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Laser heating of a finite silver sulfide slab (Ag_2S). An elegant analytical solution using Laplace integral transform method.

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ABSTRACT

The problem of heating a finite silver sulfide slab with cw laser is studied using Laplace integral transform technique. For a certain laser power irradiance an expression for the temperature profile within the slab is obtained. The temperature variations with time for the front and rear surfaces are also obtained. The critical time required to initiate melting can thus be computed. Computations for an illustrative example are given. The obtained results reveal.

KEYWORDScw laser heating, Silver sulfide target, Laplace integral transform, Analytical solution, Critical time to initiate damage.

1. Introduction

Laser heating is an accepted process for industrial applications. The ability of high-power lasers (up to 10^{12} W/m^2) makes them useful for a variety of material processing techniques, such as spot welding, scribing, drilling of holes, laser cutting, laser shock hardening, and laser glazing [1,3,4]. In the semiconductor industry many other applications of laser heating are developed, including both local diffusion and alloying to form p-n junction. Indeed laser-solid interaction have aroused considerable interest of many investigators. [1,3-8,10-13,16,17].

Different models and techniques are used to obtain solutions for topics related to such important problem. Trials are still made in order to get solutions in a well-established form, applicable for practical computations.The study of laser damage of the irradiated target is a serious problem either to avoid damage to extend their life time as in the case of laser mirrors [5], or to initiate damage (melting) when it is necessary. The importance of this problem has encouraged authors to develop several models both analytical or numerical to treat the suggested heating problem. The source function in such attempts is considered to be either cw or pulsed laser. The aim of the present article is to study the problem of laser heating of a homogenous finite slab of silver sulfide (Ag_2S) material. This compound is of great technological importance to be used as chemosensors and biosensor. [14] The optical absorption properties of the synthesized nanomaterials indicate that nanoplates can be used as optical sensors [14]

Moreover, the silver sulfide thin films have photoconductivity and photovoltaic properties [9]. Thus this material is a promising material in preparation of photovoltaic solar cells. It is an inorganic compound. Moreover, authors [18] indicated the importance of using (Ag_2S) semiconductor near-infrared (NIR) as a photocatalyst. It realized degradation rate of the methylene blue dye equal 100% within two hours under sunlight exposure, which has technological importance.

2. Theoretical approaches

In setting up the problem we shall consider a laser flux $q_\circ$(W/m²) incident in a perpendicular direction on the front surface of a silver sulfide slab of thickness "d" along the x-direction.

The heat flow is considered one dimensional [1].

The case of cooling at both the front and rear surfaces is considered.

The laser irradiance $q_\circ$ is assumed to be constant i.e CW laser is considered.

According to our model, a part of the incident laser flux will be reflected at the front surface and a part (A$q_\circ$) will be absorbed by the slab material. where “A” is the absorption coefficient, The multireflections within the slab are neglected.

The physical properties of the considered material are assumed to be temperature independent for simplicity. Heat loses arising from convention at both the front and rear surfaces are considered while losses due to thermal radiation are neglected.

The heat diffusion equation for the considered problem is written in the form:

$$\frac{\partial T(x,t)}{\partial t} = \alpha \frac{\partial^2 T(x,t)}{\partial x^2} \quad ,0 < x \le d \,, t > 0 \tag{1}$$

Where $\alpha = \frac{\lambda}{\rho c_p}$ the thermal diffusivity of the slab material in terms of the thermal conductivity λ (W/m K) and the heat capacity per unit volume $\rho\, C_p (\frac{J}{m^3 K})$, T is the excess temperature relative to the ambient temperature $T_\circ$, $T=(T^` - T_\circ)$

Equation (1) is subjected to the following initial and boundary conditions:

$$T(x, 0) = 0 \tag{2}$$

At the front surface x=0

$$-\lambda \frac{\partial T}{\partial x}|_{x=0} = q_\circ A - h_\circ T(0, t) \tag{3}$$

The Heat Balance Equation in an integral form is:

$$\int_0^t q_\circ A\, dt = \int_0^d \rho\, c_p\, T(x, t)\, dx + \int_0^t h_d\, T(d, t)\, dt + \int_0^t h_\circ\, T(0, t)\, dt \tag{4}$$

To solve equation (1) let us apply Laplace Integral transform technique, according to which, the Laplace transform of equation (1) with respect to the time gives:

$$s\overline{T}(x, s) - T(x, 0) = \alpha \frac{d^2}{dx^2} \overline{T}(x, s) \tag{5}$$

Considering condition (2), equation (5) attains the from:

$$s\overline{T}(x, s) = \alpha \frac{d^2}{dx^2} \overline{T}(x, s) \tag{6}$$

The solution of equation (6) in the usual manner attains the form:

$$\overline{T}(x, s) = C_1 \exp exp \left(\sqrt{\frac{s}{\alpha}}\, x\right) + C_2 \exp exp \left(-\sqrt{\frac{s}{\alpha}}\, x\right) \tag{7}$$

The Laplace transform of the conditions (3) and (4) gives:

$$\lambda \frac{d}{dx} \overline{T}\,(0, s) = -\frac{q_\circ A}{s} + h_\circ \overline{T}\,(0, s) \tag{8}$$

And

$$\frac{q_\circ A}{s^2} = \int_0^d \rho\, c_p\, \overline{T}(x, s)\, dx + \frac{h_d}{s} \overline{T}(d, s) + \frac{h_\circ}{s} \overline{T}(0, s) \tag{9}$$

Substitute from (7) into in equation (9) one gets

$$\frac{q_\circ A}{s^2} = \rho\, c_p \left[\sqrt{\frac{\alpha}{s}}\, C_1 \exp exp \left(\sqrt{\frac{s}{\alpha}}\, d\right) - \sqrt{\frac{\alpha}{s}}\, C_1\right] - \rho\, c_p \left[\sqrt{\frac{\alpha}{s}}\, C_2 \exp exp \left(-\sqrt{\frac{s}{\alpha}}\, d\right) - \sqrt{\frac{\alpha}{s}}\, C_2\right] + \frac{h_d}{s} C_1 \exp exp \left(\sqrt{\frac{s}{\alpha}}\, d\right) + \frac{h_d}{s} C_2 \exp exp \left(-\sqrt{\frac{s}{\alpha}}\, d\right) + \frac{h_\circ}{s} (C_1 + C_2) \tag{10}$$

While equation (8) gives:

$$\lambda \left(C_1 \sqrt{\frac{s}{\alpha}} - C_2 \sqrt{\frac{s}{\alpha}}\right) = -\frac{q_\circ A}{s} + h_\circ (C_1 + C_2) \tag{11}$$

To find C_1 *and* C_2 one has to solve both equations (10) and (11) simultaneously.

Equation (11) can be rearranged to give:

$$A\, q_{\circ} = C_1 \lambda \left[\frac{h_{\circ} s}{\lambda} - s\sqrt{\frac{s}{\alpha}}\right] + C_2 \lambda \left[s\sqrt{\frac{s}{\alpha}} + \frac{s\, h_{\circ}}{\lambda}\right] \quad (12)$$

Equation (12) can further be rewritten as:

$$A\, q_{\circ} = C_1\, a + C_2\, b \quad (13)$$

Where,

$$a = \lambda \left[\frac{h_{\circ} s}{\lambda} - s\sqrt{\frac{s}{\alpha}}\right] \quad (14)$$

and

$$b = \lambda \left[s\sqrt{\frac{s}{\alpha}} + \frac{s\, h_{\circ}}{\lambda}\right] \quad (15)$$

Equation (10) can be rewritten as:

$$\frac{q_{\circ} A}{s^2} = C_1 \left[\rho\, c_p \sqrt{\frac{\alpha}{s}}\, \exp exp \left(\sqrt{\frac{s}{\alpha}}\, d\right) - \sqrt{\frac{\alpha}{s}}\, \rho\, c_p + \frac{h_d}{s}\, \exp exp \left(\sqrt{\frac{s}{\alpha}}\, d\right) + \frac{h_{\circ}}{s}\right]$$

$$- C_2 \left[\rho\, c_p \sqrt{\frac{\alpha}{s}}\, \exp exp \left(-\sqrt{\frac{s}{\alpha}}\, d\right) - \sqrt{\frac{\alpha}{s}}\rho\, c_p - \frac{h_d}{s}\, \exp exp \left(-\sqrt{\frac{s}{\alpha}}\, d\right) - \frac{h_{\circ}}{s}\right] \quad (16)$$

Equation (16) can be rewritten as:

$$\frac{q_{\circ} A}{s^2} = C_1\, g - C_2\, h \quad (17)$$

Where,

$$g = \rho\, c_p \sqrt{\frac{\alpha}{s}}\, \exp exp \left(\sqrt{\frac{s}{\alpha}}\, d\right) - \sqrt{\frac{\alpha}{s}}\, \rho\, c_p + \frac{h_d}{s}\, \exp exp \left(\sqrt{\frac{s}{\alpha}}\, d\right) + \frac{h_{\circ}}{s} \quad (18)$$

and,

$$h = \rho\, c_p \sqrt{\frac{\alpha}{s}}\, \exp exp \left(-\sqrt{\frac{s}{\alpha}}\, d\right) - \sqrt{\frac{\alpha}{s}}\rho\, c_p - \frac{h_d}{s}\, \exp exp \left(-\sqrt{\frac{s}{\alpha}}\, d\right) - \frac{h_{\circ}}{s} \quad (19)$$

Solving equations (13) and (17) simultaneously one gets:

$$C_1 = \frac{A\, q_{\circ}\, (h + \frac{b}{s^2})}{(a\, h + g\, b)} \quad (20)$$

and

$$C_2 = \frac{A\, q_{\circ} \left(g - \frac{a}{s^2}\right)}{(a\, h + g\, b)} \quad (21)$$

Let us substitute for C_1 and C_2 into equation (7) one gets:

$$\overline{T}(x, s) = \frac{A\, q_{\circ} \left(h + \frac{b}{s^2}\right)}{(a\, h + g\, b)} \exp exp \left(\sqrt{\frac{s}{\alpha}}\, x\right) + \frac{A\, q_{\circ} \left(g - \frac{a}{s^2}\right)}{(a\, h + g\, b)} \exp exp \left(-\sqrt{\frac{s}{\alpha}}\, x\right) \quad (22)$$

Let us simplify both the numerator and the denominator in equation (22) in the following steps:

The numerator has the form:

$$A\, q_{\circ} \left[h\, \exp exp \left(\sqrt{\frac{s}{\alpha}}\, x\right) + \frac{b}{s^2} \exp exp \left(\sqrt{\frac{s}{\alpha}}\, x\right) + g\, \exp exp \left(-\sqrt{\frac{s}{\alpha}}\, x\right) - \frac{a}{s^2} \exp exp \left(-\sqrt{\frac{s}{\alpha}}\, x\right)\right] \quad (23)$$

Substituting for a,b,g and h from equations (14),(15),(18) and (19) into equation (23) one gets the numerator in the form:

$$A\, q_{\circ} \left[\rho\, c_p \sqrt{\frac{\alpha}{s}}\, \exp exp \left(-\sqrt{\frac{s}{\alpha}}\, d\right) - \sqrt{\frac{\alpha}{s}}\rho\, c_p - \frac{h_d}{s}\, \exp exp \left(-\sqrt{\frac{s}{\alpha}}\, d\right) - \frac{h_{\circ}}{s}\right] \exp exp \left(\sqrt{\frac{s}{\alpha}}\, x\right)$$

$$+ A\, q_{\circ}\left(\frac{\lambda}{s}\sqrt{\frac{s}{\alpha}} + \frac{h_{\circ}}{s}\right)\exp exp\left(\sqrt{\frac{s}{\alpha}}\, x\right)$$

$$+ A\, q_{\circ}\left[\rho\, c_p\sqrt{\frac{\alpha}{s}}\exp exp\left(\sqrt{\frac{s}{\alpha}}\, d\right) - \sqrt{\frac{\alpha}{s}}\rho\, c_p + \frac{h_d}{s}\exp exp\left(\sqrt{\frac{s}{\alpha}}\, d\right) + \frac{h_{\circ}}{s}\right]\exp exp\left(-\sqrt{\frac{s}{\alpha}}\, x\right)$$

$$- A\, q_{\circ}\exp\left(\frac{h_{\circ}}{s} - \frac{\lambda}{s}\sqrt{\frac{s}{\alpha}}\right)exp\left(-\sqrt{\frac{s}{\alpha}}\, x\right) \qquad (24)$$

Equation (24) can be rearranged in the form:

$$A\, q_{\circ}\, \rho\, c_p\sqrt{\frac{\alpha}{s}}\sqrt{\frac{s}{\alpha}}\,(x-d) + exp\ -\sqrt{\frac{s}{\alpha}}\,(x-d))$$

$$- A\, q_{\circ}\, \rho\, c_p\sqrt{\frac{\alpha}{s}}\left[\exp exp\sqrt{\frac{s}{\alpha}}\, x + exp\ -\sqrt{\frac{s}{\alpha}}\, x\right] - A\, q_{\circ}\frac{h_{\circ}}{s}\left[\exp exp\sqrt{\frac{s}{\alpha}}\, x\right.$$

$$\left.- exp\ -\sqrt{\frac{s}{\alpha}}\, x\right] + A\, q_{\circ}\frac{h_d}{s}\sqrt{\frac{s}{\alpha}}\,(d-x) - exp\ -\sqrt{\frac{s}{\alpha}}\,(d-x)] + A\, q_{\circ}\frac{h_{\circ}}{s}\left[\exp exp\sqrt{\frac{s}{\alpha}}\, x - exp\ -\sqrt{\frac{s}{\alpha}}\, x\right]$$

$$+ A\, q_{\circ}\frac{\lambda}{s}\sqrt{\frac{s}{\alpha}}\sqrt{\frac{s}{\alpha}}\, x + exp\ -\sqrt{\frac{s}{\alpha}}\, x] \qquad (25)$$

Considering the hyperbolic functions

$$\left.\begin{aligned} x &= \frac{e^{x}+e^{-x}}{2} \\ \sinh sinh\, x &= \frac{e^{x}-e^{-x}}{2} \end{aligned}\right\} \qquad (26)$$

Equation (25) can thus be rewritten as:

$$2\, A\, q_{\circ}\, \rho\, c_p\sqrt{\frac{\alpha}{s}}\, cosh\sqrt{\frac{s}{\alpha}}\,(x-d) - 2\, A\, q_{\circ}\, \rho\, c_p\sqrt{\frac{\alpha}{s}}\cosh cosh\sqrt{\frac{s}{\alpha}}\, x$$

$$- 2\, A\, q_{\circ}\frac{h_{\circ}}{s}\sinh sinh\sqrt{\frac{s}{\alpha}}\, x + 2\, A\, q_{\circ}\frac{h_d}{s}\sinh sinh\sqrt{\frac{s}{\alpha}}\,(d-x) + 2\, A\, q_{\circ}\frac{h_{\circ}}{s}\sinh sinh\sqrt{\frac{s}{\alpha}}\, x$$

$$+ 2\, A\, q_{\circ}\frac{\lambda}{s}\sqrt{\frac{s}{\alpha}}\cosh cosh\sqrt{\frac{s}{\alpha}}\, x \qquad (27)$$

We finally obtained the numerator in the form:

$$2\left(A\, q_{\circ}\, \rho\, c_p\sqrt{\frac{\alpha}{s}}\right)cosh\sqrt{\frac{s}{\alpha}}\,(x-d) + 2\left(A\, q_{\circ}\frac{\lambda}{s}\sqrt{\frac{s}{\alpha}} - A\, q_{\circ}\, \rho\, c_p\sqrt{\frac{\alpha}{s}}\right)\cosh cosh\sqrt{\frac{s}{\alpha}}\, x$$

$$+ 2\left(A\, q_{\circ}\frac{h_d}{s}\right)\sinh sinh\sqrt{\frac{s}{\alpha}}\,(d-x) \qquad (28)$$

From equation (22), the denominator equals

(ah+bg)

To simplify the denominator, let us substitute for a,b,g and h once more from equations (14),(15),(18) and (19) it attains the form:

$$\left(h_{\circ}s - \lambda\, s\sqrt{\frac{s}{\alpha}}\right)\left[\rho\, c_p\sqrt{\frac{\alpha}{s}}\exp exp\left(-\sqrt{\frac{s}{\alpha}}\, d\right) - \sqrt{\frac{\alpha}{s}}\rho\, c_p - \frac{h_d}{s}\exp exp\left(-\sqrt{\frac{s}{\alpha}}\, d\right) - \frac{h_{\circ}}{s}\right]$$

$$+ \left(\lambda\, s\sqrt{\frac{s}{\alpha}} + h_{\circ}s\right)\left[\rho\, c_p\sqrt{\frac{\alpha}{s}}\exp exp\left(\sqrt{\frac{s}{\alpha}}\, d\right) - \sqrt{\frac{\alpha}{s}}\rho\, c_p + \frac{h_d}{s}\exp exp\left(\sqrt{\frac{s}{\alpha}}\, d\right) + \frac{h_{\circ}}{s}\right] \qquad (29)$$

Equation (29) can be rearranged to give:

$$h_{\circ}s\left[\rho\, c_p\sqrt{\frac{\alpha}{s}}\exp exp\left(-\sqrt{\frac{s}{\alpha}}\, d\right) - \sqrt{\frac{\alpha}{s}}\rho\, c_p - \frac{h_d}{s}\exp exp\left(-\sqrt{\frac{s}{\alpha}}\, d\right) - \frac{h_{\circ}}{s}\right]$$

$$- \lambda s\sqrt{\frac{s}{\alpha}}\left[\rho c_p \sqrt{\frac{\alpha}{s}} \exp exp\left(-\sqrt{\frac{s}{\alpha}} d\right) - \sqrt{\frac{\alpha}{s}}\rho c_p - \frac{h_d}{s} \exp exp\left(-\sqrt{\frac{s}{\alpha}} d\right) - \frac{h_\circ}{s}\right]$$
$$+ \lambda s\sqrt{\frac{s}{\alpha}}\left[\rho c_p \sqrt{\frac{\alpha}{s}} \exp exp\left(\sqrt{\frac{s}{\alpha}} d\right) - \sqrt{\frac{\alpha}{s}} \rho c_p + \frac{h_d}{s} \exp exp\left(\sqrt{\frac{s}{\alpha}} d\right) + \frac{h_\circ}{s}\right] + h_\circ s$$
$$\left[\rho c_p \sqrt{\frac{\alpha}{s}} \exp exp\left(\sqrt{\frac{s}{\alpha}} d\right) - \sqrt{\frac{\alpha}{s}} \rho c_p + \frac{h_d}{s} \exp exp\left(\sqrt{\frac{s}{\alpha}} d\right) + \frac{h_\circ}{s}\right] \tag{30}$$

Equation (30) can further be rearranged to be in the form:

$$h_\circ s \rho c_p \sqrt{\frac{\alpha}{s}}\left(exp\left(\sqrt{\frac{s}{\alpha}} d\right) + exp\left(-\sqrt{\frac{s}{\alpha}} d\right)\right) - 2 h_\circ s \rho c_p \sqrt{\frac{\alpha}{s}}$$
$$+ h_\circ h_d\left(exp\left(\sqrt{\frac{s}{\alpha}} d\right) - exp\left(-\sqrt{\frac{s}{\alpha}} d\right)\right) + \lambda s\rho c_p \left(exp\left(\sqrt{\frac{s}{\alpha}} d\right) - exp\left(-\sqrt{\frac{s}{\alpha}} d\right)\right)$$
$$+ \lambda s\sqrt{\frac{s}{\alpha}}\frac{h_d}{s}\left(exp\left(\sqrt{\frac{s}{\alpha}} d\right) + exp\left(-\sqrt{\frac{s}{\alpha}} d\right)\right) + 2 h_\circ \lambda \sqrt{\frac{s}{\alpha}} \tag{31}$$

Equation (31) can be rewritten in the form:

$$2 h_\circ s \rho c_p \sqrt{\frac{\alpha}{s}} \cosh cosh\left(\sqrt{\frac{s}{\alpha}} d\right) - 2 h_\circ s \rho c_p \sqrt{\frac{\alpha}{s}} + 2h_\circ h_d \sinh sinh\left(\sqrt{\frac{s}{\alpha}} d\right) + 2 \lambda s\rho c_p \left(\sqrt{\frac{s}{\alpha}} d\right)$$
$$+ 2\lambda \sqrt{\frac{s}{\alpha}} h_d \cosh cosh\left(\sqrt{\frac{s}{\alpha}} d\right) + 2 h_\circ \lambda \sqrt{\frac{s}{\alpha}} \tag{32}$$

Taking into account that:

$$2 h_\circ \lambda \sqrt{\frac{s}{\alpha}} = 2 h_\circ \sqrt{\lambda s \rho c_p} \tag{33}$$

and

$$- 2 h_\circ s \rho c_p \sqrt{\frac{\alpha}{s}} =- 2 h_\circ \sqrt{\lambda s \rho c_p} \tag{34}$$

Moreover, it is worth to note that.

$$2 h_\circ s \rho c_p \sqrt{\frac{\alpha}{s}} = 2 h_\circ \sqrt{\frac{s^2 \rho^2 c_p{}^2 \lambda}{s \rho c_p}} = 2 h_\circ \sqrt{\lambda s \rho c_p} \tag{35}$$

and

$$2 h_d \lambda \sqrt{\frac{s}{\alpha}} = 2 h_d \sqrt{\frac{s \rho c_p \lambda^2}{\lambda}} = 2 h_d \sqrt{\lambda s \rho c_p} \tag{36}$$

This makes it possible to write equation (32) for the denominator in the form:

$$2 \cosh cosh\left(\sqrt{\frac{s}{\alpha}} d\right)\left(h_\circ + h_d\right)\sqrt{s \lambda \rho c_p} + 2 \sinh sinh\left(\sqrt{\frac{s}{\alpha}} d\right)\left(h_\circ h_d + \lambda s\rho c_p \right) \tag{37}$$

Finally we get the following expression for $\overline{T}(x, s)$ from equations (28) and (37) in the form:

$$= \frac{2\left(A q_\circ \rho c_p \sqrt{\frac{\alpha}{s}}\right) cosh \sqrt{\frac{s}{\alpha}} (x-d) + 2\left(A q_\circ \frac{\lambda}{s}\sqrt{\frac{s}{\alpha}} - A q_\circ \rho c_p \sqrt{\frac{\alpha}{s}}\right) cosh \sqrt{\frac{s}{\alpha}}\, x + 2\sqrt{\frac{s}{\alpha}}\, (d-x)}{2 \cosh cosh\left(\sqrt{\frac{s}{\alpha}} d\right)\sqrt{s \lambda \rho c_p}\,(h_\circ + h_d) + 2 \sinh sinh\left(\sqrt{\frac{s}{\alpha}} d\right)\left(h_\circ h_d + \lambda s\rho c_p \right)} \tag{38}$$

We cannot find from the standard tables, the inverse Laplace transform for equation (38).

On the basis of mathematical logic one can simplify the expression of the denominator equation (37) as follows:

Since the heat transfer coefficient $h_\circ \cong 10^5 \frac{W}{m^2 K}$ also $h_d \cong 10^5 \frac{W}{m^2 K}$

One can conclude that

$h_{\circ} h_{d} \gg (h_{\circ} + h_{d})$

Moreover, $\lambda\, s\, \rho\, c_p > \sqrt{s\, \lambda\, \rho\, c_p}$

This makes it possible to put the denominator as follows:

$$= 2\left(\sqrt{\frac{s}{\alpha}}\, d\right) \quad (39)$$

We still cannot find the Laplace inverse transform for $\overline{T}(x, s)$ that is because the denominator consists of two terms:

Let us proceed as follows:

Since one cannot evaluate the order of magnitude of the term $\lambda\, s\rho\, c_p$ relative to the term $h_{\circ} h_d$, we shall consider two probabilities as follows:

(i) The case for which $h_{\circ} h_{d} \gg \lambda\, s\rho\, c_p$,

(ii) The case for which $\lambda\, s\rho\, c_p \gg h_{\circ} h_d$

Thus we shall have eight expressions for both probabilities, that is because the numerator consists of four terms.

For the first probability $(h_{\circ} h_d >> \lambda\, s\rho\, c_p)$

$$let, \quad \overline{T}(x, s) = \overline{T}_1(x, s) + \overline{T}_2(x, s) + \overline{T}_3(x, s) + \overline{T}_4(x, s) \quad (40)$$

For the second probability $(\lambda\, s\rho\, c_p >> h_{\circ} h_d)$

$$let, \quad \overline{T}^{*}(x, s) = \overline{T}_1^{*}(x, s) + \overline{T}_2^{*}(x, s) + \overline{T}_3^{*}(x, s) + \overline{T}_4^{*}(x, s) \quad (41)$$

where

$$\overline{T}_1(x, s) = \frac{2\left(A\, q_{\circ}\, \rho\, c_p \sqrt{\frac{\alpha}{s}}\right) cosh\sqrt{\frac{s}{\alpha}}\,(x-d)}{2h_{\circ} h_d\, \text{sinh}sinh\left(\sqrt{\frac{s}{\alpha}}\, d\right)}, \quad (42)$$

$$\overline{T}_2(x, s) = \frac{2\ \left(A\, q_{\circ} \frac{\lambda}{s} \sqrt{\frac{s}{\alpha}}\right) cosh\sqrt{\frac{s}{\alpha}}\, x}{2h_{\circ} h_d \text{sinh}sinh\left(\sqrt{\frac{s}{\alpha}}\, d\right)}, \quad (43)$$

$$\overline{T}_3(x, s) = \frac{-2\ \ \left(A\, q_{\circ}\, \rho\, c_p \sqrt{\frac{\alpha}{s}}\right) cosh\sqrt{\frac{s}{\alpha}}\, x}{2\text{sinh} h_{\circ} h_d sinh\left(\sqrt{\frac{s}{\alpha}}\, d\right)}, \quad (44)$$

$$\overline{T}_4(x, s) = \frac{2\ \left(A\, q_{\circ} \frac{h_d}{s}\right) \text{sinh}sinh\sqrt{\frac{s}{\alpha}}\ (d-x)}{2h_{\circ} h_d \text{sinh}sinh\left(\sqrt{\frac{s}{\alpha}}\, d\right)} \quad (45)$$

For the second probability $(\lambda\, s\rho\, c_p >> h_{\circ} h_d)$

$$\overline{T}_1^{*}(x, s) = \frac{2\left(A\, q_{\circ}\, \rho\, c_p \sqrt{\frac{\alpha}{s}}\right) cosh\sqrt{\frac{s}{\alpha}}\,(x-d)}{2\left(\lambda\, s\rho\, c_p\right)\text{sinh}sinh\left(\sqrt{\frac{s}{\alpha}}\, d\right)}, \quad (46)$$

$$\overline{T}_2^{*}(x, s) = \frac{2\ \ \left(A\, q_{\circ} \frac{\lambda}{s} \sqrt{\frac{s}{\alpha}}\right) cosh\sqrt{\frac{s}{\alpha}}\, x}{2\left(\lambda\, s\rho\, c_p\right)\text{sinh}sinh\left(\sqrt{\frac{s}{\alpha}}\, d\right)}, \quad (47)$$

$$\overline{T}_3^{\,*}(x,s)=\frac{-2\,(A\,q_\circ\,\rho\,c_p\sqrt{\frac{\alpha}{s}})\,cosh\sqrt{\frac{s}{\alpha}}x}{2\left(\lambda\,s\rho\,c_p\right)\mathrm{sinh}sinh\left(\sqrt{\frac{s}{\alpha}}d\right)}, \quad (48)$$

$$\overline{T}_4^{\,*}(x,s)=\frac{2\,(A\,q_\circ\,\frac{h_d}{s})\mathrm{sinh}sinh\sqrt{\frac{s}{\alpha}}\,(d-x)}{2\mathrm{sinh}\left(\lambda\,s\rho\,c_p\right)sinh\left(\sqrt{\frac{s}{\alpha}}d\right)} \quad (49)$$

According to the standard tables of special Laplace transform:[17]

One finds the following transformations:

$$(1)\frac{\mathrm{cosh}cosh\,x\sqrt{s}}{\sqrt{s}\;\mathrm{sinh}sinh\,a\sqrt{s}}=\frac{1}{a}+\frac{2}{a}\sum_{n=1}^{\infty}(-1)^n e^{\frac{-n^2\pi^2t}{a^2}}\,.\,cos\frac{n\pi x}{a} \quad (50)$$

$$(2)\frac{\mathrm{sinh}sinh\,x\sqrt{s}}{s^2\mathrm{sinh}sinh\,a\sqrt{s}}=\frac{x\,t}{a}+\frac{2\,a^2}{\pi^3}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}\left(1-e^{\frac{-n^2\pi^2t}{a^2}}\right).\,sin\frac{n\pi x}{a} \quad (51)$$

$$(3)\frac{\mathrm{sinh}sinh\,x\sqrt{s}}{s\mathrm{sinh}sinh\,a\sqrt{s}}=\frac{x}{a}+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}e^{\frac{-n^2\pi^2t}{a^2}}\,.\,sin\frac{n\pi x}{a} \quad (52)$$

These make it possible to get the inverse transform of equations (42)-(49) in the form:

For the first probability $(h_\circ h_d >> \lambda\,s\rho\,c_p)$

$$T_1(x,t)=\frac{A\,q_\circ\,\rho\,c_p\,\alpha}{h_\circ h_d\;\;d}+\frac{2\,A\,q_\circ\,\rho\,c_p\,\alpha}{h_\circ h_d d}\sum_{n=1}^{\infty}(-1)^n e^{\frac{-n^2\pi^2t\alpha}{d^2}}\;cos\frac{n\pi(x-d)}{d} \quad (53)$$

$$T_2(x,t)=\frac{A\,q_\circ\,\lambda}{h_\circ h_d}\left[\frac{1}{d}+\frac{2}{d}\sum_{n=1}^{\infty}(-1)^n e^{\frac{-n^2\pi^2t\alpha}{d^2}}\;cos\frac{n\pi x}{d}\right] \quad (54)$$

$$T_3(x,t)=\frac{-A\,q_\circ\,\rho\,c_p}{h_\circ h_d}\left[\frac{\alpha}{d}+\frac{2\alpha}{d}\sum_{n=1}^{\infty}(-1)^n e^{\frac{-n^2\pi^2t\alpha}{d^2}}\;cos\frac{n\pi x}{d}\right] \quad (55)$$

$$T_4(x,t)=\frac{A\,q_\circ}{h_\circ}\left[\frac{(d-x)}{d}+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}e^{\frac{-n^2\pi^2t\alpha}{d^2}}\;sin\frac{n\pi(d-x)}{d}\right] \quad (56)$$

For the second probability $(\lambda\,s\rho\,c_p >> h_\circ h_d)$

$$T_1^{\,*}(x,t)=\frac{A\,q_\circ\,\alpha\,t}{\lambda\,d}+\frac{2A\,q_\circ\,d}{\lambda\,\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{-n^2}\left[e^{\frac{-n^2\pi^2t\alpha}{d^2}}-1\right]cos\frac{n\pi(x-d)}{d} \quad (57)$$

Note:

$$L^{-1}\left\{\frac{f(s)}{s}\right\}=\int_0^t F(u)du \quad [2] \quad (58)$$

$$T_2^{\,*}(x,t)=\frac{A\,q_\circ}{\rho\,c_p}\left\{\frac{t}{d}+\frac{2d}{\pi^2\alpha}\sum_{n=1}^{\infty}\frac{(-1)^n}{-n^2}\left[e^{\frac{-n^2\pi^2t\alpha}{d^2}}-1\right]cos\frac{n\pi x}{d}\right\} \quad (59)$$

$$T_3^{\,*}(x,t)=\frac{-A\,q_\circ}{\lambda}\left\{\frac{\alpha\,t}{d}+\frac{2d}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{-n^2}\left[e^{\frac{-n^2\pi^2t\alpha}{d^2}}-1\right]cos\frac{n\pi x}{d}\right\} \quad (60)$$

$$T_4^*(x,t) = \frac{A\,q_\circ h_d}{\lambda\,\rho\,c_p}\left\{\frac{(d-x)\,t}{d} + \frac{2d^2}{\alpha\,\pi^3}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}\left[1 - e^{\frac{-n^2\pi^2 t\alpha}{d^2}}\right]sin\frac{n\pi(d-x)}{d}\right\} \quad (61)$$

Thus the temperature profile for the first probability $(h_\circ h_d >> \lambda\, s\rho\, c_p)$ can be written in the form:

$$T(x,t) = \frac{A\,q_\circ\,\rho\,c_p\,\alpha}{h_\circ h_d\quad d} + \left\{\frac{2\,A\,q_\circ\,\rho\,c_p\,\alpha}{h_\circ h_d d}\sum_{n=1}^{\infty}(-1)^n e^{\frac{-n^2\pi^2 t\alpha}{d^2}}\ cos\frac{n\pi(x-d)}{d}\right\} + \frac{A\,q_\circ\,\lambda}{h_\circ h_d}\left\{\frac{1}{d} + \frac{2}{d}\sum_{n=1}^{\infty}(-1)^n e^{\frac{-n^2\pi^2 t\alpha}{d^2}}\ cos\frac{n\pi x}{d}\right\}$$

$$-\frac{A\,q_\circ\,\rho\,c_p}{h_\circ h_d}\left\{\frac{\alpha}{d} + \frac{2\alpha}{d}\sum_{n=1}^{\infty}(-1)^n e^{\frac{-n^2\pi^2 t\alpha}{d^2}}\ cos\frac{n\pi x}{d}\right\} + \frac{A\,q_\circ}{h_\circ}\left\{\frac{(d-x)}{d} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}e^{\frac{-n^2\pi^2 t\alpha}{d^2}}\ sin\frac{n\pi(d-x)}{d}\right\} \quad (62)$$

Equation (62) can be rearranged to give:

$$T(x,t) = \frac{A\,q_\circ\,\rho\,c_p\,\alpha}{h_\circ h_d\quad d} + \left\{\frac{2\,A\,q_\circ\,\rho\,c_p\,\alpha}{h_\circ h_d\,d}\sum_{n=1}^{\infty}(-1)^n e^{\frac{-n^2\pi^2 t\alpha}{d^2}}\ cos\frac{n\pi(x-d)}{d}\right\} + \frac{A\,q_\circ}{h_\circ}\left\{\frac{(d-x)}{d} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}e^{\frac{-n^2\pi^2 t\alpha}{d^2}}\ sin\frac{n\pi(d-x)}{d}\right\}$$

The temperature profile for the second probability

$(\lambda\, s\rho\, c_p >> h_\circ h_d)$ can be written as:

$$T(x,t) = \frac{Aq_\circ\alpha\, t}{\lambda\, d} + \frac{2\,Aq_\circ\, d}{\lambda\,\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{-n^2}\left[e^{\frac{-n^2\pi^2 t\alpha}{d^2}} - 1\right]cos\frac{n\pi(x-d)}{d}$$

$$+\frac{A\,q_\circ}{\rho\,c_p}\left\{\frac{t}{d} + \frac{2d}{\pi^2\alpha}\sum_{n=1}^{\infty}\frac{(-1)^n}{-n^2}[e^{\frac{-n^2\pi^2 t\alpha}{d^2}} - 1]cos\frac{n\pi x}{d}\right\}$$

$$-\frac{A\,q_\circ}{\lambda}\left\{\frac{\alpha\, t}{d} + \frac{2d}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{-n^2}(e^{\frac{-n^2\pi^2 t\alpha}{d^2}} - 1)cos\frac{n\pi x}{d}\right\}$$

$$+\frac{A\,q_\circ h_d}{\lambda\,\rho\,c_p}\left\{\frac{(d-x)t}{d} + \frac{2\,d^2}{\alpha\,\pi^3}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}(1 - e^{\frac{-n^2\pi^2 t\alpha}{d^2}})sin\frac{n\pi(d-x)}{d}\right\} \quad (64)$$

This profile can further be simplified to be in the form

$$T(x,t) = \frac{A\,q_\circ\,\alpha\, t}{\lambda\, d} + \frac{2A\,q_\circ\, d}{\lambda\,\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{-n^2}\left[e^{\frac{-n^2\pi^2 t\alpha}{d^2}} - 1\right]cos\frac{n\pi(x-d)}{d}$$

$$+\frac{A\,q_\circ h_d}{\lambda\,\rho\,c_p}\left\{\frac{(d-x)\,t}{d} + \frac{2\,d^2}{\alpha\,\pi^3}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}(1 - e^{\frac{-n^2\pi^2 t\alpha}{d^2}})sin\frac{n\pi(d-x)}{d}\right\} \quad (65)$$

Expression (65) indicates that as the coefficient of cooling h_d increases the temperature increases. This contradicts the well known physical behavior according to which as cooling increases the temperature of the target decreases.

To overcome this situation one has to modify the original expression for $\overline{T}_4^*(x,s)$ written in the form:

$$\overline{T}_4^*(x,s) = \frac{2\,(A\,q_\circ\frac{h_d}{s})\sinh sinh\sqrt{\frac{s}{\alpha}}\,(d-x)}{2\sinh\left(h_\circ h_d + \lambda\, s\rho\, c_p\right)sinh\left(\sqrt{\frac{s}{\alpha}}\,d\right)} \quad (66)$$

Let us divide both the numerator and the denominator in equation (66) by h_d, one gets:

$$\overline{T}_4^*(x,s) = \frac{2\,(A\,q_\circ)\sinh sinh\sqrt{\frac{s}{\alpha}}\,(d-x)}{2\sinh\left(h_\circ\, s + \frac{\lambda\, s^2\rho\, c_p}{h_d}\right)sinh\left(\sqrt{\frac{s}{\alpha}}\,d\right)} \quad (67)$$

Discussing the order of magnitude of different terms in equation (67) one finds that:

$$h_\circ\, s \gg \frac{\lambda\, s\rho\, c_p}{h_d}\,,$$

Since $h_\circ = h_d = 10^6\, w/m^2K$

Thus one can write for $\overline{T}_4^{\,*}(x, s)$ the following expression

$$\overline{T}_4^{\,*}(x, s) = \frac{2\,A\,q_\circ \sinh sinh\sqrt{\frac{s}{\alpha}}\,(d-x)}{2\left(\sqrt{\frac{s}{\alpha}}\,d\right)} \tag{68}$$

The Laplace inverse transform of equation (68) gives: [2]

$$\overline{T}_4^{\,*} = \frac{A\,q_\circ}{h_\circ}\{\frac{(d-x)}{d} +$$

$$+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\,e^{\frac{-n^2\pi^2 t\alpha}{d^2}}\,sin\frac{n\pi(d-x)}{d}\} \tag{69}$$

Finally for the second probability one gets the modified expression in the form:

$$T(x,t) = \frac{A\,q_\circ\,\alpha\,t}{\lambda\,d} + \frac{2A\,q_\circ\,d}{\lambda\,\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{-n^2}\left[e^{\frac{-n^2\pi^2 t\alpha}{d^2}} - 1\right]cos\frac{n\pi(x-d)}{d}$$

$$+\frac{A\,q_\circ}{h_\circ}\left\{\frac{(d-x)}{d} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\,e^{\frac{-n^2\pi^2 t\alpha}{d^2}}\,sin\frac{n\pi(d-x)}{d}\right\} \tag{70}$$

Equation (70) does not satisfy the condition (2) for which T(x,0)=0 (2) ,

instead one gets:

$$T(x,0) = \frac{A\,q_\circ}{h_\circ}\left\{\frac{(d-x)}{d} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\,sin\frac{n\pi(d-x)}{d}\right\}$$

(71)

Thus to satisfy the condition (2) one has to subtract equation (71) from equation (70) to get the profile for the second probability in the form:

$$T(x,t) = \frac{A\,q_\circ\,\alpha\,t}{\lambda\,d} + \frac{2A\,q_\circ\,d}{\lambda\,\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{-n^2}\left[e^{\frac{-n^2\pi^2 t\alpha}{d^2}} - 1\right]cos\frac{n\pi(x-d)}{d} + \frac{A\,q_\circ}{h_\circ}\,\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\,(e^{\frac{-n^2\pi^2 t\alpha}{d^2}} - 1)\;sin\frac{n\pi(d-x)}{d} \tag{72}$$

Moreover, Equation (63) for the first probability does not also satisfy the condition in equation (2), T(x,0)=0.

Instead one gets:

$$T(x,0) = \frac{A\,q_\circ\,\rho\,c_p\,\alpha}{h_\circ h_d\quad d} + \left\{\frac{2\,A\,q_\circ\,\rho\,c_p\,\alpha}{h_\circ h_d\,d}\sum_{n=1}^{\infty}(-\,1)^n\,cos\frac{n\pi(x-d)}{d}\right\} + \frac{A\,q_\circ}{h_\circ}\left\{\frac{(d-x)}{d} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\,sin\frac{n\pi(d-x)}{d}\right\} \tag{73}$$

Thus we have to subtract equation (73) form equation (63) to get the profile for the first probability $(h_\circ h_d > \lambda\, s\rho\, c_p)$ in the form:

$$T(x,t) = \left\{\frac{2\,A\,q_\circ\,\rho\,c_p\,\alpha}{h_\circ h_d\,d}\sum_{n=1}^{\infty}(-\,1)^n\,(e^{\frac{-n^2\pi^2 t\alpha}{d^2}} - 1)\,cos\frac{n\pi(x-d)}{d}\right\} + \frac{A\,q_\circ}{h_\circ}\left\{\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\,(e^{\frac{-n^2\pi^2 t\alpha}{d^2}} - 1)\,sin\frac{n\pi(d-x)}{d}\right\} \tag{74}$$

Thus the total profile for the problem for both probabilities is the sum of equations (72) and (74).

This gives

$$T(x,t) = \left\{\frac{2\,A\,q_\circ\,\rho\,c_p\,\alpha}{h_\circ h_d\,d}\sum_{n=1}^{\infty}(-\,1)^n\,(e^{\frac{-n^2\pi^2 t\alpha}{d^2}} - 1)\,cos\frac{n\pi(x-d)}{d}\right\} + \frac{A\,q_\circ}{h_\circ}\left\{\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\,(e^{\frac{-n^2\pi^2 t\alpha}{d^2}} - 1)\,sin\frac{n\pi(d-x)}{d}\right\}$$

$$+\frac{A\,q_\circ\,\alpha\,t}{\lambda\,d} + \frac{2A\,q_\circ\,d}{\lambda\,\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\left[1 - e^{\frac{-n^2\pi^2 t\alpha}{d^2}}\right]cos\frac{n\pi(x-d)}{d} \tag{75}$$

3. Computations

As an illustrative example computations for a slab of silver sulfide material of thickness d= $1\times10^{-3}\ m$, irradiated by a constant laser irradiance (cw laser) of value $q_{\circ} = 0.5 \times10^{8}\ W/m^2$.

The thermal and optical properties of the slab material are assumed to be temperature independent and are given in table 1 [15].

Cooling conditions at the front and rear surfaces of the slab are considered through the coefficient of heat transfer by convection at the front surface $h_{\circ} = 10^{6}\ w/m^{2}K$ and that at the rear surface $h_{d} = 10^{6}\ w/m^{2}K$.

Table I. The physical, thermal and optical properties of the silver sulfide (Ag_2S) slab material [15]

Material	ρ (kg/m^3)	λ (W/m.K)	c_p (J/ kg.K)	α (m^2/sec)	T_m (K)	A
Silver sulfide	7234	25	309	1.12×10^{-5}	1098	0.7

Computations using (equation 75) for the profiles T(0.t),T(d,t) and T(x,t) are performed with the aid of MATLAB-2022 version.

The obtained results are given in tables 2,3 and 4 and are illustrated graphically in Fig.1,2 and 3 respectively.

Table 2. The front surface temperature $T(0, t),\ K$.

Time (t) /second	T(0, t),K
0	0
0.5×10^{-2}	314.8418
1×10^{-2}	469.0836
1.5×10^{-2}	587.8606
2×10^{-2}	689.1478
2.5×10^{-2}	780.6792
3×10^{-2}	866.6308
3.5×10^{-2}	949.3755
4×10^{-2}	1030.3
4.2×10^{-2}	1062.3
4.4×10^{-2}	1094.2
4.425×10^{-2} (t_{cr})	1098.2

Table 3. The rear surface temperature $T(d, t),\ K$.

Time (t)/second	T(d,t),K
0	0
1×10^{-2}	13.1563
1.5×10^{-2}	52.6594
2×10^{-2}	108.3344
2.5×10^{-2}	173.6207
3×10^{-2}	244.4711
3.5×10^{-2}	318.5266
4×10^{-2}	394.4267

Table 4. The temperature profile $T(x, t),\ K\ at\ t\ =\ 3x10^{-2}\ sec.$

X /m	T(x, t),K
0	866.6308
0.2×10^{-3}	746.9854
0.3×10^{-3}	612.5593
0.4×10^{-3}	523.4662
0.5×10^{-3}	454.2109
0.6×10^{-3}	379.5121
0.7×10^{-3}	320.4793
0.8×10^{-3}	289.4523
0.9×10^{-3}	265.8939
1×10^{-3}	244.4711

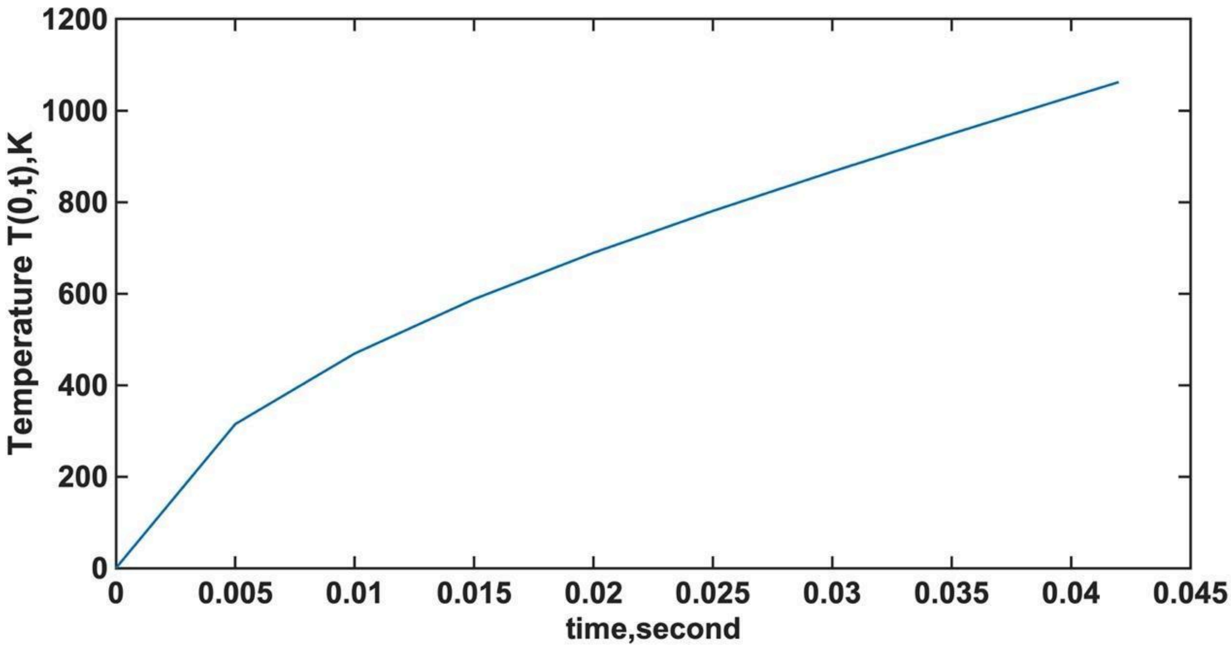

Figure 1. The temperature of the front surface $T(0, t)$, K .

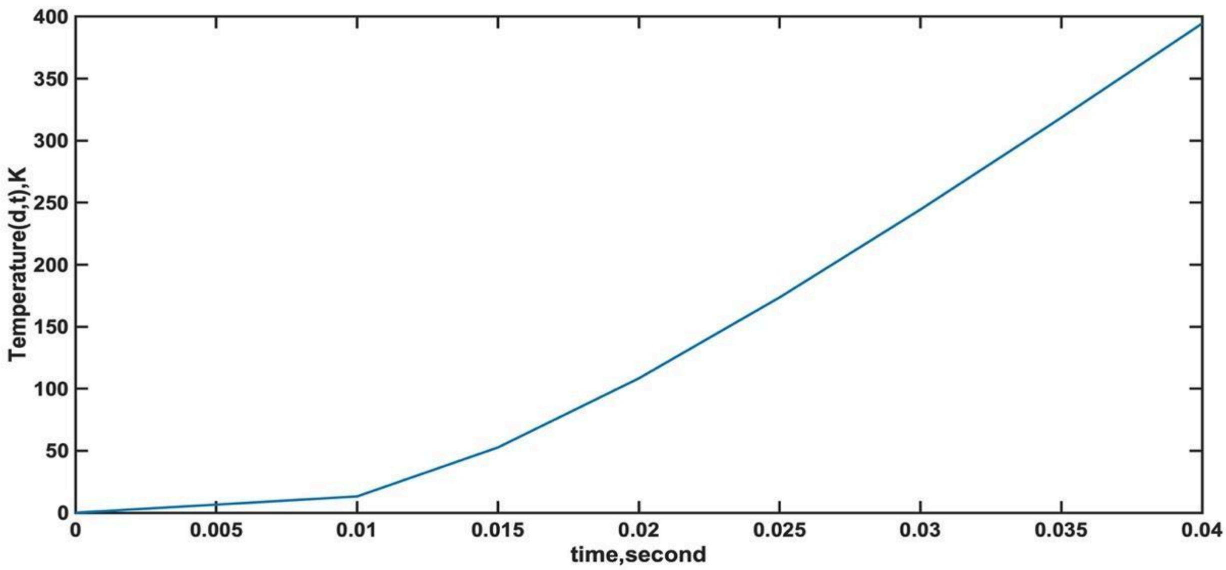

Figure 2. The temperature of the rear surface $T(d, t)$, K.

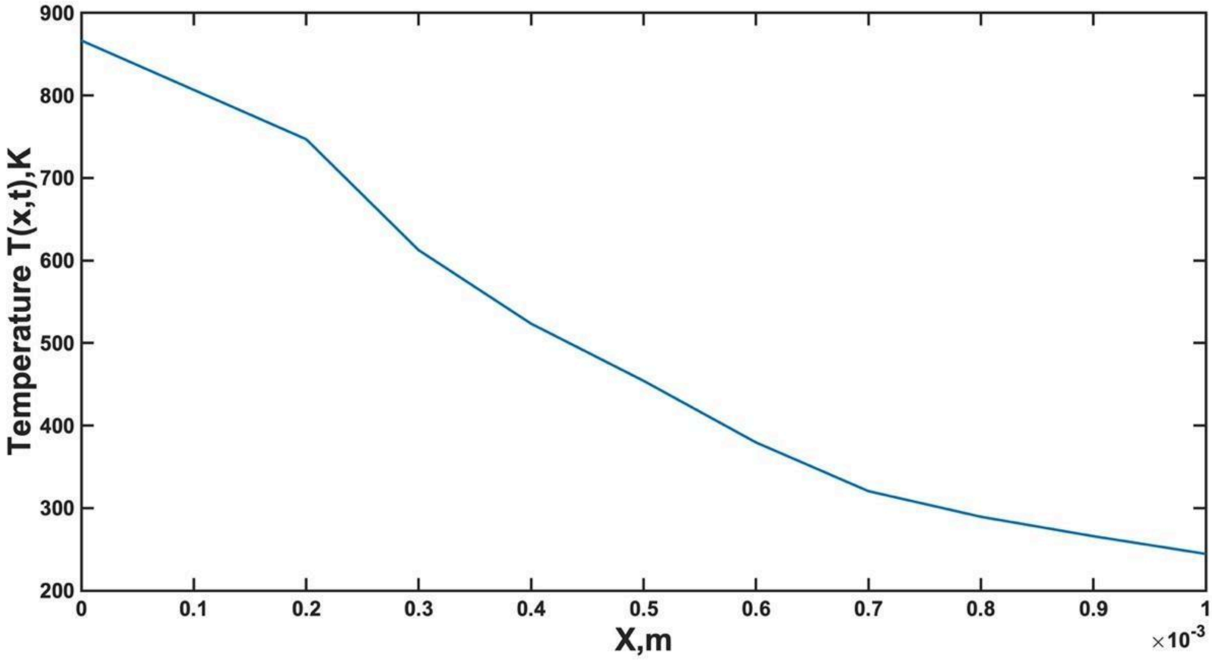

Figure 3. The temperature profile $T(x,t),\ K\ at\ t\ =\ 3x10^{-2}\ sec.$

4. Conclusions

The obtained results make it possible to highlight the following conclusions:

1- Laplace integral transfer technique is promising in solving heat transfer problems though it is somewhat cumbersome.

2- The obtained mathematical expressions for the profiles reveal that the dependence of these profiles on both the laser irradiance $q_{\circ}\ W/m^2$ and the optical absorption A is linear.

3- The dependence of these profiles on the other physical and thermal parameters is not linear.

4- The expressions for the profiles T(0.t), T(d,t) and T(x,t) are obtained in a simple form. This is important for practical and technological applications.

5- The critical time (t_{cr}) required to initiate melting (damage) at the front surface can easily be obtained for which one can put T(0, t_{cr})=T_m , where T_m is the melting temperature of the slab material. For the considered target $t_{cr}\ =\ 4.425\times10^{-2}$ second.

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