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Derivation the availability of a system with one unit in normal and abnormal weather conditions by using Laplace transform with application to case of Normal Weather

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Abstract

In this paper We will study the system in good and bad weather by using Laplace transform, the system have one unite in operative, partial failure and total failure modes in two weather conditions normal and stormy resolved by Laplace transform and application to the case of normal Weather only considering existence of preventive maintenance.

Keywords: Reliability, Steady-state Availability, weather conditions, Laplace transform.

1.1 Introduction

Reliability study of the repair problem of the device is more important in our lives where it is used widely in the industrial system and the manufacturing. We will study the system in good and bad weather by using Laplace transform, the system have one unite in operative, partial failure and total failure modes in two weather conditions normal and stormy resolved by Laplace transform.

The comparison of reliability characteristics of two systems with preventive maintenance and different mode by Khaled Moh. El-Said and Rabaa Abd El-Hamid (2008). Some characteristics of two dissimilar unit cold standby redundant system with three modes by G.S (1997). Mokaddis and M.L. Tawfek.

Stochastic behavior of man machine system operating under different weather condition by L.R. Goel, A. kumar and A.K. Rastogi (1985). Analysis of a Repairable System Operating Under Different Weather Conditions by Ashish K. Barak and M.S. Barak (2013). Cost analysis of a system with partial failure mode and abnormal weather conditions by L.R. Goel and P. Gupta (1984).

1.2 system description

The system is analyzed under the following practical assumptions:

1-The system unit contains to operative, partial failure and total failure modes.
2- The system fails totally only through the partial failure mode.
3- The distribution of failure time and time of the change of weather condition is exponential distribution.
4- A single repair facility is available to repair facility is not always with the system but it can be made available instantaneously whenever needed.
5- The failure rates of the failed system are different whether the weather is normal or abnormal.
6- Repaired system is as good as new.

A unit description

N: Operative mode.
W: Normal weather.
$\overline{W}$: Stormy weather.
$PF/\overline{PF}$: Partial failure mode in normal and stormy weather.
$TF/\overline{TF}$: Total failure mode in normal and stormy weather.

Notations and system states:

λ_1 : Rate of partial failure in normal weather.
λ_2 : Rate of total failure in normal weather.
λ_3 : Rate of partial failure in stormy weather.
λ_4 : Rate of total failure in stormy weather.
μ_1 : Repair of total failure in normal weather.
μ_2 : Repair of total failure in stormy weather.
μ_3 : Constant rate of change of change of weather from normal to stormy.
μ_4 : Constant rate of change of change of weather from stormy to normal.

$p_i(t)$: Probability that the system in stat $= 0, 1, 2, 3, 4, 5$
$\overline{p_i(s)}$: Laplace transform of $p_i(t)$
$A(t)$: Functions of availability
Where Laplace transform of $p_i(t)$ is

$$\overline{p_i(s)} = \int_0^\infty e^{-st} P_i(t)dt$$

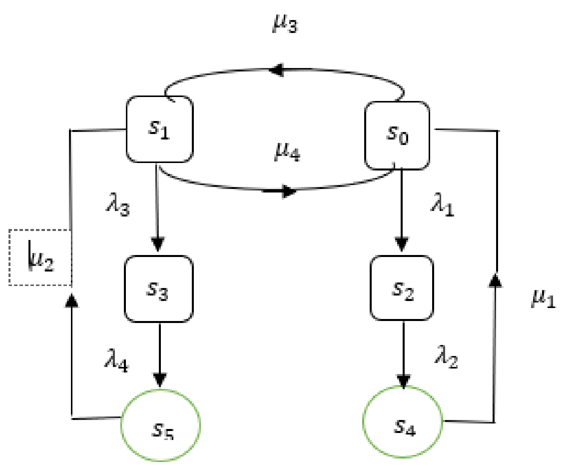

Fig. 1.1: state transition diagram

The system may be one of the following states
Up states $s_0 = (N, W),\ s_1 = (N, \overline{W}),\ s_2 = (PF, W),\ s_4 = (\overline{PF}, \overline{W})$
Down states $s_4 = (TF, W),\ s_5 = (\overline{TF}, \overline{W})$

1.3 Mathematical model description

This part showing the Differential equation for the system of Fig (1.1) Transition

$$\frac{dp_0(t)}{dt} =- \left(\mu_3 - \lambda_1\right)p_0(t) + \mu_4 p_1(t) + \mu_1 p_4(t) \qquad (1.3.1)$$

$$\frac{dp_1(t)}{dt} =- \left(\mu_4 - \lambda_3\right)p_1(t) + \mu_3 p_0(t) + \mu_2 p_5(t) \qquad (1.3.2)$$

$$\frac{dp_2(t)}{dt} =- \lambda_2 p_2(t) + \lambda_1 p_0(t) \qquad (1.3.3)$$
$$\frac{dp_3(t)}{dt} =- \lambda_4 p_3(t) + \lambda_3 p_1(t) \qquad (1.3.4)$$
$$\frac{dp_4(t)}{dt} =- \mu_1 p_4(t) + \lambda_2 p_2(t) \qquad (1.3.5)$$
$$\frac{dp_5(t)}{dt} =- \mu_2 p_5(t) + \lambda_4 p_3(t) \qquad (1.3.6)$$

Initial condition :

$p_0(0) = 1 \quad , \ p_1(0) = \ p_2(0) = \ p_3(0) = \ p_4(0) = \ p_5(0) = 0$

Take Laplace transform :

$$\left(s + \mu_3 + \lambda_1\right)p_0(s) = \mu_4 p_1(s) + \mu_1 p_4(s) + 1$$
$$\left(s + \mu_4 + \lambda_3\right)p_1(s) = \mu_3 p_0(s) + \mu_2 p_5(s)$$
$$\left(s + \lambda_2\right)p_2(s) = \lambda_1 p_0(s)$$
$$\left(s + \lambda_4\right)p_3(s) =- \lambda_3 p_1(s)$$
$$(s + \mu_1)p_4(s) =- \lambda_2 p_2(s)$$
$$(s + \mu_2)p_5(s) =- \lambda_4 p_3(s)$$

Solving this equation simultaneously to get :

$\overline{p_0(s)}, \ \overline{p_1(s)}, \overline{p_2(s)}, \ \overline{p_3(s)}, \ \overline{p_4(s)}, \ \overline{p_5(s)}$

Then calculating probability of up states as follows :

$$\overline{p_{up}(s)} = \ \overline{p_0(s)} + \ \overline{p_1(s)} + \overline{p_2(s)} + \ \overline{p_3(s)}$$

$$\begin{aligned}
\overline{p_{up}(s)} = {} & (-((s+\lambda_2)(s+\mu_1)(-\lambda_3\lambda_4\mu_2+(s+\lambda_4)(s+\mu_2)(s+\lambda_3 \\
& +\mu_4)))/(\lambda_1\lambda_2\mu_1(-\lambda_3\lambda_4\mu_2+(s+\lambda_4)(s+\mu_2)(s+\lambda_3+\mu_4))+(s+\lambda_2)(s \\
& +\mu_1)((s+\lambda_4)(s+\mu_2)\mu_3\mu_4-(s+\lambda_1+\mu_3)(-\lambda_3\lambda_4\mu_2+(s+\lambda_4)(s+\mu_2)(s \\
& +\lambda_3+\mu_4))))) + (-\frac{1}{\mu_4} + ((s+\lambda_2)(s+\mu_1)(\frac{\lambda_1\lambda_2\mu_1}{(s+\lambda_2)(s+\mu_1)\mu_4} \\
& -\frac{s+\lambda_1+\mu_3}{\mu_4})(-\lambda_3\lambda_4\mu_2+(s+\lambda_4)(s+\mu_2)(s+\lambda_3+\mu_4)))/(\lambda_1\lambda_2\mu_1(-\lambda_3\lambda_4\mu_2 \\
& +(s+\lambda_4)(s+\mu_2)(s+\lambda_3+\mu_4))+(s+\lambda_2)(s+\mu_1)((s+\lambda_4)(s+\mu_2)\mu_3\mu_4 \\
& -(s+\lambda_1+\mu_3)(-\lambda_3\lambda_4\mu_2+(s+\lambda_4)(s+\mu_2)(s+\lambda_3+\mu_4))))) + (-(\lambda_1(s \\
& +\mu_1)(-\lambda_3\lambda_4\mu_2+(s+\lambda_4)(s+\mu_2)(s+\lambda_3+\mu_4)))/(\lambda_1\lambda_2\mu_1(-\lambda_3\lambda_4\mu_2+(s \\
& +\lambda_4)(s+\mu_2)(s+\lambda_3+\mu_4))+(s+\lambda_2)(s+\mu_1)((s+\lambda_4)(s+\mu_2)\mu_3\mu_4-(s \\
& +\lambda_1+\mu_3)(-\lambda_3\lambda_4\mu_2+(s+\lambda_4)(s+\mu_2)(s+\lambda_3+\mu_4)))))) \\
& +(-\frac{(s+\mu_2)(s+\lambda_3+\mu_4)}{\lambda_4\mu_2\mu_4}+((s+\lambda_2)(s+\mu_1)(-\lambda_3\lambda_4\mu_2+(s+\lambda_4)(s \\
& +\mu_2)(s+\lambda_3+\mu_4))(\frac{(s+\mu_2)\mu_3}{\lambda_4\mu_2}+\frac{\lambda_1\lambda_2\mu_1(s+\mu_2)(s+\lambda_3+\mu_4)}{(s+\lambda_2)\lambda_4(s+\mu_1)\mu_2\mu_4} \\
& -\frac{(s+\mu_2)(s+\lambda_1+\mu_3)(s+\lambda_3+\mu_4)}{\lambda_4\mu_2\mu_4}))/(\lambda_1\lambda_2\mu_1(-\lambda_3\lambda_4\mu_2+(s+\lambda_4)(s \\
& +\mu_2)(s+\lambda_3+\mu_4))+(s+\lambda_2)(s+\mu_1)((s+\lambda_4)(s+\mu_2)\mu_3\mu_4-(s+\lambda_1 \\
& +\mu_3)(-\lambda_3\lambda_4\mu_2+(s+\lambda_4)(s+\mu_2)(s+\lambda_3+\mu_4)))))
\end{aligned}$$

Since, The equation appears after taking inverse Laplace transform is too big, we consider the probability of up states considering specific value as follows :

$$\lambda_1 := 0.4\,, \lambda_2 := 0.2\,, \lambda_3 := 0.25\,, \lambda_4 := 0.3\,, \mu_1 := 0.1\,, \mu_2 := 0.3\;\;, \mu_3 := 0.5\,, \mu_4 := 0.7$$

$$\begin{aligned} A(t) &= 0.4839572192513438 + 0.0007177844390693882e^{-1.5070309184172936t} \\ &+ 0.44115405936358254e^{-0.2725133687353162t} \\ &+ 0.03138890732732236e^{-0.12689073889336594t} \\ &+ e^{-0.42178248697701215t}(0.04278202960625794Cos[0.19340103084403665t] \\ &+ 0.7411013998033563Sin[0.19340103084403665t]) \end{aligned}$$

1.4 Application to the case of Normal Weather only considering existence of preventive maintenance

1.4.1 Model description and Assumptions:

1-The system consists of one unit, Initially operates.
2- The system fails totally only through partial failure mode.
3-system goes to preventive maintenance.
4- All random variables are mutually independent.

1.4.2 States description:

1- s_0 : system operate
2- s_1 : system operate with partial failure
3- s_2 : system is totally failure
4- s_3: system at preventive maintenance.

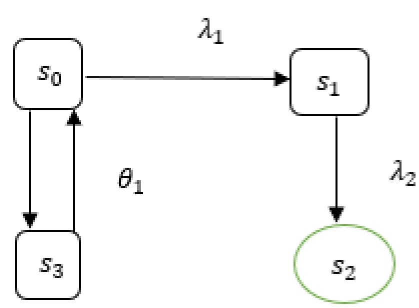

Fig. 1.2: state transition diagram

λ_1 : Rate of partial failure
λ_2 : Rate of total failure.
θ_1 : Rate of time of taking a unit into preventive maintenance.

θ_2 : Rate of preventive maintenance time.

1.4.3 Model solution:

Let $P_i(t)$ probability that the system is in state S_i . Then the initial condition when $t = 0$

$p_0(0) = 1 \quad and\ p_1(0) = p_2(0) = p_3(0) = 0$

from the diagram the diffraction equation of the system is given by:

$$\frac{dp_0(t)}{dt} = -\left(\lambda_1 + \theta_2\right)p_0(t) + \theta_1 p_3(t) \tag{1.4.3.1}$$

$$\frac{dp_1(t)}{dt} = -\lambda_2 p_1(t) + \lambda_1 p_0(t) \tag{1.4.3.2}$$

$$\frac{dp_2(t)}{dt} = -\lambda_2 p_1(t) \tag{1.4.3.3}$$

$$\frac{dp_3(t)}{dt} = -\theta_1 p_3(t) + \theta_2 p_0(t) \tag{1.4.3.4}$$

Taking Laplace transform:

$$\left(s + \lambda_1 + \theta_2\right)\overline{p_0(s)} = \theta_1\overline{p_3(s)} + p_0(0)$$

$$\left(s + \lambda_2\right)\overline{p_1(s)} = \lambda_1\overline{p_0(s)} + p_1(0)$$

$$s\,\overline{p_2(s)} = \lambda_2\overline{p_1(s)} + p_2(0)$$

$$\left(s + \theta_1\right)\overline{p_3(s)} = \theta_2\overline{p_0(s)} + p_3(0)$$

$$\therefore p_0(0) = 1 \quad and\ p_1(0) = p_2(0) = p_3(0) = 0$$

Now

$$\overline{p_0(s)} = \frac{1}{s+\lambda_1+\theta_2}[\theta_1\overline{p_3(s)}]$$

$$\overline{p_1(s)} = \frac{\lambda_1}{s+\lambda_2}\overline{p_0(s)}$$

$$\overline{p_2(s)} = \frac{\lambda_2}{s}\overline{p_0(s)}$$

$$\overline{p_3(s)} = \frac{\theta_2}{s+\theta_1}\overline{p_0(s)}$$

By substitution

$$\left(s + \lambda_1 + \theta_2\right)\overline{p_0(s)} = \frac{\theta_1\theta_2}{s+\theta_1}\overline{p_0(s)} + 1$$

$$\left(s + \lambda_1 + \theta_2 - \frac{\theta_1\theta_2}{s+\theta_1}\right)\overline{p_0(s)} = 1$$

$$\overline{p_0(s)} = \frac{s+\theta_1}{s^2+\lambda_1 s+\theta_2 s+\theta_1 s+\lambda_1\theta_1}$$

$$\overline{p_0(s)} = \frac{s+\theta_1}{s^2+(\lambda_1+\theta_2+\theta_1)s+\lambda_1\theta_1}$$

$$\overline{p_{up}(s)} = \overline{p_0(s)} + \overline{p_1(s)} + \overline{p_3(s)}$$

$$P_{up} = \left(1 + \frac{\lambda_1}{s+\lambda_2} + \frac{\theta_2}{s+\theta_1}\right)\left(\frac{s+\theta_1}{s^2+(\lambda_1+\theta_1+\theta_2)s+\lambda_1\theta_1}\right) \tag{1.4.3.5}$$

Take Inverse Laplace and put

λ_1=0.4

λ_2 =0.1

θ_1 =0.2

θ_2 =0.3

Then Availability of the system is given by

$A(t) =- 0.0611856\, e^{-0.8t} + (1.06119 + 0.05758t)e^{-0.1t}$

in case without preventive maintenance

θ_1=0

θ_2=0

$A(t) =- 0.3333333e^{-0.4t} + 1.33333333e^{-0.1t}$

Time t	Availability With PM	Availability Without PM
0	1	1
1	0.984809	0.98301
2	0.950758	0.941865
3	0.908564	0.88736
4	0.863229	0.826461
5	0.817142	0.763596
6	0.771492	0.70151
7	0.726897	0.641844
8	0.683699	0.585518
9	0.642094	0.532985
10	0.602194	0.484401
11	0.564063	0.439736
12	0.527733	0.398849
13	0.493207	0.361537
14	0.460472	0.327563
15	0.4295	0.296681

Table (1.1)

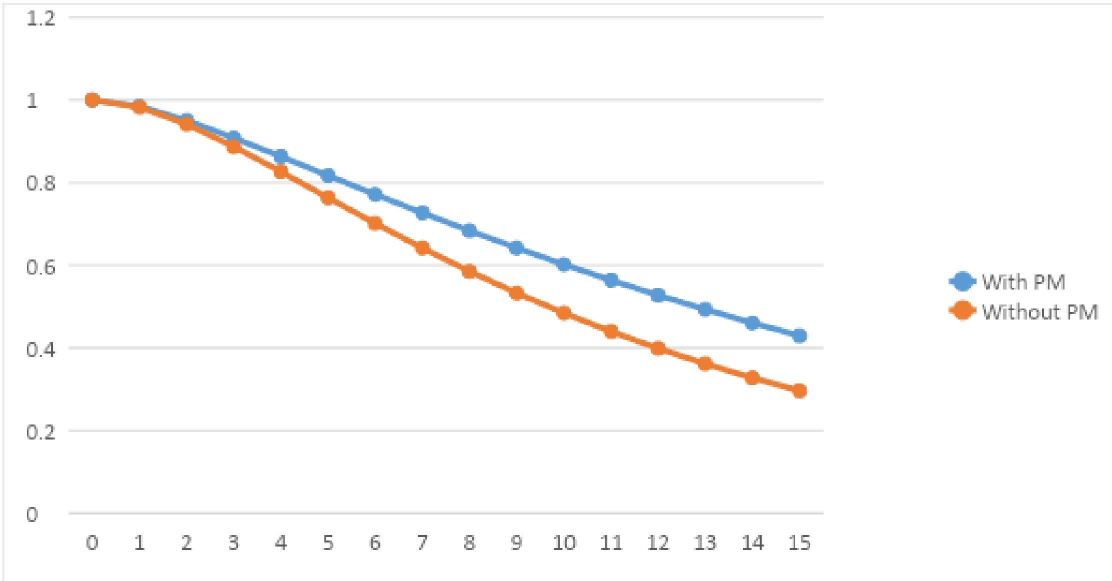

Fig. 2.2: Steady –state availability with normal weather (with PM /without PM)

Conclusion

This study showed that the Availability of the system with preventive maintenance (PM) were higher than those for system without preventive maintenance (PM).

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