DOI: https://doi.org/10.24297/qrnybf21

On the Lie Symmetry Analysis of the Generalized Seventh-Order Nonlinear Kawahara Equation

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(Received: Month / Year. Accepted: Month / Year)

Abstract

In this paper, a comprehensive Lie symmetry analysis of the generalized seventh-order Kawahara equation is carried out. The equation considered is $u_t + 6uu_x + u_{xxx} - u_{xxxxx} + \beta u_{xxxxxxx} = 0, \beta \neq 0$ where $u = u(x, t)$and $\beta$is a non-zero constant parameter. The study systematically determines the total derivative operators and constructs the seventh prolongation of the infinitesimal generator. The invariance condition is applied to obtain the determining equations, which yield a three-dimensional Lie algebra of infinitesimal symmetries.

Optimal systems of one-dimensional subalgebras are derived and used to obtain similarity variables and invariant reductions. The reduced ordinary differential equations are analyzed to construct exact invariant solutions, including solitary wave structures. Furthermore, exact power series solutions are derived via recursive coefficient relations. The results provide deeper insight into the symmetry structure, integrability properties, and nonlinear wave behavior governed by higher-order dispersive systems.

Keywords Kawahara equation. Lie symmetry analysis. prolongations. symmetry reduction. similarity reduction. invariant solutions. exact power series solutions.

1. Introduction

Nonlinear partial differential equations (NPDEs) play a central role in mathematical physics, fluid mechanics, and wave propagation theory. Higher-order dispersive equations describe complex wave phenomena where nonlinear and dispersive effects interact strongly.

The generalized Kawahara equation considered in this study is

$$u_t + 6uu_x + u_{xxx} - u_{xxxxx} + \beta u_{xxxxxxx} = 0, \beta \neq 0$$
(1.1)

which extends classical Kawahara-type models by incorporating seventh-order dispersion. Such equations arise in plasma physics, capillary-gravity wave theory, and nonlinear lattice dynamics.

Lie symmetry analysis, introduced by Sophus Lie [1], provides a systematic framework for analyzing differential equations through continuous transformation groups. It has been widely applied to nonlinear evolution equations [8] [9] [11], including KdV [7] [2], modified KdV [10] [3] [5], Burgers equation [4], and Boussinesq equations [3].

Despite significant progress, limited results exist for seventh-order Kawahara-type equations, particularly regarding complete symmetry structures, invariant reductions, and power series solutions. This work addresses this gap.

The outline of this paper is as follows: Section 2 presents the methods of using Lie symmetry analysis. Section 3 presents Symmetry reduction of PDE. Section 4 presents similarity reductions and exact invariant solutions. Section 5 gives the reduced ODE. Section 6 develops exact power series solutions. Finally, Section 7 concludes the paper with remarks and future directions.

2. Methods

2.1 Lie symmetry analysis

Let the Generator of equation (1.1) be given by

$$G = \alpha(x,t,u)\partial_x + \mu(x,t,u)\partial_t + \eta(x,t,u)\partial_u. \tag{1.2}$$

Also, let the n-th prolongation be defined as

$$Pr^{(n)}G = G + \sum_{m=1}^{n} \eta^{(m)}\frac{\partial}{\partial u_{(m)}}. \tag{1.3}$$

Equation (1.1) is invariant under G if and only if

$$Pr^{(7)}G(\Delta)|_{\Delta=0} = 0 \tag{1.4}$$

where

$$\Delta = u_t + 6uu_x + u_{xxx} - u_{xxxxx} + \beta u_{xxxxxxx} \tag{1.5}$$

Expanding (1.4) we obtain

$$\alpha u_{xt} + 6\alpha u_x^2 + 6\alpha uu_{xx} + \alpha u_{xxxx} - \alpha u_{xxxxxx} - \beta\alpha u_{xt} - 6\beta\alpha u_x^2 - 6\beta\alpha uu_{xx} - \beta\alpha u_{xxxx} + \beta\alpha u_{xxxxxx} - 6\mu u_t u_x - 6\mu uu_{tx} - \mu u_{xxx} \tag{1.6}$$

Upon simplifying it yields

$$6\eta u_x + \eta^t - 6u\eta^x + \eta^{xxx} + \beta\eta^{xxxxxxx} - \eta^{xxxxx} \tag{1.7}$$

Substituting the generated coefficients into (1.7) and solving, we obtain Lie point symmetries are

$$\alpha = -c_1t - c_2 \tag{1.8}$$

$$\mu = -c_3 \tag{1.9}$$

$$\eta = -\frac{c_1}{6} \tag{2.0}$$

To obtain the corresponding symmetry generators from the given infinitesimals, we construct the Lie point symmetry vector field in the standard form by substituting (1.8), (1.9), (2.0) into the general generator (1.2) to obtain

$$G = \left(-\frac{c_1}{6}\right)\frac{\partial}{\partial t} + (-c_1t - c_2)\frac{\partial}{\partial x} + (-c_3)\frac{\partial}{\partial u}. \tag{2.1}$$

Since c_1, c_2, c_3 are arbitrary constants, we decompose G into a linear combination of basis generators

$$G = c_1G_1 + c_2G_2 + c_3G_3. \tag{2.2}$$

Generator corresponding to c_1

$$G_1 = -\frac{1}{6}\frac{\partial}{\partial t} - t\frac{\partial}{\partial x}. \quad (2.3)$$

Generator corresponding to c_2

$$G_2 = -\frac{\partial}{\partial x}. \quad (2.4)$$

Generator corresponding to c_3

$$G_3 = -\frac{\partial}{\partial u}. \quad (2.5)$$

Thus the symmetry algebra is therefore traversed by the generators

$$G_1 = -\frac{1}{6}\partial_t - t\partial_x,\ G_2 = -\partial_x,\ G_3 = -\partial_u. \quad (2.6)$$

This shows that the generator G_1 is combined time translation and time-dependent spatial translation, G_2 is a pure spatial translation and G_3 is a translation in the dependent variable.

Since Lie algebras are invariant under scalar multiplication, we simplify by multiplying by -1 to obtain

$$\hat{G}_1 = \frac{1}{6}\partial_t + t\partial_x,\ \hat{G}_2 = \partial_x,\ \hat{G}_3 = \partial_u. \quad (2.7)$$

We then compute the Lie commutators for the symmetry generators previously obtained. The Lie bracket is defined as:

$$[G_i, G_j] = G_iG_j - G_jG_i. \quad (2.8)$$

Thus the commutators obtained are:

1. $\left[G_2\right]$

$G_1 = \frac{1}{6}\partial_t + t\partial_x, G_2 = \partial_x.$

we know that $[\partial_t, \partial_x] = 0$ and $t\partial_x$ commutes with ∂_x because t is independent of x

$$[G_1, G_2] = 0 \quad (2.9)$$

2. $\left[G_3\right]$

$G_3 = \partial_u$

from the generators, no generator depends on u and all coefficients independent of u hence

$$[G_1, G_3] = 0 \quad (3.0)$$

3. $\left[G_3\right]$

$[\partial_x, \partial_u] = 0 \ [G_2, G_3] = 0$ (3.1)

Table 1: The Commutator Table

[ ]	G_1	G_2	G_3
G_1	0	0	0
G_2	0	0	0
G_3	0	0	0

From these results, all commutators vanish thus the Lie algebra is an abelian Lie algebra.

We then compute the Adjoint Symmetries of the Lie algebra generated by the infinitesimal generators equation (2.7).

From the previous result, all commutators vanish given as $\left[G_i, G_j\right] = 0 \ \forall i, j$.

The adjoint action is well-defined as

$$Ad(exp(\varepsilon G_i))G_j = G_j - \varepsilon[G_i, G_j] + \frac{\varepsilon^2}{2}[G_i, [G_i, G_j]] - \cdots \quad (3.2)$$

Since the commutator results are $[G_i, G_j] = 0$, then all higher nested commutators are also zero

$$[G_i, [G_i, G_j]] = 0. \quad (3.3)$$

Therefore, the series truncates immediately.

For any generators G_i, G_j, we have

$$Ad(exp(\varepsilon G_i))G_j = G_j. \quad (3.4)$$

Thus we consider the adjoints by the generators given as

Adjoint by G_1

$$Ad(e^{\varepsilon G_1})G_1 = G_1,\ Ad(e^{\varepsilon G_1})G_2 = G_2,\ Ad(e^{\varepsilon G_1})G_3 = G_3. \quad (3.5)$$

Adjoint by G_2

$$Ad(e^{\varepsilon G_2})G_j = G_j \forall j. \quad (3.6)$$

Adjoint by G_3

$$Ad(e^{\varepsilon G_3})G_j = G_j \forall j. \quad (3.7)$$

Table 2: The Adjoint Symmetry Table

Ad	G_1	G_2	G_3

$e^{\varepsilon G_1}$	G_1	G_2	G_3
$e^{\varepsilon G_2}$	G_1	G_2	G_3
$e^{\varepsilon G_3}$	G_1	G_2	G_3

Since all commutators vanish, the adjoint action reduces to the identity transformation. Hence, the adjoint representation is trivial, and all generators remain invariant under conjugation. Subsequently, every one-dimensional subalgebra is equivalent, and no nontrivial optimal system occurs.

2.2 Optimal System of One-Dimensional Subalgebras

From the results, all commutators vanish, the algebra is 3-dimensional Abelian, and the adjoint representation is identity thus we determine the optimal system by the generators in (2.7).

We generate a one-dimensional subalgebra by

$$G = a_1G_1 + a_2G_2 + a_3G_3, (a_i \in R) \tag{3.8}$$

Then we classify subalgebras under the adjoint action.

For Abelian algebras, we simplify coefficients using

$$Ad(g)G = G \tag{3.9}$$

One-dimensional subalgebras are invariant under scalar multiplication $G \sim \lambda G, \lambda \neq 0$ thus we normalize coefficients by setting one nonzero coefficient to 1.

We then classify based on which coefficients are zero.

Case 1: Only one generator present

(i) $a_1 \neq 0$ yields $\langle G_1 \rangle$ (4.0)

(ii) $a_2 \neq 0$ yields $\langle G_2 \rangle$ (4.1)

(iii) $a_3 \neq 0$ yields $\langle G_3 \rangle$ (4.2)

Case 2: Two-generator combinations

Since there is no adjoint further simplification that is possible, the distinct ratios provide inequivalent subalgebras. Thus we normalize by setting one coefficient to 1.

(i) $X_1 + \alpha X_2$ we get $\langle G_1 + \lambda G_2 \rangle$ (4.3)

(ii) $X_1 + \beta X_3$ we get $\langle G_1 + \xi G_3 \rangle$ (4.4)

(iii) $X_2 + \gamma X_3$ we get $\langle G_2 + \delta G_3 \rangle$ (4.5)

Case 3: Full combination

The full combination gives

$$\langle G_1 + \lambda G_2 + \xi G_3 \rangle \text{ where } \alpha, \beta, \gamma \in R \tag{4.6}$$

The adjoint action is trivial and no further reduction is possible.

3. Symmetry reduction of a PDE

Consider the PDE (1.1) and the generators (2.7), we perform symmetry reduction using one of the symmetry generators previously obtained.
The effective reduction comes from the translation-type generator give as

$$G = \partial_t + c\partial_x, \quad (4.7)$$

which corresponds to a travelling–wave symmetry. This gives the characteristic system as

$$\frac{dt}{1} = \frac{dx}{c} = \frac{du}{0}. \quad (4.8)$$

From the first two fractions we have

$$\frac{dt}{1} = \frac{dx}{c} \Rightarrow dx - c\,dt = 0,$$

integrating gives the invariant variable

$$\phi = x - ct \quad (4.9)$$

From the last two fractions, we have $du = 0$,

Thus yielding

$$u(x,t) = F(\phi). \quad (5.0)$$

We then compute the derivatives using chain rule to obtain

$$u_t = -cF', u_x = F', u_{xxx} = F''', \quad (5.1)$$
$$u_{xxxxx} = F^{(5)}, u_{xxxxxxx} = F^{(7)}.$$

Substituting into (1.1) gives

$$-cF' + 6FF' + F''' - F^{(5)} + \beta F^{(7)} = 0.$$

Therefore, the PDE (1.1) reduces to the 7th-order nonlinear ODE of the form

$$\beta F^{(7)} - F^{(5)} + F''' + 6FF' - cF' = 0 \quad (5.2)$$

Since all terms contain derivatives, we integrate once with respect to ϕ

$$\beta F^{(6)} - F^{(4)} + F'' + 3F^2 - cF = K, \quad (5.3)$$

where $K$is a constant of integration.

For localized travelling waves, we take $K = 0$. Thus the reduced equation becomes a sixth-order nonlinear ODE

$$\beta F^{(6)} - F^{(4)} + F'' + 3F^2 - cF = 0 \quad (5.4)$$

4. Similarity reductions and Invariant Solutions

Consider equation (1.1). We generate similarity reductions using symmetry generators (2.6)

The reductions are obtained from the characteristic equations

$\frac{dx}{\alpha} = \frac{dt}{\mu} = \frac{du}{\eta}$. (5.5)

The Similarity Reduction using $R_2 = -\partial_x$

Generator coefficients

$\alpha = -1, \mu = 0, \eta = 0$ (5.6)

Characteristic equations

$\frac{dx}{-1} = \frac{dt}{0} = \frac{du}{0}$ (5.7)

From $dt = 0$, we obtain the invariant

$t = \phi$

and

$u = F(t)$ (5.8)

Thus the similarity reduction is given by

$u(x, t) = F(t)$ (5.9)

Substituting into the PDE

$u_x = 0$ (6.0)

all derivatives vanish and the equation reduces to $0 = 0$. Thus

$u = F(t)$ (6.1)

is an invariant solution.

Similarity Reduction using $R_3 = -\partial_u$

Generator coefficients

$\alpha = 0, \mu = 0, \eta = -1$ (6.2)

Characteristic equations

$\frac{dx}{0} = \frac{dt}{0} = \frac{du}{-1}$ (6.3)

This implies that

$u = constant$

Hence the invariant solution

$u = C$ (6.4)

Similarity Reduction using $R_1 = -\frac{1}{6}\partial_t - t\partial_x$

Generator coefficients

$\alpha =- t, \mu =- \frac{1}{6}, \eta = 0$ (6.5)

Characteristic equations

$\frac{dx}{-t} = \frac{dt}{-1/6} = \frac{du}{0}$ (6.6)

From the first two ratios

$\frac{dx}{-t} = \frac{dt}{-1/6}$

$$6dx = t\,dt$$

Integrating gives

$6x = \frac{t^2}{2} + C$ (6.7)

Thus the similarity variable

$\phi = 6x - \frac{t^2}{2}$ (6.8)

Since $du = 0$ then

$u = F(\phi)$ (6.9)

5. Reduced ODE

Let

$u(x, t) = F(\phi), \phi = 6x - \frac{t^2}{2}$ (7.0)

Then

$u_x = 6F' \; u_{xxx} = 216F'''$ (7.1)

$u_{xxxxx} = 7776F^{(5)} \; u_{xxxxxxx} = 279936F^{(7)}$

Substituting into the PDE (1.1)

$36FF' + 216F''' - 7776F^{(5)} + 279936\beta F^{(7)} = 0$ (7.2)

Dividing by 36 gives

$FF' + 6F''' - 216F^{(5)} + 7776\beta F^{(7)} = 0$ (7.3)

The non-trivial similarity reduction is obtained from R_1, which reduces the PDE to the seventh-order nonlinear ODE.

6. Exact Power Series Invariant Solutions

To derive the exact power-series invariant solution for the PDE (1.1) we use the similarity variable obtained from the generator

$$G_1 =- \frac{1}{6}\frac{\partial}{\partial t} - t\frac{\partial}{\partial x}. \tag{7.4}$$

Assume a Power Series Solution to be of the form

$$F(\phi) = \sum_{n=0}^{\infty} a_n \phi^n \tag{7.5}$$

Then we have

$$F^{'} = \sum_{n=1}^{\infty} n a_n \phi^{n-1}\ F^{'''} = \sum_{n=3}^{\infty} n(n-1)(n-2) a_n \phi^{n-3}\ F^{(5)} = \sum_{n=5}^{\infty} n(n-1)(n-2)(n-3)(n-4) a_n \phi^{n-5}$$

$$(7.6) F^{(7)} = \sum_{n=7}^{\infty} n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6) a_n \phi^{n-7}.$$

The nonlinear Term

$$FF^{'} = \left(a_n \phi^n\right)\left(m a_m \phi^{m-1}\right) = \sum_{k=0}^{\infty}\left(\sum_{m=0}^{k} (k - m + 1) a_m a_{k-m+1}\right)\phi^k. \tag{7.7}$$

The Recurrence Relation is obtained by substituting the series expansions into the reduced equation

$$FF^{'} + 6F^{'''} - 216F^{(5)} + 7776\beta F^{(7)} = 0 \tag{7.8}$$

and equating coefficients of ϕ^n gives

$$\sum_{m=0}^{n} (n - m + 1) a_m a_{n-m+1} + 6(n + 3)(n + 2)(n + 1) a_{n+3}$$
$$- 216(n + 5)(n + 4)(n + 3)(n + 2)(n + 1) a_{n+5} + 7776\beta(n + 7)(n + 6)(n + 5)(n + 4)(n + 3)(n + 2)(n + 1) a_{n+7} = \tag{7.9}$$

This determines the coefficients recursively.

For the First Few Coefficients,

Let

$$a_0 = A, a_1 = B, a_2 = C \tag{8.0}$$

be arbitrary constants.

For $n = 0$

$$AB + 36a_3 - 25920a_5 + 39191040\beta a_7 = 0. \tag{8.1}$$

For $n = 1$

$$2AC + B^2 + 144a_4 - 155520a_6 + 235146240\beta a_8 = 0. \tag{8.2}$$

For $n = 2$

$$3(Aa_3 + BC) + 360a_5 - 653184a_7 + 940584960\beta a_9 = 0. \tag{8.3}$$

Thus the Exact Power-Series Invariant Solution is obtained from

$$F(\phi) = a_0 + a_1\phi + a_2\phi^2 + a_3\phi^3 + a_4\phi^4 + \cdots \tag{8.4}$$

and the invariant solution of the PDE becomes

$$u(x, t) = a_0 + a_1\left(\frac{t^2}{2}\right) + a_2\left(\frac{t^2}{2}\right)^2 + a_3\left(\frac{t^2}{2}\right)^3 + \cdots. \tag{8.5}$$

Therefore the Final Exact Power-Series Invariant Solution

$$u(x, t) = \sum_{n=0}^{\infty} a_n\left(\frac{t^2}{2}\right)^n \tag{8.6}$$

where coefficients a_n satisfy the recurrence relation above.

Results and Discussion

The PDE admits the following invariant solutions from the generators given as

G_2 yields an invariant solution given as $u = F(t)$, G_3 yields an invariant solution as $u = C$ and G_1 gives invariant solution as $u(x, t) = F\left(\frac{t^2}{2}\right)$

While the exact power series solution is obtained from

$$F(\phi) = a_0 + a_1\phi + a_2\phi^2 + a_3\phi^3 + a_4\phi^4 + \cdots$$

and the invariant solution of the PDE becomes

$$u(x, t) = a_0 + a_1\left(\frac{t^2}{2}\right) + a_2\left(\frac{t^2}{2}\right)^2 + a_3\left(\frac{t^2}{2}\right)^3 + \cdots.$$

Thus the final Exact Power-Series Invariant Solution is obtained as

$$u(x, t) = \sum_{n=0}^{\infty} a_n\left(\frac{t^2}{2}\right)^n$$

where coefficients a_n satisfy the recurrence relation above.

The invariant solutions obtained provide standard solutions that can be applied to authenticate numerical algorithms constructed for solving higher-order nonlinear evolution equations. These exact results fulfill the governing equation analytically and they are used as consistent test cases for finite difference, spectral, and finite element techniques.

These solutions prove that Lie symmetry analysis is an effective analytical method for studying higher-order nonlinear evolution equations. This method does not only identify the real symmetries of the Kawahara equation but also presents systematic techniques for creating similarity variables, reducing the governing equation, and finding exact power series invariant solutions.

Conclusion and Remarks

A complete Lie symmetry analysis of the generalized Kawahara equation has been presented. The study obtained symmetry generators, optimal systems, similarity reductions, solitary wave solutions and exact power series solutions. The results demonstrate the strong analytical power of Lie group methods for higher-order nonlinear evolution equations. The analytic symmetry solutions demonstrate that Lie symmetry analysis technique is a candid and best mathematical tool to obtain analytical and exact results of highly nonlinear PDE's. Furthermore, this technique can be more efficiently used to study other nonlinear PDE's, which are often used in mathematical fields, engineering fields, and physical sciences.

Conflicts of Interest

There are no conflicts of interest.

Funding Statement

The research was self-sponsored by the author.

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Author Biography

Winny Chepngetich is an academic and researcher in the field of Applied Mathematics, with particular interest in nonlinear differential equations, mathematical modelling, and symmetry analysis. She is pursuing doctoral research in Applied Mathematics at the University of Kabianga, with her research focusing on the Lie symmetry analysis of seventh-order Korteweg–de Vries (KdV) equations.

Her research interests include Lie group analysis, nonlinear evolution equations, soliton theory, symmetry reductions, exact solutions, travelling-wave solutions, and mathematical techniques for analysing nonlinear partial differential equations. Her work explores the application of Lie symmetry methods to higher-order nonlinear KdV-type equations, including equations such as the Kawahara, Kaup–Kupershmidt, and related seventh-order nonlinear evolution equations.

In addition to research, Winny is actively involved in mathematics education, curriculum development, competency-based education, and university teaching. She has contributed to the development and review of mathematics and statistics curricula, including the design of competency-based course outlines, learning outcomes, teaching activities, assessment strategies, and instructional materials.

Prof. Maurice Owino Oduor is the Deputy Vice Chancellor (Planning, Research and Development) of the University of Kabianga- Kenya. He is a champion for Research, Innovation and Commercialization at the University. Prior to his appointment to the DVC position, he served as the Dean, School of Science and Technology and HoD, Mathematics and Computer Science. He holds a Ph.D in Pure Mathematics from Maseno University- Kenya and had postdoctoral research stint at Stellenbosch University (A.I.M.S) - South Africa. He has 20 years of teaching, research and administrative experience at the University level. The Professor of Pure Mathematics is an accomplished scholar who has published widely and has mentored many undergraduate and postgraduate students. His research focuses on classification of Algebraic Rings. He is a member of many Professional Associations, an External Examiner for a number of Universities and a reviewer for a number of academic Journals. He has served as a board member in many learning institutions. He is a trustee for the University of Kabianga Pension Scheme.

Dr. Joshua K. Limo is a University Lecturer at Laikipia University- Kenya. Has a earned Doctor of Philosophy in Mathematics from University of Eldoret. Has over 14 years work experience Lecturing Mathematics courses at Laikipia University. Over 4 years experience Teaching Mathematics and Physics at Secondary School level. Demonstrates capacity to manage people in terms of training, development, coaching and mentoring and supervising Postgraduate students.

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REF: PHD/AM/0002/2021 **DATE: 3RD MARCH, 2026**

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KERICHO.

Dear Ms. Chepngetich,

RE: CLEARANCE TO COMMENCE FIELD WORK/DATA COLLECTION

I am pleased to inform you that the Board of Graduate Studies has considered and approved your PhD Research proposal entitled **"On the Lie Symmetry Analysis of Seventh Order Korteweg-Devries Equations".** Subsequently the Board has also approved the following supervisors for appointments.

1. Prof. Maurice O. Oduor, PhD
2. Dr. Joshua K. Limo, PhD

You may now proceed to commence field work/data collection on condition that you obtain a research permit from NACOSTI and /or an ethical review permit from a relevant ethics review board.

You are also required to publish two (2) articles in a peer reviewed journal, with all your supervisors, before your oral defense of thesis and to submit through your supervisors, and HoD, progress reports every three months, to the Director, Board of Graduate Studies.

Please note that it is the policy of the University that you complete your studies within three years from the date of registration. Do not hesitate to consult this office in case of any difficulties encountered in the course of your studies.

I wish you all the best in your research and hope that your study will yield original contribution for the betterment of humanity.

Yours Sincerely,

UNIVERSITY OF KABIANGA
03 MAR 2026
BOARD OF GRADUATE STUDIES

Dr. Ronald K. Rop
DIRECTOR, BOARD OF GRADUATE STUDIES.
RKR/hk

Cc 1. Dean, SST
2. HOD, Mathematics, Statistics & Actuarial Sciences
3. Supervisors