cos\Theta_v \frac{M_{\Theta_{4W_1}} - \cos\Theta_v\, M_{\phi_{4W_1}}}{I_{\breve{W}} \sin^2\Theta_v} \tag{36}$$

$$\dot{\phi}_{4W_2} = \frac{M_{\phi_{4W_2}}}{I_{\breve{S}}} - \cos\Theta_v \frac{M_{\Theta_{4W_2}} - \cos\Theta_v\, M_{\phi_{4W_2}}}{I_{\breve{W}} \sin^2\Theta_v} \tag{37}$$

Substituting right sides of equations (30), (31), (32), (33) , (36) and (37) into equation (22), we obtain the explicit form of the Hamiltonian $H$ of the space channel moving freely with velocity $v$:

$$\begin{aligned} H(\boldsymbol{p},\boldsymbol{q}) = \frac{{M_{\psi}}^2}{2\, I_{\breve{S}}} + \frac{{M_{\Theta_v}}^2}{2\, I_{\breve{W}}} + \frac{{M_{\phi_{4W_1}}}^2}{2\, I_{\breve{S}}} + \frac{{M_{\phi_{4W_2}}}^2}{2\, I_{\breve{S}}} + \frac{\left(M_{\Theta_{4W_1}} - \cos\Theta_v\, M_{\phi_{4W_1}}\right)^2}{2\, I_{\breve{W}}\ \sin^2\Theta_v} \\ + \frac{\left(M_{\Theta_{4W_2}} - \cos\Theta_v\, M_{\phi_{4W_2}}\right)^2}{2\, I_{\breve{W}} \sin^2\Theta_v} + 4m_0c^2 \sec\Theta_v \sin^4\frac{\Theta_v}{2} \end{aligned} \tag{38}$$

In the cited Theory of Space the solution to the system of Hamilton's equations is given:

$$\frac{\boldsymbol{dp}}{\boldsymbol{dt}} = \frac{\partial H}{\partial \boldsymbol{q}} \tag{39}$$

$$\frac{\boldsymbol{dq}}{\boldsymbol{dt}} = \frac{\partial H}{\partial \boldsymbol{p}} \tag{40}$$

, using the fact that the angles $(\psi, \Theta_{4W_1}, \Theta_{4W_2}, \phi_{4W_1}, \phi_{4W_2})$ are cyclic coordinates, namely:

$$\Theta_v(t) = \Theta_{v0} + C_2 \sin\omega_0\, t \tag{41}$$

, where $\omega_0$ is equal[8]:

$$\omega_0{}^2 = \frac{(2+\cos 2\,\Theta_{v0})}{I_{\breve{W}}{}^2 \sin^4 \Theta_{v0}}\left(M_{\Theta 4W_1}{}^2 + M_{\Theta 4W_2}{}^2 + M_{\phi 4W_1}{}^2 + M_{\phi 4W_2}{}^2\right) - \frac{(23\cos\Theta_{v0} + \cos 3\,\Theta_{v0})}{4I_{\breve{W}}{}^2 \sin^4 \Theta_{v0}}\left(M_{\Theta 4W_1}M_{\phi 4W_1} + M_{\Theta 4W_2}M_{\phi 4W_2}\right) + \frac{m_0 c^2 (5+\cos 2\,\Theta_{v0})\tan^2\Theta_{v0}}{2I_{\breve{W}}\cos\Theta_{v0}} \tag{42}$$

$$C_2 \approx \frac{2c^2 I_{\breve{W}} m_0}{3\left(\left(M_{\Theta 4W_1} - M_{\phi 4W_1}\right)^2 + \left(M_{\Theta 4W_2} - M_{\phi 4W_2}\right)^2\right)}\Theta_{v0}{}^7 \tag{43}$$

The remaining solutions can be easily determined from equation (42) by substituting $\Theta_v$ into the right side of equation (41), as presented in the Theory of Space (Sobolewski D. S., Theory of Space, 2016) (Sobolewski D. S., Theory of Space, 2017) (Sobolewski D. S., Theory of Space, 2024).

Using the estimated distance between boundary hypersurfaces on the Earth's surface, we have:

$$^{\alpha}\tau \geq \sqrt{\frac{1}{3}\frac{h\,\bar{s}}{m_0 c}} \tag{44}$$

, in which $\bar{s}$ denotes $1[m]$, we can estimate the constant $C_2$ from the formula (Sobolewski D. S., Theory of Space, 2016) (Sobolewski D. S., Theory of Space, 2017) (Sobolewski D. S., Theory of Space, 2024):

$$C_2 \leq {}^{\alpha}\tau\, tg\Theta_{v0} \quad ^{9} \tag{45}$$

To sum up, the space vortices oscillate (nutation) around the equilibrium position $\Theta_{v0}$, between the critical angles $\Theta_1, \Theta_2$, namely: $\Theta_1 <= \Theta_{v0} <= \Theta_2$ and perform precession, which can be determined from the equations:

$$\dot{\phi}_{4W_1} = \frac{M_{\phi 4W_1}}{I_{\breve{S}}} - \cos\Theta_v \frac{M_{\Theta 4W_1} - \cos\Theta_v M_{\phi 4W_1}}{I_{\breve{W}}\sin^2\Theta_v} \tag{46}$$

$$\dot{\phi}_{4W_2} = \frac{M_{\phi 4W_2}}{I_{\breve{S}}} - \cos\Theta_v \frac{M_{\Theta 4W_2} - \cos\Theta_v M_{\phi 4W_2}}{I_{\breve{W}}\sin^2\Theta_v} \tag{47}$$

$$\dot{\Theta}_{4W_1} = \frac{M_{\Theta 4W_1} - \cos\Theta_v M_{\phi 4W_1}}{I_{\breve{W}}\sin^2\Theta_v} \tag{48}$$

$$\dot{\Theta}_{4W_2} = \frac{M_{\Theta 4W_2} - \cos\Theta_v M_{\phi 4W_2}}{I_{\breve{W}}\sin^2\Theta_v} \tag{49}$$

In turn, from equation (31), we have:

[8] In the cited publication "Theory of Space" an approximation based on linearization of the Lagrangian system was also used.

[9] The subscript $v0$ for the equilibrium angle $\Theta$ is not related to the initial velocity $v_0$.

$$\psi(t) = C_\psi + \frac{M_\psi}{I_{\breve{S}}} \tag{50}$$

, from which we can determine the angular velocity $\omega_\psi = \frac{2\pi}{T} = 2\pi \frac{I_{\breve{S}}}{M_\psi}$, assuming $\psi(0) = 0$.

**Hamiltonian-Jacobi function of a single vortex connecting the hypersurfaces $^{\alpha}\aleph^{\alpha}$ and $^{\beta}\aleph^{\beta}$ moving freely**

Using the Hamiltonian function from equation (38) we can write the Hamiltonian–Jacobi equation for the action function $S(\boldsymbol{q}, t)$:

$$\frac{\partial S}{\partial t} = - H(\boldsymbol{p}, \boldsymbol{q}, t) \tag{51}$$

, where $\boldsymbol{p} = \frac{\partial S}{\partial \boldsymbol{q}}$:

$$\frac{\partial S}{\partial t} = -\frac{1}{2\, I_{\breve{S}}}\left(\frac{\partial S}{\partial \psi}\right)^2 - \frac{1}{2\, I_{\breve{W}}}\left(\frac{\partial S}{\partial \Theta_v}\right)^2 - \frac{1}{2\, I_{\breve{S}}}\left(\frac{\partial S}{\partial \phi_{4W_1}}\right)^2 - \frac{1}{2\, I_{\breve{S}}}\left(\frac{\partial S}{\partial \phi_{4W_2}}\right)^2 - \frac{\left(\frac{\partial S}{\partial \Theta_{4W_1}} - \cos\Theta_v \frac{\partial S}{\partial \phi_{4W_1}}\right)^2}{2\, I_{\breve{W}}\ \sin^2\Theta_v} - \frac{\left(\frac{\partial S}{\partial \Theta_{4W_2}} - \cos\Theta_v \frac{\partial S}{\partial \phi_{4W_2}}\right)^2}{2\, I_{\breve{W}} \sin^2\Theta_v} - 4 m_0 c^2 \sec\Theta_v \sin^4\frac{\Theta_v}{2} \tag{52}$$

, where:

- $\Theta_v, \psi, \Theta_{4W_1}, \Theta_{4W_2}, \phi_{4W_1}, \phi_{4W_2}$ – Euler angles in $SO(4)$,
- $\cos\Theta_v = \sqrt{1 - \frac{v^2}{c^2}}$ - the relationship between the orientation of a spatial vortex and its velocity.

The estimated $I_{\breve{W}}$ in the book "Theory of Space" is (Sobolewski D. S., Theory of Space, 2016) (Sobolewski D. S., Theory of Space, 2017) (Sobolewski D. S., Theory of Space, 2024):

$$I_{\breve{W}} < \frac{3c^2h^2}{2\pi^2 m_0 v^4}\left(1 - \frac{v^2}{c^2}\right) \tag{53}$$

The solution of the nonlinear partial equation (44) is the action function $S(\Theta_v, \psi, \Theta_{4W_1}, \Theta_{4W_2}, \phi_{4W_1}, \phi_{4W_2}, t)$, which can be identified with the wave function $\breve{\psi}(\Theta_v, \psi, \Theta_{4W_1}, \Theta_{4W_2}, \phi_{4W_1}, \phi_{4W_2}, t)$ used in classical quantum mechanics[10].

Let's write equation (52) in operator form

$$-\frac{\partial \breve{\psi}}{\partial t} = \hat{H}\breve{\psi} \tag{54}$$

, where:

[10] We have added an accent to the notation of the wave function $\breve{\psi}$ in order to distinguish it from the generalized coordinate $\psi$.

$$\widehat{H} = \frac{1}{2\, I_{\breve{S}}}\left(\frac{\partial S}{\partial \psi}\right)^2 + \frac{1}{2\, I_{\breve{W}}}\left(\frac{\partial S}{\partial \Theta_v}\right)^2 + \frac{1}{2\, I_{\breve{S}}}\left(\frac{\partial S}{\partial \phi_{4W_1}}\right)^2 + \frac{1}{2\, I_{\breve{S}}}\left(\frac{\partial S}{\partial \phi_{4W_2}}\right)^2 + \frac{\left(\frac{\partial S}{\partial \Theta_{4W_1}} - \cos\Theta_v \frac{\partial S}{\partial \phi_{4W_1}}\right)^2}{2\, I_{\breve{W}}\, \sin^2\Theta_v} + \frac{\left(\frac{\partial S}{\partial \Theta_{4W_2}} - \cos\Theta_v \frac{\partial S}{\partial \phi_{4W_2}}\right)^2}{2\, I_{\breve{W}}\, \sin^2\Theta_v} + 4 m_0 c^2 \sec\Theta_v \sin^4\frac{\Theta_v}{2} \tag{55}$$

Note that in the Hamiltonian (55) there appear mixed partial derivatives $\frac{\partial}{\partial \Theta_{4W_1}}\frac{\partial}{\partial \phi_{4W_1}}$ and $\frac{\partial}{\partial \Theta_{4W_2}}\frac{\partial}{\partial \phi_{4W_2}}$, which makes it impossible to reduce the solution of the nonlinear differential equation (54) to the form:

$$-\frac{\partial S(\boldsymbol{r},t)}{\partial t} = \frac{\left(\nabla S(\boldsymbol{r},t)\right)^2}{2\,\mu} + U(\boldsymbol{r}) \tag{56}$$

, which is cited in many publications[11] emphasizing the differences between quantum and classical mechanics to the detriment of classical mechanics, while it is the other way around provided that the four-dimensional structure of space and elementary particles is taken into account.

**Approximation used in classical quantum mechanics**

As already mentioned in the introduction, the vortices $\breve{U}_i$ moving freely with velocity v deform the boundary hypersurfaces and thus oscillate around the equilibrium position $\Theta_{v_0}$ performing precession and nutation identified with matter waves, as shown in Figure 1.

Therefore, reducing the description of the vortex trajectories $\breve{U}_i$ to points located on the three-dimensional boundary hypersurface $^{\beta}\aleph^{\beta}$ and taking into account the deformation of this hypersurface itself, we can speak of their wave-like propagation, which is consistent with the approximation used by classical quantum mechanics[12] using plane de Broglie waves.

Let us note that the wave function $\breve{\psi}$ for a freely moving vortex with velocity $v$, i.e. inclined at an angle $\Theta_{v0} = arcsin\left(\frac{v}{c}\right)$, cannot be represented in the form:

$$\breve{\psi} = exp[\imath\,(\boldsymbol{p}\,\boldsymbol{q} - E\,t)] \tag{57}$$

, where:

- $\boldsymbol{q} = \left(\Theta_v, \psi, \Theta_{4W_1}, \Theta_{4W_2}, \phi_{4W_1}, \phi_{4W_2}\right)$,
- $\boldsymbol{p} = \left(M_{\Theta_v}, M_\psi, M_{\Theta_{4W_1}}, M_{\Theta_{4W_2}}, M_{\phi_{4W_1}}, M_{\phi_{4W_2}}\right)$.

**Kinetic energy of the centre of gravity of a spatial vortex in four-dimensional space**

Let us determine the kinetic energy $T_\omega$ of a single spatial vortex of mass m in the spherical coordinate system restricted to a three-dimensional subspace, as shown in Figure 3.

[11] See the chapter entitled "The Relationship of Quantum Mechanics to Classical Mechanics" (Dawydow, 1967).
[12] See the chapter entitled "The Essence of the Quantum Mechanics" in „Theory of Space" (Sobolewski D. S., Theory of Space, 2016) (Sobolewski D. S., Theory of Space, 2017) (Sobolewski D. S., Theory of Space, 2024).

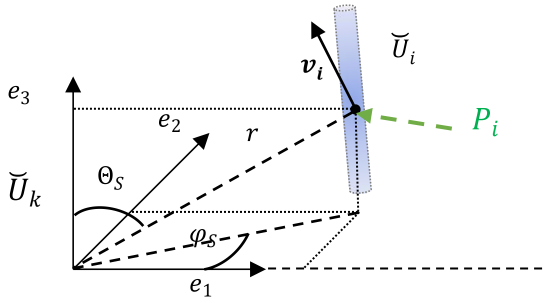



Figure 3. Description of the motion of the centre of gravity $P_i$ of the spatial vortex $\breve{U}_i$ in spherical coordinates

In the Cartesian coordinate system the kinetic energy $T_\omega$ is equal to:

$$T_\omega = \frac{1}{2} m\left(\dot{x}_1{}^2 + \dot{x}_2{}^2 + \dot{x}_3{}^2\right) \tag{58}$$

In turn, Cartesian coordinates are expressed in the spherical coordinate system as follows:

$$x_1 = r\, Sin[\Theta_S]\, Cos[\phi_S] \tag{59}$$

$$x_2 = r\, Sin[\Theta_S] Sin[\phi_S] \tag{60}$$

$$x_3 = r\, Cos[\Theta_S] \tag{61}$$

$$\dot{x}_1 = r\mathrm{Cos}[\Theta_S]\mathrm{Cos}[\phi_S]\dot{\Theta}_S + \mathrm{Cos}[\phi_S]\mathrm{Sin}[\Theta_S]\dot{r} - r\mathrm{Sin}[\Theta_S]\mathrm{Sin}[\phi_S]\dot{\phi}_S \tag{62}$$

$$\dot{x}_2 = rCos[\phi_S]Sin[\Theta_S]\dot{\phi}_S + rCos[\Theta_S]Sin[\phi_S]\dot{\Theta}_S + Sin[\Theta_S]Sin[\phi_S]\dot{r} \tag{63}$$

$$\dot{x}_3 = Cos[\Theta_S]\dot{r} - rSin[\Theta_S]\dot{\Theta}_S \tag{64}$$

, where we will accept $\Theta_S \in < 0, \pi)$ and $\phi_S \in < 0, 2\pi)$

Ultimately, we get:

$$T_\omega = \frac{1}{2} m\left(\dot{x}_1{}^2 + \dot{x}_2{}^2 + \dot{x}_3{}^2\right) = \frac{1}{2} m\left(\dot{r}^2 + r^2\dot{\Theta}_S{}^2 + r^2\, \mathrm{Sin}[\Theta_S]\, \dot{\phi}_S{}^2\right) \tag{65}$$

Applying the Legendre transformation to $T_\omega(\dot{\boldsymbol{q}})$ with respect to $\dot{\boldsymbol{q}} = \left(\dot{r}, \dot{\Theta}_S, \dot{\phi}_S\right)$:

$$T_\omega(\boldsymbol{p}) = \boldsymbol{p}\,\dot{\boldsymbol{q}} - T_\omega(\dot{\boldsymbol{q}}) \tag{66}$$

and assuming $\boldsymbol{p} = \left(p, M_\Theta, M_\phi\right)$, we get:

$$p = m\,\dot{r}\,, \qquad \dot{r} = \frac{p}{m} \tag{67}$$

$$M_\Theta = m\, r^2\, \dot{\Theta}_S\,, \qquad \dot{\Theta}_S = \frac{M_\Theta}{m\, r^2} \tag{68}$$

$$M_{\phi} = m\, r^2\, \mathrm{Sin}[\Theta_S]\, \dot{\phi}_S \;, \qquad \dot{\phi}_S = \frac{M_{\phi}}{m\, r^2\, \mathrm{Sin}[\Theta_S]} \tag{69}$$

Substituting $\dot{\boldsymbol{q}}$ into formula (66) expressed in terms of generalized momenta we have:

$$T_{\omega}(\boldsymbol{p},\boldsymbol{q}) = \frac{p^2}{2\, m} + \frac{{M_{\Theta}}^2}{2\, m\; r^2} + \frac{{M_{\phi}}^2}{2\, m\, r^2 \mathrm{Sin}[\Theta_S]} \tag{70}$$

## Hydrogen atom

We will simplify the analysis of the hydrogen atom using the approximation in which the structure of the electron and proton is reduced to single spatial vortices in a four-dimensional space of the type $\breve{U}_1$ or $\breve{U}_2$ shown in the figure Figure 1.

The configuration space of the hydrogen atom nucleus is a Cartesian product $SO(4) \times SO(4) \times \mathbb{R}^+ \times < 0, 2\pi) \times < 0, \pi)$, which means that any position can be denoted as $\boldsymbol{q}$:

$$\boldsymbol{q} = \left(\boldsymbol{q}_{\boldsymbol{E_C}}, \boldsymbol{q}_{\boldsymbol{E_E}}, \boldsymbol{q}_{\boldsymbol{S}}\right) = \begin{pmatrix} \Theta_{C,\ v_C}, \psi_C, \Theta_{C,4W_1}, \Theta_{C,4W_2}, \phi_{C,4W_1}, \phi_{C,4W_2}, \\ \Theta_{E,\ v_E}, \psi_E, \Theta_{E,4W_1}, \Theta_{E,4W_2}, \phi_{E,4W_1}, \phi_{E,4W_2}, \\ r, \phi_S, \Theta_S \end{pmatrix} \tag{71}$$

, where the subscript $C$ denotes the generalized coordinates of the nucleus of the hydrogen atom and the subscript E denotes the generalized coordinates of the electron, i.e. the extended Euler angles for the case of four-dimensional space, and the spherical coordinates of the electron relative to the nucleus of the atom by $r, \phi_S, \Theta_S$ [13].

Due to the definition of Euler angles in four-dimensional space, let us assume the following relations between angles:

$$\Theta_{4W_1} = \Theta_S \tag{72}$$

$$\Theta_{4W_2} = \phi_S \tag{73}$$

, which follows directly from the definition of the angle $\Theta_{4W_1}$, as the angle measured around the invariant hyperplane $e_2 \wedge e_4$ and the angle $\Theta_{4W_2}$ as the angle measured around the invariant hyperplane $e_3 \wedge e_4$.

To sum up, the configuration space of the system of interacting elementary particles in the hydrogen atom is simplified to the form $\boldsymbol{q} \in SO(4) \times SO(4) \times \mathbb{R}^+$:

$$\boldsymbol{q} = \left(\boldsymbol{q}_{\boldsymbol{E_C}}, \boldsymbol{q}_{\boldsymbol{E_E}}, \boldsymbol{q}_{\boldsymbol{S}}\right) = \begin{pmatrix} \Theta_{C,\ v_C}, \psi_C, \Theta_{C,4W_1}, \Theta_{C,4W_2}, \phi_{C,4W_1}, \phi_{C,4W_2}, \\ \Theta_{E,\ v_E}, \psi_E, \Theta_{E,4W_1}, \Theta_{E,4W_2}, \phi_{E,4W_1}, \phi_{E,4W_2}, \\ r \end{pmatrix} = \tag{74}$$

, where $\Theta_{E,4W_1} = \Theta_S \in < 0{,}2\pi)$[14] and $\Theta_{E,4W_2} = \phi_S \in < 0{,}2\pi)$.

[13] Coordinates describe the motion of an electron around the nucleus of an atom.

[14] We have not reduced the interval to π as we did when recalling spherical coordinates, because we are considering the space SO(4) and not $E^3$.

It is important to remember the relationships between angles in the spherical coordinate system and Euler angles, as shown in the equations (72) and (73).

Thus, the hydrogen atom Hamiltonian $H_H(\boldsymbol{p},\boldsymbol{q})$ is in the general case the sum of the hydrogen nucleus spin Hamiltonian $H_C(\boldsymbol{p},\boldsymbol{q})$ from equation (38) and the electron Hamiltonian $H_E(\boldsymbol{p},\boldsymbol{q})$ which must be determined from the Lagrangian function $\mathcal{L}_E$ in the form presented in equation (21) supplemented with the kinetic energy of the electron moving around the nucleus $T_\omega(\boldsymbol{p},\boldsymbol{q})$ from equation (70):

$$H_H(\boldsymbol{p},\boldsymbol{q}) = H_C(\boldsymbol{p},\boldsymbol{q}) + H_E(\boldsymbol{p},\boldsymbol{q}) \tag{75}$$

The Lagrange function $\mathcal{L}_E$ for an electron in a hydrogen atom is as follows[15]:

$$\begin{gathered}\mathcal{L}_E = T - U = \frac{1}{2} I_{\breve{S}}\left(\dot{\psi}^2 + \left(\dot{\phi}_{4W_1} + \dot{\Theta}_{4W_1} \cos\Theta_v\right)^2 + \left(\dot{\phi}_{4W_2} + \dot{\Theta}_{4W_2}\cos\Theta_v\right)^2\right) + \\ + \frac{1}{2} I_{\breve{W}}\left(\dot{\Theta}_{4W_1}{}^2 \sin^2\Theta_v + \dot{\Theta}_{4W_2}{}^2 \sin^2\Theta_v + \dot{\Theta}_v{}^2\right) + \frac{1}{2} m_0\left(\dot{r}^2 + r^2\dot{\Theta}_{4W_1}{}^2 + r^2 \sin\Theta_{4W_1}\dot{\Theta}_{4W_2}{}^2\right) \\ + \frac{1}{4\,\pi\,\varepsilon\,\varepsilon_0}\frac{\breve{q}^2}{r} - m_0 c^2(\cos^{-1}\Theta_v + \cos\Theta_v - 2)\end{gathered} \tag{76}$$

, where $\boldsymbol{q} \stackrel{\text{def}}{=} \left(\Theta_v, \psi, \Theta_{4W_1}, \Theta_{4W_2}, \phi_{4W_1}, \phi_{4W_2}, r\right)$.

Determining the generalized momenta from equation (23) we have:

$$M_{\Theta_v} = \frac{\partial\mathcal{L}(\boldsymbol{q},\dot{\boldsymbol{q}},t)}{\partial\dot{\Theta}_v} = I_{\breve{W}}\,\dot{\Theta}_v \tag{77}$$

$$M_\psi = \frac{\partial\mathcal{L}(\boldsymbol{q},\dot{\boldsymbol{q}},t)}{\partial\dot{\psi}} = I_{\breve{S}}\,\dot{\psi} \tag{78}$$

$$M_{\Theta_{4W_1}} = \frac{\partial\mathcal{L}(\boldsymbol{q},\dot{\boldsymbol{q}},t)}{\partial\dot{\Theta}_{4W_1}} = I_{\breve{W}}\sin^2\Theta_v\,\dot{\Theta}_{4W_1} + \cos\Theta_v\, I_{\breve{S}}\left(\dot{\phi}_{4W_1} + \cos\Theta_v\,\dot{\Theta}_{4W_1}\right) + m_0\, r^2\dot{\Theta}_{4W_1} \tag{79}$$

$$M_{\Theta_{4W_2}} = \frac{\partial\mathcal{L}(\boldsymbol{q},\dot{\boldsymbol{q}},t)}{\partial\dot{\Theta}_{4W_2}} = I_{\breve{W}}\sin^2\Theta_v\,\dot{\Theta}_{4W_2} + \cos\Theta_v\, I_{\breve{S}}\left(\dot{\phi}_{4W_2} + \cos\Theta_v\,\dot{\Theta}_{4W_2}\right) + m_0\, r^2\sin\Theta_{4W_1}\,\dot{\Theta}_{4W_2} \tag{80}$$

$$M_{\phi_{4W_1}} = \frac{\partial\mathcal{L}(\boldsymbol{q},\dot{\boldsymbol{q}},t)}{\partial\dot{\phi}_{4W_1}} = I_{\breve{S}}\left(\dot{\phi}_{4W_1} + \cos\Theta_v\,\dot{\Theta}_{4W_1}\right) \tag{81}$$

$$M_{\phi_{4W_2}} = \frac{\partial\mathcal{L}(\boldsymbol{q},\dot{\boldsymbol{q}},t)}{\partial\dot{\phi}_{4W_2}} = I_{\breve{S}}\left(\dot{\phi}_{4W_2} + \cos\Theta_v\,\dot{\Theta}_{4W_2}\right) \tag{82}$$

$$p_r = \frac{\partial\mathcal{L}(\boldsymbol{q},\dot{\boldsymbol{q}},t)}{\partial\dot{r}} = m_0\dot{r} \tag{83}$$

, where the generalized momentum is defined as follows $\boldsymbol{p} = \left(M_{\Theta_v},\ M_\psi,\ M_{\Theta_{4W_1}},\ M_{\Theta_{4W_2}},\ M_{\phi_{4W_1}}, M_{\phi_{4W_2}},\ p_r\right)$.

Determining $\dot{\boldsymbol{q}}$ from the equations, we have:

$$\dot{\Theta}_v = \frac{M_{\Theta_v}}{I_{\breve{W}}} \tag{84}$$

[15] We added an accent to the charge quantities notation $\breve{q}$ to distinguish it from generalized coordinates $\boldsymbol{q}$.

$$\dot{\psi} = \frac{M_{\psi}}{I_{\breve{S}}} \tag{85}$$

$$\dot{\Theta}_{4W_1} = \frac{M_{\Theta_{4W_1}} - \cos\Theta_v M_{\phi_{4W_1}}}{I_{\breve{W}} \sin^2\Theta_v + m_0 r^2} \tag{86}$$

$$\dot{\Theta}_{4W_2} = \frac{M_{\Theta_{4W_2}} - \cos\Theta_v M_{\phi_{4W_2}}}{I_{\breve{W}} \sin^2\Theta_v + m_0 r^2 \sin\Theta_{4W_1}} \tag{87}$$

$$\dot{\phi}_{4W_1} = \frac{M_{\phi_{4W_1}}}{I_{\breve{S}}} - \cos\Theta_v \frac{M_{\Theta_{4W_1}} - \cos\Theta_v M_{\phi_{4W_1}}}{I_{\breve{W}} \sin^2\Theta_v + m_0 r^2} \tag{88}$$

$$\dot{\phi}_{4W_2} = \frac{M_{\phi_{4W_2}}}{I_{\breve{S}}} - \cos\Theta_v \frac{M_{\Theta_{4W_2}} - \cos\Theta_v M_{\phi_{4W_2}}}{I_{\breve{W}} \sin^2\Theta_v + m_0 r^2 \sin\Theta_{4W_1}} \tag{89}$$

$$\dot{r} = \frac{p_r}{m_0} \tag{90}$$

Substituting right sides of equations (84), (85), (86), (87), (88), (89) and (90) into equation (22), we obtain the explicit form of the Hamiltonian $H_E(\boldsymbol{p},\boldsymbol{q})$ of the electron:

$$\begin{aligned} H_E(\boldsymbol{p},\boldsymbol{q}) = {} & M_{\Theta_v}\frac{M_{\Theta_v}}{I_{\breve{W}}} + M_{\psi}\frac{M_{\psi}}{I_{\breve{S}}} + M_{\Theta_{4W_1}}\frac{M_{\Theta_{4W_1}} - \cos\Theta_v M_{\phi_{4W_1}}}{I_{\breve{W}} \sin^2\Theta_v + m_0 r^2} \\ & + M_{\Theta_{4W_2}}\frac{M_{\Theta_{4W_2}} - \cos\Theta_v M_{\phi_{4W_2}}}{I_{\breve{W}} \sin^2\Theta_v + m_0 r^2 \sin\Theta_{4W_1}} \\ & + M_{\phi_{4W_1}}\left(\frac{M_{\phi_{4W_1}}}{I_{\breve{S}}} - \cos\Theta_v \frac{M_{\Theta_{4W_1}} - \cos\Theta_v M_{\phi_{4W_1}}}{I_{\breve{W}} \sin^2\Theta_v + m_0 r^2}\right) \\ & + M_{\phi_{4W_2}}\left(\frac{M_{\phi_{4W_2}}}{I_{\breve{S}}} - \cos\Theta_v \frac{M_{\Theta_{4W_2}} - \cos\Theta_v M_{\phi_{4W_2}}}{I_{\breve{W}} \sin^2\Theta_v + m_0 r^2 \sin\Theta_{4W_1}}\right) + p_r\frac{p_r}{m_0} \\ & - \frac{1}{2} I_{\breve{S}}\left(\left(\frac{M_{\psi}}{I_{\breve{S}}}\right)^2 + \left(\frac{M_{\phi_{4W_1}}}{I_{\breve{S}}} - \cos\Theta_v \frac{M_{\Theta_{4W_1}} - \cos\Theta_v M_{\phi_{4W_1}}}{I_{\breve{W}} \sin^2\Theta_v + m_0 r^2}\right.\right. \\ & \left. + \frac{M_{\Theta_{4W_1}} - \cos\Theta_v M_{\phi_{4W_1}}}{I_{\breve{W}} \sin^2\Theta_v + m_0 r^2}\cos\Theta_v\right)^2 \\ & + \left(\frac{M_{\phi_{4W_2}}}{I_{\breve{S}}} - \cos\Theta_v \frac{M_{\Theta_{4W_2}} - \cos\Theta_v M_{\phi_{4W_2}}}{I_{\breve{W}} \sin^2\Theta_v + m_0 r^2 \sin\Theta_{4W_1}}\right. \\ & \left.\left. + \frac{M_{\Theta_{4W_2}} - \cos\Theta_v M_{\phi_{4W_2}}}{I_{\breve{W}} \sin^2\Theta_v + m_0 r^2 \sin\Theta_{4W_1}}\cos\Theta_v\right)^2\right) \\ & - \frac{1}{2} I_{\breve{W}}\left(\left(\frac{M_{\Theta_{4W_1}} - \cos\Theta_v M_{\phi_{4W_1}}}{I_{\breve{W}} \sin^2\Theta_v + m_0 r^2}\right)^2 \sin^2\Theta_v\right. \\ & \left. + \left(\frac{M_{\Theta_{4W_2}} - \cos\Theta_v M_{\phi_{4W_2}}}{I_{\breve{W}} \sin^2\Theta_v + m_0 r^2 \sin\Theta_{4W_1}}\right)^2 \sin^2\Theta_v + \left(\frac{M_{\Theta_v}}{I_{\breve{W}}}\right)^2\right) \\ & - \frac{1}{2} m_0\left(\left(\frac{p_r}{m_0}\right)^2 + r^2\left(\frac{M_{\Theta_{4W_1}} - \cos\Theta_v M_{\phi_{4W_1}}}{I_{\breve{W}} \sin^2\Theta_v + m_0 r^2}\right)^2\right. \\ & \left. + r^2 \sin\Theta_{4W_1}\left(\frac{M_{\Theta_{4W_2}} - \cos\Theta_v M_{\phi_{4W_2}}}{I_{\breve{W}} \sin^2\Theta_v + m_0 r^2 \sin\Theta_{4W_1}}\right)^2\right) - \frac{1}{4\pi\varepsilon\varepsilon_0}\frac{\breve{q}^2}{r} \\ & + m_0 c^2(\cos^{-1}\Theta_v + \cos\Theta_v - 2) \end{aligned} \tag{91}$$

After calculations we get the final form of the Hamilton function for the electron:

$$H_E(\boldsymbol{p},\boldsymbol{q}) = \frac{{M_{E,\Theta_v}}^2}{2I_{E,\breve{W}}} + \frac{{M_{E,\psi}}^2}{2I_{E,\breve{S}}} + \frac{{M_{E,\phi_{4W_1}}}^2}{2I_{E,\breve{S}}} + \frac{{p_{E,r}}^2}{2m_{0,E}} + \frac{{M_{E,\phi_{4W_2}}}^2}{2I_{E,\breve{S}}} + \frac{\left(M_{E,\Theta_{4W_1}} - cos\,\Theta_{v_E} M_{E,\phi_{4W_1}}\right)^2}{I_{E,\breve{W}}\, sin^2\,\Theta_v + m_{0,E}\, r^2} + \frac{\left(M_{E,\Theta_{4W_2}} - cos\,\Theta_{v_E} M_{E,\phi_{4W_2}}\right)^2}{I_{E,\breve{W}}\, sin^2\,\Theta_{v_E} + m_{0,E}\, r^2 \sin\Theta_{E,4W_1}} - \frac{1}{2} m_{0,E}\, r^2 \left(\frac{M_{E,\Theta_{4W_1}} - cos\,\Theta_{v_E} M_{E,\phi_{4W_1}}}{I_{E,\breve{W}}\, sin^2\,\Theta_{v_E} + m_{0,E}\, r^2}\right)^2 - \frac{1}{2} m_{0,E}\, r^2 \sin\Theta_{4W_1} \left(\frac{M_{E,\Theta_{4W_2}} - cos\,\Theta_{v_E} M_{E,\phi_{4W_2}}}{I_{E,\breve{W}}\, sin^2\,\Theta_{v_E} + m_{0,E}\, r^2 \sin\Theta_{E,4W_1}}\right)^2 - \frac{1}{4\,\pi\,\varepsilon\,\varepsilon_0} \frac{\breve{q}^2}{r} + 4 m_{0,E} c^2\, sec\,\Theta_{v_E}\, sin^4 \frac{\Theta_{v_E}}{2} \tag{92}$$

Substituting the explicit forms of the Hamiltonians $H_C(\boldsymbol{p},\boldsymbol{q})$ and $H_E(\boldsymbol{p},\boldsymbol{q})$ into equation (75) we obtain:

$$H_H(\boldsymbol{p},\boldsymbol{q}) = \frac{{M_{C,\psi}}^2}{2\,I_{C,\breve{S}}} + \frac{{M_{C,\Theta_{v_C}}}^2}{2\,I_{C,\breve{W}}} + \frac{{M_{C,\Theta_{4W_1}}}^2}{2\,I_{C,\breve{S}}} + \frac{{M_{C,\Theta_{4W_2}}}^2}{2\,I_{C,\breve{S}}} + \frac{\left(M_{C,\phi_{4W_1}} - cos\,\Theta_{v_C}\, M_{C,\Theta_{4W_1}}\right)^2}{2\,I_{C,\breve{W}}\, sin^2\,\Theta_{v_C}} + \frac{\left(M_{C,\phi_{4W_2}} - cos\,\Theta_{v_C}\, M_{C,\Theta_{4W_2}}\right)^2}{2\,I_{C,\breve{W}}\, sin^2\,\Theta_{v_C}} + 4 m_{0,C} c^2\, sec\,\Theta_{v_C}\, sin^4 \frac{\Theta_{v_C}}{2} + \frac{{M_{E,\Theta_v}}^2}{2I_{E,\breve{W}}} + \frac{{M_{E,\psi}}^2}{2I_{E,\breve{S}}} + \frac{{M_{E,\phi_{4W_1}}}^2}{2I_{E,\breve{S}}} + \frac{{p_{E,r}}^2}{2m_{0,E}} + \frac{{M_{E,\phi_{4W_2}}}^2}{2I_{E,\breve{S}}} + \frac{\left(M_{E,\Theta_{4W_1}} - cos\,\Theta_{v_E} M_{E,\phi_{4W_1}}\right)^2}{I_{E,\breve{W}}\, sin^2\,\Theta_{v_E} + m_{0,E}\, r^2} + \frac{\left(M_{E,\Theta_{4W_2}} - cos\,\Theta_{v_E} M_{E,\phi_{4W_2}}\right)^2}{I_{E,\breve{W}}\, sin^2\,\Theta_{v_E} + m_E\, r^2 \sin\Theta_{E,4W_1}} - \frac{1}{2} m_{0,E}\, r^2 \left(\frac{M_{E,\Theta_{4W_1}} - cos\,\Theta_{v_E} M_{E,\phi_{4W_1}}}{I_{E,\breve{W}}\, sin^2\,\Theta_{v_E} + m_{0,E}\, r^2}\right)^2 - \frac{1}{2} m_{0,E}\, r^2 \sin\Theta_{E,4W_1} \left(\frac{M_{E,\Theta_{4W_2}} - cos\,\Theta_{v_E} M_{E,\phi_{4W_2}}}{I_{E,\breve{W}}\, sin^2\,\Theta_{v_E} + m_{0,E}\, r^2 \sin\Theta_{E,4W_1}}\right)^2 - \frac{1}{4\,\pi\,\varepsilon\,\varepsilon_0} \frac{\breve{q}^2}{r} + 4 m_{0,E} c^2\, sec\,\Theta_{v_E}\, sin^4 \frac{\Theta_{v_E}}{2} \tag{93}$$

Using the Hamilton function $H_H(\boldsymbol{p},\boldsymbol{q})$ from equation (93), we can write the system of Hamilton equations:

$$\dot{\boldsymbol{p}} = -\frac{\partial H}{\partial \boldsymbol{q}} \;,\; \dot{\boldsymbol{q}} = \frac{\partial H}{\partial \boldsymbol{p}} \tag{94}$$

, where:

- $\boldsymbol{q} = \boldsymbol{q_C} \times \boldsymbol{q_E} \times \boldsymbol{r} = \begin{pmatrix} \Theta_{\,v_C}, \psi_C, \Theta_{C,4W_1}, \Theta_{C,4W_2}, \phi_{C,4W_1}, \phi_{C,4W_2}, \\ \Theta_{\,v_E}, \psi_E, \Theta_{E,4W_1}, \Theta_{E,4W_2}, \phi_{E,4W_1}, \phi_{E,4W_2}, \\ r \end{pmatrix}$
- $\boldsymbol{p} = \begin{pmatrix} M_{C,\Theta_{v_C}}, M_{C,\psi}, M_{C,\Theta_{4W_1}}, M_{C,\Theta_{4W_2}}, M_{C,\phi_{4W_1}}, M_{C,\phi_{4W_2}}, \\ M_{E,\Theta_{v_E}}, M_{E,\psi}, M_{E,\Theta_{4W_1}}, M_{E,\Theta_{4W_2}}, M_{E,\phi_{4W_1}}, M_{E,\phi_{4W_2}}, \\ p_{E,r} \end{pmatrix}$

Based on Hamiltonian function $H_H(\boldsymbol{p},\boldsymbol{q})$ (93), similarly to what was done earlier, we can write the Hamiltonian-Jacobi equations for the action function $S_H(\boldsymbol{q},t)$:

$$
\begin{aligned}
-\frac{\partial S}{\partial t} = {} & \frac{1}{2\,I_{C,\breve{S}}}\left(\frac{\partial S}{\partial \psi_C}\right)^2 + \frac{1}{2\,I_{C,\breve{W}}}\left(\frac{\partial S}{\partial \Theta_{v_C}}\right)^2 + \frac{1}{2\,I_{C,\breve{S}}}\left(\frac{\partial S}{\partial \Theta_{C,4W_1}}\right)^2 + \frac{1}{2\,I_{C,\breve{S}}}\left(\frac{\partial S}{\partial \Theta_{C,4W_2}}\right)^2 \\
& + \frac{\left(\frac{\partial S}{\partial \phi_{C,4W_1}} - \cos\Theta_{v_C}\frac{\partial S}{\partial \Theta_{C,4W_1}}\right)^2}{2\,I_{C,\breve{W}}\ \sin^2\Theta_{v_C}} + \frac{\left(\frac{\partial S}{\partial \phi_{C,4W_2}} - \cos\Theta_{v_C}\frac{\partial S}{\partial \Theta_{C,4W_2}}\right)^2}{2\,I_{C,\breve{W}}\sin^2\Theta_{v_C}} \\
& + 4m_{0,C}c^2\sec\Theta_{v_C}\sin^4\frac{\Theta_{v_C}}{2} + \frac{1}{2I_{E,\breve{W}}}\left(\frac{\partial S}{\partial \Theta_{v_E}}\right)^2 + \frac{1}{2I_{E,\breve{S}}}\left(\frac{\partial S}{\partial \psi_E}\right)^2 \\
& + \frac{1}{2I_{E,\breve{S}}}\left(\frac{\partial S}{\partial \phi_{E,4W_1}}\right)^2 + \frac{1}{2m_{0,E}}\left(\frac{\partial S}{\partial r}\right)^2 + \frac{1}{2I_{E,\breve{S}}}\left(\frac{\partial S}{\partial \phi_{E,4W_2}}\right)^2 \\
& + \frac{\left(\frac{\partial S}{\partial \Theta_{E,4W_1}} - \cos\Theta_{v_E}\frac{\partial S}{\partial \phi_{E,4W_1}}\right)^2}{I_{\breve{W}}\sin^2\Theta_v + m_{0,E}\,r^2} + \frac{\left(\frac{\partial S}{\partial \Theta_{E,4W_2}} - \cos\Theta_{v_E}\frac{\partial S}{\partial \phi_{E,4W_2}}\right)^2}{I_{E,\breve{W}}\sin^2\Theta_{v_E} + m_{0,E}\,r^2\sin\Theta_{E,4W_1}} \\
& - \frac{1}{2}m_{0,E}\,r^2\left(\frac{\frac{\partial S}{\partial \Theta_{E,4W_1}} - \cos\Theta_{v_E}\frac{\partial S}{\partial \phi_{E,4W_1}}}{I_{E,\breve{W}}\sin^2\Theta_{v_E} + m_{0,E}\,r^2}\right)^2 \\
& - \frac{1}{2}m_{0,E}\,r^2\sin\Theta_{E,4W_1}\left(\frac{\frac{\partial S}{\partial \Theta_{E,4W_2}} - \cos\Theta_{v_E}\frac{\partial S}{\partial \phi_{E,4W_2}}}{I_{E,\breve{W}}\sin^2\Theta_{v_E} + m_{0,E}\,r^2\sin\Theta_{E,4W_1}}\right)^2 - \frac{1}{4\,\pi\,\varepsilon\,\varepsilon_0}\frac{\breve{q}^2}{r} \\
& + 4m_{0,E}c^2\sec\Theta_{v_E}\sin^4\frac{\Theta_{v_E}}{2}
\end{aligned}
\tag{95}
$$

Note that equation (95) can be significantly simplified by assuming that the atomic nucleus is stationary and replacing the electron mass $m_{0,E}$ with the reduced mass $m_r = \frac{m_E\, m_{0,C}}{m_E + m_{0,C}}$, but without the rest mass present in the component defining the strain energy of the boundary hypersurfaces, which gives the following equation:

$$
\begin{aligned}
-\frac{\partial S'}{\partial t} = {} & \frac{1}{2I_{E,\breve{W}}}\left(\frac{\partial S'}{\partial \Theta_{v_E}}\right)^2 + \frac{1}{2I_{E,\breve{S}}}\left(\frac{\partial S'}{\partial \psi_E}\right)^2 + \frac{1}{2I_{E,\breve{S}}}\left(\frac{\partial S'}{\partial \phi_{E,4W_1}}\right)^2 + \frac{1}{2m_r}\left(\frac{\partial S'}{\partial r}\right)^2 + \frac{1}{2I_{E,\breve{S}}}\left(\frac{\partial S'}{\partial \phi_{E,4W_2}}\right)^2 \\
& + \frac{\left(\frac{\partial S'}{\partial \Theta_{E,4W_1}} - \cos\Theta_{v_E}\frac{\partial S'}{\partial \phi_{E,4W_1}}\right)^2}{I_{E,\breve{W}}\sin^2\Theta_v + m_r\,r^2} + \frac{\left(\frac{\partial S'}{\partial \Theta_{E,4W_2}} - \cos\Theta_{v_E}\frac{\partial S'}{\partial \phi_{E,4W_2}}\right)^2}{I_{E,\breve{W}}\sin^2\Theta_{v_E} + m_r\,r^2\sin\Theta_{E,4W_1}} \\
& - \frac{1}{2}m_r\,r^2\left(\frac{\frac{\partial S'}{\partial \Theta_{E,4W_1}} - \cos\Theta_{v_E}\frac{\partial S'}{\partial \phi_{E,4W_1}}}{I_{E,\breve{W}}\sin^2\Theta_{v_E} + m_r\,r^2}\right)^2 \\
& - \frac{1}{2}m_r\,r^2\sin\Theta_{E,4W_1}\left(\frac{\frac{\partial S'}{\partial \Theta_{E,4W_2}} - \cos\Theta_{v_E}\frac{\partial S'}{\partial \phi_{E,4W_2}}}{I_{E,\breve{W}}\sin^2\Theta_{v_E} + m_r\,r^2\sin\Theta_{E,4W_1}}\right)^2 - \frac{1}{4\,\pi\,\varepsilon\,\varepsilon_0}\frac{\breve{q}^2}{r} \\
& + 4m_{0,E}c^2\sec\Theta_{v_E}\sin^4\frac{\Theta_{v_E}}{2}
\end{aligned}
\tag{96}
$$

Let us pay attention to the fact that in four-dimensional space the axis of rotation is not a straight line but a hyperplane, which is why the end of the oscillating spatial vortex outlines complex hypersurfaces and not lines, which after a full rotation of the electron around the nucleus of the hydrogen atom should hit exactly the same phase, because otherwise the electron will interact with itself and the sought state of the atom will not be stable, as shown in the Figure 4.

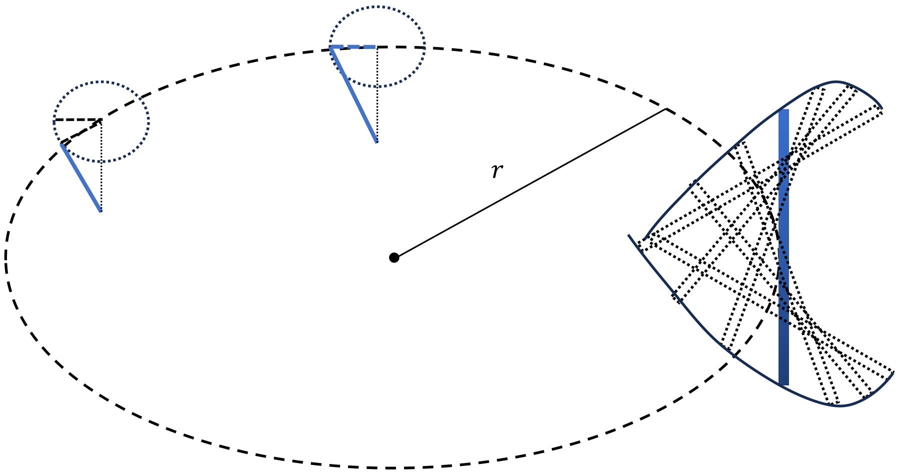



Figure 4. Fragment of the complex hyperplanes traced by the ends of the electron's spatial vortex in orbit around the nucleus of a hydrogen atom.

Note that both the electron and the nucleus of the atom move around the common centre of mass performing complex nutations $\Theta_v(t)$ and precessions, which were revealed by the analyses of the freely moving space vortex – see equations (34), (35), (36), (37) and (41).

Therefore, when looking for stationary states of the hydrogen atom, one should look for solutions of equation (95) or (96) in which the angular velocities are equal or multiple, so that after the electron and the nucleus of the atom rotate around the centre of mass after time $t - t_0 = T$ the spatial vortices overlap in the phase $\boldsymbol{q}(t) = \boldsymbol{q}(t_0)$ and do not lead to self-interaction of the electron or self-interaction of the nucleus of the atom.

Therefore, taking into account the fact that oscillations in the form of nutation and precession are performed in the configuration space $SO(4) \times SO(4) \times \mathbb{R}^+$, we have the following conditions:

$$\begin{aligned}\boldsymbol{q}(t_0 + T) = \boldsymbol{q}(t_0) \Leftrightarrow \int_{\boldsymbol{0}}^{\boldsymbol{T}} \dot{\Theta}_{v_C} dt = 2\pi k_{\Theta_{v_C}} \wedge \int_0^T \dot{\psi}_C dt = 2\pi k_{\psi_C} \wedge \int_0^T \dot{\Theta}_{C,4W_1} dt \\ = 2\pi k_{\Theta_{C,4W_1}} \wedge \int_0^T \dot{\Theta}_{C,4W_2} dt = 2\pi k_{\Theta_{C,4W_2}} \wedge \int_0^T \dot{\phi}_{C,4W_1} dt \\ = 2\pi k_{\phi_{C,4W_1}} \wedge \int_0^T \dot{\phi}_{C,4W_2} dt = 2\pi k_{\phi_{C,4W_2}} \wedge \int_0^T \dot{\Theta}_{v_E} dt = 2\pi k_{\Theta_{v_E}} \wedge \int_0^T \dot{\psi}_E dt \\ = 2\pi k_{\psi_E} \wedge \int_0^T \dot{\Theta}_{E,4W_1} dt = 2\pi k_{\Theta_{E,4W_1}} \wedge \int_0^T \dot{\Theta}_{E,4W_2} dt \\ = 2\pi k_{\Theta_{E,4W_2}} \wedge \int_0^T \dot{\phi}_{E,4W_1} dt = 2\pi k_{\phi_{E,4W_1}} \wedge \int_0^T \dot{\phi}_{E,4W_2} dt = 2\pi k_{\phi_{E,4W_2}}\end{aligned} \tag{97}$$

, where $k_{\Theta_{v_C}} \in \mathbb{N}$, $k_{\psi_C} \in \mathbb{N}$, $k_{\Theta_{C,4W_1}} \in \mathbb{N}$, $k_{\Theta_{C,4W_2}} \in \mathbb{N}$, $k_{\phi_{C,4W_1}} \in \mathbb{N}$, $k_{\phi_{C,4W_2}} \in \mathbb{N}$, $k_{\Theta_{v_E}} \in \mathbb{N}$, $k_{\psi_E} \in \mathbb{N}$, $k_{\Theta_{E,4W_1}} \in \mathbb{N}$, $k_{\Theta_{E,4W_2}} \in \mathbb{N}$, $k_{\phi_{E,4W_1}} \in \mathbb{N}$, $k_{\phi_{E,4W_2}} \in \mathbb{N}$.

The introduced coefficients k can be identified with quantum numbers, because they correspond to different energy levels of the hydrogen atom.

Note that the velocity of the end of the spatial vortex can be determined from the formula (Sobolewski D. S., Theory of Space, 2016) (Sobolewski D. S., Theory of Space, 2017) (Sobolewski D. S., Theory of Space, 2024):

$$\boldsymbol{v} = \boldsymbol{v}_S + *^T \overset{2}{\boldsymbol{\omega}} \wedge \boldsymbol{x}' \tag{98}$$

, where:

- $\boldsymbol{v}_S$ - the velocity of the origin of the coordinate system $\Theta'$ in rotation,
- $*^T$ - transposed Hodge star operator,
- $\overset{2}{\boldsymbol{\omega}}$ – angular velocity bivector,
- $\boldsymbol{x}'$ - coordinates of the point in system $\Theta'$.

The velocity value $\|\boldsymbol{v}_S\|$ of the origin of the coordinate system $\Theta'$ can be determined from the formula $v = c \sin\theta_v$ revealed in the equation (19), while its components are determined from spherical coordinates, i.e. based on equations (62), (63) and (64) and the adopted equalities (98):

$$\boldsymbol{v}_{S,1} = r\mathrm{Cos}[\Theta_{4W_1}]\mathrm{Cos}[\Theta_{4W_2}]\dot{\Theta}_{4W_1} + \mathrm{Cos}[\Theta_{4W_2}]\mathrm{Sin}[\Theta_{4W_1}]\dot{r} - r\mathrm{Sin}[\Theta_{4W_1}]\mathrm{Sin}[\Theta_{4W_2}]\dot{\Theta}_{4W_2} \tag{99}$$

$$\boldsymbol{v}_{S,2} = rCos[\Theta_{4W_2}]Sin[\Theta_{4W_1}]\dot{\Theta}_{4W_2} + rCos[\Theta_{4W_1}]Sin[\Theta_{4W_2}]\dot{\Theta}_{4W_1} + Sin[\Theta_{4W_1}]Sin[\Theta_{4W_2}]\dot{r} \tag{100}$$

$$\boldsymbol{v}_{S,3} = Cos[\Theta_{4W_1}]\dot{r} - rSin[\Theta_{4W_1}]\dot{\Theta}_{4W_1} \tag{101}$$

$$\boldsymbol{v}_{S,4} = 0 \tag{102}$$

Therefore, by substituting $\boldsymbol{x}'$ for the coordinates of the end of the space vortex of the hydrogen atom nucleus and the electron in equation (103) and $\overset{2}{\boldsymbol{\omega}}$ for the angular velocities, we obtain the equations:

$$\begin{aligned}\boldsymbol{v_C} = \boldsymbol{v}_{C,S} + *^T\big(=-\dot{\psi}_C e_1 \wedge e_4 - \big(\dot{\Theta}_{C,4W_1} \cos\Theta_{v_C} + \dot{\phi}_{C,4W_1}\big)e_2 \wedge e_4 - \dot{\Theta}_{C,4W_1} \sin\Theta_{v_C}\, e_2 \wedge e_1 \\ - \dot{\Theta}_{C,4W_2} \sin\Theta_{v_C}\, e_3 \wedge e_1 - \big(\dot{\Theta}_{C,4W_2} \cos\Theta_{v_C} + \dot{\phi}_{C,4W_2}\big)e_3 \wedge e_4 + \dot{\Theta}_{v_C} e_2 \wedge e_3\big) \\ \wedge \left(e'_4 \frac{1}{2}\sqrt{\frac{1}{3}\frac{h\,\bar{s}}{m_{0,C}c}}\right)\end{aligned} \tag{103}$$

$$\begin{aligned}\boldsymbol{v}_E = \boldsymbol{v}_{E,S} + *^T\big(=-\dot{\psi}_E e_1 \wedge e_4 - \big(\dot{\Theta}_{E,4W_1} \cos\Theta_{v_E} + \dot{\phi}_{E,4W_1}\big)e_2 \wedge e_4 - \dot{\Theta}_{E,4W_1} \sin\Theta_{v_E}\, e_2 \wedge e_1 \\ - \dot{\Theta}_{E,4W_2} \sin\Theta_{v_E}\, e_3 \wedge e_1 - \big(\dot{\Theta}_{E,4W_2} \cos\Theta_{v_E} + \dot{\phi}_{E,4W_2}\big)e_3 \wedge e_4 + \dot{\Theta}_{v_E} e_2 \wedge e_3\big) \\ \wedge \left(e'_4 \frac{1}{2}\sqrt{\frac{1}{3}\frac{h\,\bar{s}}{m_{0,E}c}}\right)\end{aligned} \tag{104}$$

, where the benchmark $(e_1, e_2, e_3, e_4)$ is in the Euler coordinate system, therefore, in order to determine the four components of the velocities $\boldsymbol{v}_E$ and $\boldsymbol{v}_C$, it is necessary to apply the transformation formulas for going from the system $(e'_1, e'_2, e'_3, e'_4)$ to the Euler system $(e_1, e_2, e_3, e_4)$, which we will do in the next publications.

Note that having the component velocities $\boldsymbol{v}_E$ and $\boldsymbol{v}_C$ we can determine the components of the four-dimensional wave vectors $\boldsymbol{k}_E$ and $\boldsymbol{k}_C$ for the ends of the space channels:

$$\forall_{\boldsymbol{i}\in(\boldsymbol{1,2,3,4})}\, k_i = \frac{2\pi}{\lambda_i} = \frac{2\pi}{\int_0^T v_i dt} \tag{105}$$

, which means that the de Broglie equation (20) can be derived directly from the analysis of the motion of the space vortex tip

**Conclusions**

The publication formulates Hamilton-Jacobi equations for a freely moving elementary particle and for a hydrogen atom based on the structure of space and elementary particles revealed in the Theory of Space, which will be the subject of analyses in subsequent publications.

It also reveals the limitations of the applicability of classical quantum mechanics, including the Schrödinger equations, due to their far-reaching approximations that come down to the analysis of the deformations of the boundary hypersurface ${}^{\beta}\aleph^{\beta}$ by one of the ends of the spatial vortex, ignoring its complex oscillations around the equilibrium position in $SO(4)$ space – another article by the HTS physics department is in the works.

In subsequent publications we will also provide equations for photons ${}^{\beta}\gamma_1, {}^{\beta}\gamma_2, {}^{\beta}\gamma_3$ – see e.g. (Sobolewski, Sobolewski i Sobolewski, 2017) , (Sobolewska, Sobolewska, Sobolewski, Sobolewski i Sobolewski, New Generations of Rocket Engines, 2020), (Sobolewski D. S., Theory of Space, 2016), (Sobolewski D. S., Theory of Space, 2017), (Sobolewski D. S., Theory of Space, 2024), (Sobolewska, Sobolewska, Sobolewski, Sobolewski i Sobolewski, New Generations of Rocket Engines, 2021).

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