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Controlling Chaos in the Lozi Map Using Hidden Variables: A Novel Approach for Stability Enhancement

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Abstract

In this paper, we provide a novel 3D system that demonstrates the existence of a hidden attractor. Although equilibrium points are present in the system, this hidden attractor cannot be found by examining equilibrium points or their surroundings, in contrast to self-excited attractors. Differential equation theory explains this behavior by showing that the system has intricate dynamics that are not directly dependent on equilibrium points. We added a hidden variable to the system to make it more complex and improve its stability by adding the following equation to the original system: $z_{n+1}=wz_n+r3x_n$, where the hidden variable interacts with the original system through control parameters r1, r2, and r3, increasing chaos or improving the system stability. where it appeared us more complex dynamic behaviors.

Keywords: Hidden Variables, Chaos Control, Bifurcation Points, Iterative Equations, Lyapunov exponent.

Introduction

The Lü system [3] and the Chen system [11] are two examples of independent three-dimensional chaotic oscillators that have been described. Many new chaotic systems have been introduced in a variety of scientific and engineering applications [6,7]. In recent years, chaotic attractors of differential systems have been described by self-excited and hidden attractors. [8, 1] Not much research has been done on memristor-based discrete-time chaotic systems [4,5]. It is intriguing to examine the complex dynamics of the recently developed discrete-time chaotic systems and successfully apply memristors to them because these attractors belong to a class that lacks basins of attraction connected to the neighbors of unstable equilibria. [1,10] The discrete-time chaotic systems, on the other hand, are a unique class of dynamical systems that describe the instantaneous [2] This study presents a memristor-based 3D Lozi map by incorporating a second memristor into the original 2D map[9]. The suggested map is a hidden chaotic map since it lacks fixed points but can produce hidden hyperchaos .

One of the main research priorities has been to build discrete-time chaotic systems and study the bifurcation mechanism of hidden attractors creating unnoticed chaos. The purpose of this new map is to increase the complexity of chaotic behavior and offer another illustration of how discrete memristor-based chaotic systems can be used.

In Section 1, the new Lozi map model based on memristors is introduced, and the existence and characteristics of fixed points are examined.

Section 2: Examines the memristor's dynamical impact on the suggested map, utilizing bifurcation analysis and a study of Lyapunov exponents.

1. System dynamics analysis

Properties 1.1

the fixed point of m. lozi map $IF\ X > 0$ then $\left(\frac{1-w}{(1-w)+ax(1-w)-b(1-w)-r1r3} \quad \frac{m(1-w)+r2r3(1-w)^2}{(1-w)+nx(1-w)-m(1-w)-r1r3} \quad \frac{r3(1-c)^2}{(1-w)+nx(1-w)-m(1-w)-r1r3}\right)$and $if\ x < 0$ then $\left(\frac{1-w}{(1-w)-nx(1-w)-m(1-w)-r1r3} \quad \frac{m(1-w)+r2r3(1-w)^2}{(1-w)-nx(1-w)-m(1-w)-r1r3} \quad \frac{r3(1-w)^2}{(1-w)-nx(1-w)-m(1-w)-r1r3}\right)$

Proof

$$x = 1 - nx + y + r_{1z}$$

$$y = mx + r_2 z$$

$$z = wz + r_{3x}$$

then $z = wz + r_{3x}$, $z - wz = r_{3x}$ therefore $z(1-w) = r_{3x}$ then $z = \frac{xr_3}{1-w}$ We substitute it into the equation $y = mx + r_2\frac{xr_3}{1-w}$ therefore $x = 1 - n|x| + mx + r_1\frac{xr_3}{1-w}$ $if\ x > 0$ then $x - 1 + nx - mx - \frac{xr1r_3}{1-w} = 0$ therefor $\frac{x(1-w)-(1-w)+nx(1-w)-mx(1-w)-xr1r3}{1-w} = 0$ hence $x(1-w) + nx(1-w) - mx(1-w) - xr1r3 = 1 - w$

$x((1-w)+nx-m(1-w)-r_1r_3)=1-w$ therefore $x=\frac{1-w}{(1-w)+nx-m(1-w)-r1r3}$,$y=\frac{m(1-w)+r2r3(1-w)^2}{(1-w)+nx(1-w)-m(1-w)-r1r3}$, $z=\frac{r3(1-w)^2}{(1-w)+nx(1-w)-m(1-w)-r1r3}$ $if\ x<0$ then fixed point is $then\ z=wz+r_{3x}$, $z-wz=r_{3x}$ therefore $z(1-w)=r_{3x}$ then $z=\frac{xr_3}{1-w}$ We substitute it into the equation $y=mx+r_2\frac{xr_3}{1-w}$ then $x-1-nx-mx-\frac{xr1r_3}{1-w}=0$ therefor $\frac{x(1-w)-(1-w)-nx(1-w)-mx(1-w)-xr1r3}{1-w}=0$ hence $x(1-w)-nx(1-w)-mx(1-w)-xr1r3=1-w$,
$x((1-w)-nx-m(1-w)-r_1r_3)=1-w$ therefore $x=\frac{1-w}{(1-w)-nx-m(1-w)-r1r3}$, $y=\frac{m(1-w)+r2r3(1-w)^2}{(1-w)-nx(1-w)-m(1-w)-r1r3}$
$z=\frac{r3(1-w)^2}{(1-w)-nx(1-w)-m(1-w)-r1r3}$

Proposition 1.2 : The Jacobin of m.lozi map if x>0 then and if x<0 then is r_2r_3-wm
Proof

J=$\left[-n.sign(x)\ 1\ r_1\ m\ 0\ r_2\ r_3\ 0\ w\right]$, if x<0 and if x>0 then J=r_2r_3-wm

Proposition 1.3 : Eigenvalues of m.lozi map is $\lambda^3+(a-c)\lambda^2+(nw-m-r_1r_2)\lambda+(bmw-r_3r_2)=0$

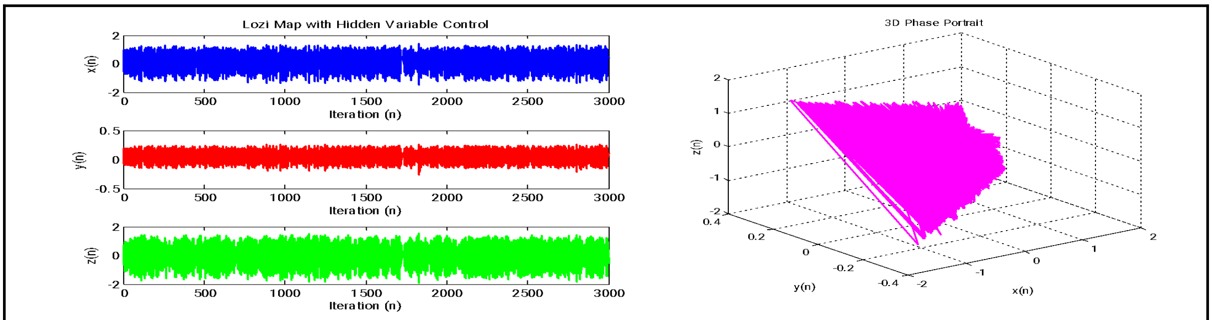
Proof

$$J=\left[n\ 1\ r_1\ m\ 0\ r_2\ r_3\ 0\ w\right] \text{ then } J=\left[n-\lambda\ 1\ r1\ m\ 0-\lambda\ r_2\ r_3\ 0\ w-\lambda\right]$$

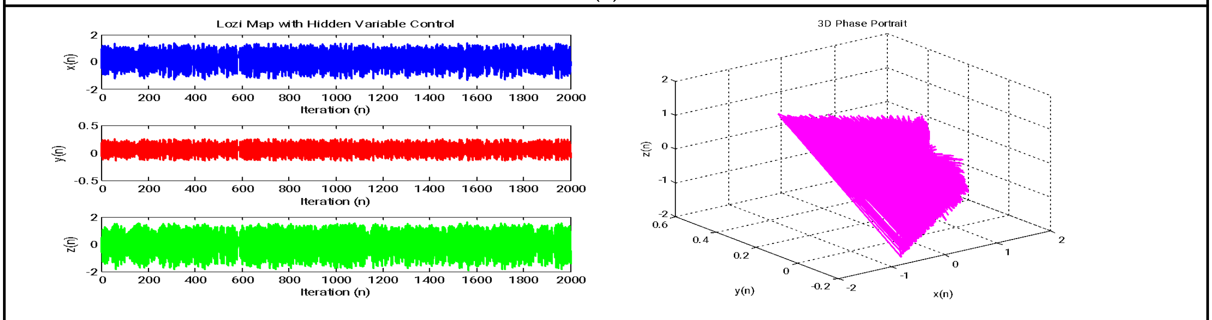
Therefore $\lambda^3+(n-w)\lambda^2+(nw-m-r1\ r2)\lambda+(mw-r3r2)=0$ then if $\left|\lambda_1,\lambda_2,\lambda_3\right|<1$ then map stable and if $\left|\lambda_1,\lambda_2,\lambda_3\right|>1$ then map unstable .

2. m.Lozi Map with Hidden Variable Control

This non-linear dynamic system, which is based on simple recurring equations, can be used as a model to understand chaos in discrete systems. Its hidden variable, z, adds complexity to the system, and its effect is controlled by the coefficients r_1, r_2, and r_3.



(a)



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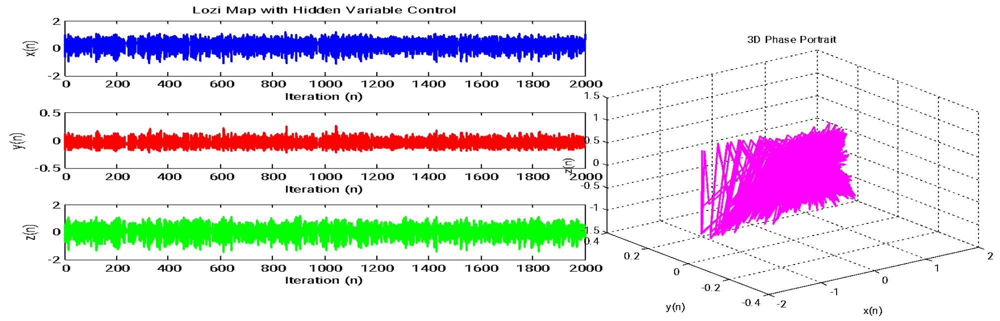

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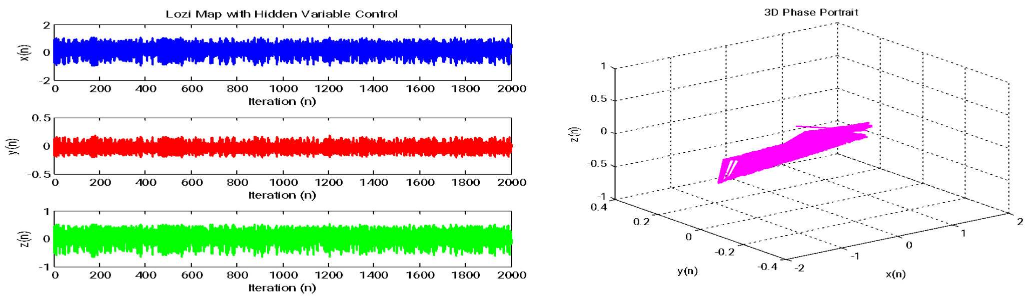


(d)

Fig.1: **a.** Chaotic interactions with sharp inflation of X(n), Y(n), and Z(n) values because of latent variable enhancement (W = 0.5) and negative control coefficient R_1 = −0.8 **b.** negative control coefficients (R_1 = −0.8) and normalized values (n = 1.6, m= 0.2), which cause chaotic interactions in phase space (x(n),y(n)) and a sharp collapse of z(n) values.**c.** Chaotic interactions in phase space (x(n) and y(n)) and sharp collapse of z(n) and X(n) values due to the negative control parameter R_1 = −0.8 and the effect of the enhanced hidden variable **d.** Chaotic interaction between x(n) and y(n) in phase space, with the values of X(n), Y(n), and Z(n) being inflated iterations due to positive control coefficients (R_1 = 0.6, w = 0.5).

3. lyapunov exponents

Lyapunov exponents are mathematical metrics that quantify how quickly adjacent paths in a dynamical system diverge or converge. They are classified as follows:
The divergence of paths, which is a requirement for chaos to exist, is indicated by a positive Lyapunov exponent.
When the Lyapunov exponent is negative, it means that the paths are convergent (system stable).
The Lyapunov exponent of zero denotes a neutral state, like uniform rotation.
By the table the results show a positive baseline $\lambda_1>0$ this confirms the presence of chaos, which is often associated with critical branching points.
by use matlab then we get :

n	M	w	R_1	R_2	R_3	L1	L2	L3
0.8	2.2	**-0.8**	**0.5**	**0.1,**	**0.4**	1.0368	-0.8335	-2.7787
1.5	2.2	**-0.8**	**0.5**	**0.1,**	**0.4**	0.8354	-0.8656	-4.4943
2.8	2.2	**-0.8**	**0.5**	**0.1,**	**0.4**	0.6132	-0.8641	-8.9457
1.11	2.2	**-0.8**	**0.5**	**0.1,**	**0.4**	0.9400	-0.8480	-3.3559
2.9	1.2	**-0.3**	**0.5**	**0.1,**	**0.4**	2.6168	-0.0898	-0.6767

4. Bifurcation : Frequent regions of branching appear at specific values of r_2 (e.g., 0.5 < r_2 < 0.8), indicating a transition from stability to chaos . The nonlinear interactions that lead to the formation of the hidden attractor are induced by critical values of r_2, such as $r_2 \approx$ 0.7. The complexity of the system increases as r_2 increases due to increased coupling of variables, making the bifurcation regions and hidden attractor more apparent.

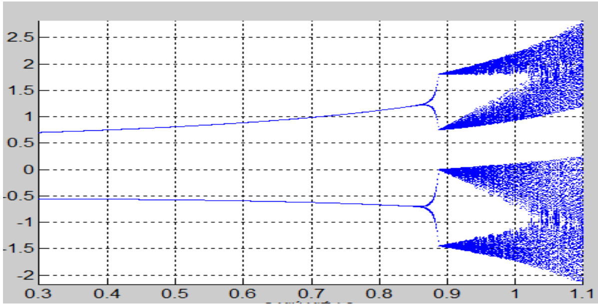
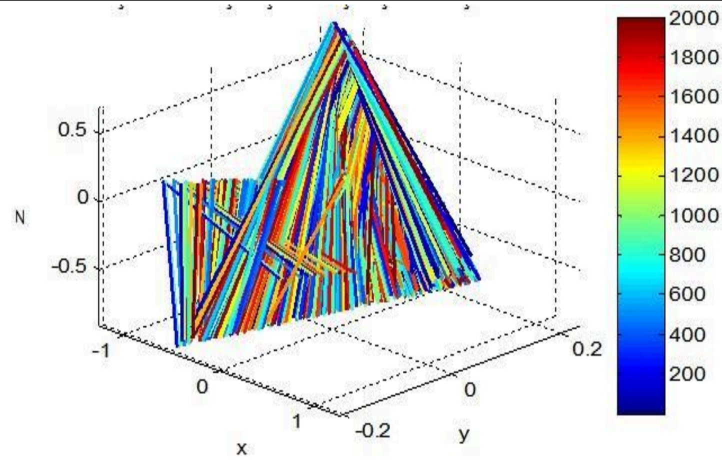


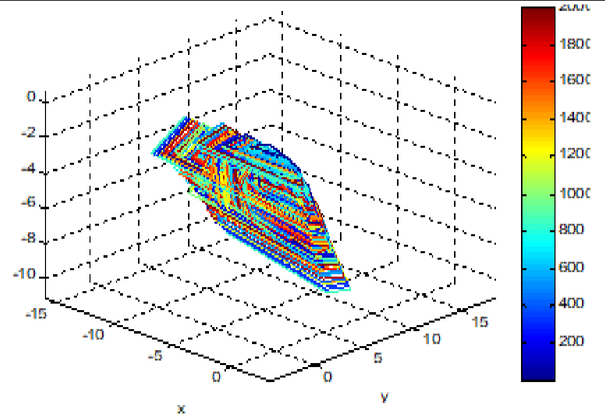
Fig.2: The nonlinear interactions that lead to the formation of the hidden attractor are induced by critical values of r_2, such as $r_2 \approx 0.7$. The complexity of the system increases as r_2 increases due to increased coupling of variables, making the bifurcation regions and hidden attractor more apparent.

5. Hidden attractor

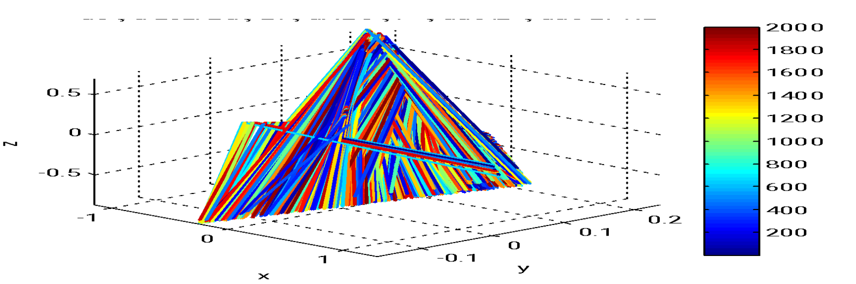
strange attractor: A specific path that the dynamical system (such as a physical or mathematical system) tends to take over time, but which only appears if the system starts from initial conditions that are far from known stable regions. Unlike a classical attractor (such as a fixed point or a regular cycle), a hidden attractor: Cannot be easily predicted from nearby initial conditions. Exhibits complex behavior (such as tangling of irregular paths). Often associated with chaos in nonlinear systems.



(a)



(b)



(c)

Fig(3): **a** .The system is not stable in classical cases because of complex coupling (r_2 = 0.3, 1.1) and strong damping (w = 0.7), which results in the appearance of the hidden attractor . **b**. A hidden attractor or unknown chaos is strongly suggested by the graph's complex chaotic behavior, which includes extreme negative values and enhanced nonlinear interactions (n=1.6, m=-1.2, w=0.6).**c** .The values n=1.7, m=0.2, w=-0.6, R_3=0.5, R_2=0.1, and R_3=0.3 exhibit complex nonlinear interactions with strong damping (w=-0.6) and moderate coupling, which induces chaotic behavior and unusual bifurcations, strongly suggesting the presence of a hidden attractor in the dynamical system.

6. Conclusions

A new three-dimensional almond-shaped map based on discrete memristor resistance is proposed. The proposed function contains fixed points, making it a chaotic system with complex dynamic behavior. The results show that the introduction of the memristor complicated the dynamic behavior of the original map. A positive Lyapunov exponent for hyperchaos was found and The system's dynamic behavior is examined using bifurcation diagrams.

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