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A Modern Technique for Evaluating the Square Root of a Complex Number

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Abstract

The subject of complex numbers issue is very significant because of its wide utility, especially in the engineering circuits representation. In this paper, a modern method to find the square root of the complex number has been analyzed, and some examples on the subject were presented.

Keywords: Complex Number; real part; imaginary part; Square Root; Completed Square

1.introduction

The complex numbers are very important issue in a lot of situations such as controlling circuits representation, electricity circuits representation and used in advanced calculus.

The complex number is a number that is written in the form a + i b, where a and b are real numbers while i is the imaginary one and equal $\sqrt{-1}$. If the complex number is z, then z=a+ib, where a is named as the real part and is written as Re(z) and b, is named the imaginary part and can be put in form I'm(z) [1].

In this paper, a new technique has been invented for evaluating the square root of the complex numbers, and it was proved by many examples.

2. Analysis of the Square Root of the Complex Numbers

Let z=a+ib (1)

Then, $\sqrt{z}= \sqrt{a+ib}$

$i^2=-1$ then multiplying it by c and adding to it c, the result will be zero, i.e. c i^2+c =0 (2)

Putting eq.(2) inside the root of z, then

$$\sqrt{z} = \sqrt{a+ib+ci^2+c} \quad (3)$$

Rearranging of eq.(3) then

$$\sqrt{z} =\sqrt{ci^2+ib+a+c} \quad (4)$$

The form of expression inside root is a second order equation of the form Ax^2+Bx+C, where, A=c, B= b and C= a+c . If we consider the equation is a completed square then the equation becomes $(\sqrt{A}x+\sqrt{C}\,)2$ but B must equals $\sqrt{4AC}$, that is the condition of the completed square.

Hence from above conservations and substituting (i) in place of (x) then:

$$\sqrt{z} = \sqrt{(\sqrt{c}i \mp \sqrt{a+c}\,)2} \quad (5)$$

Then, $\sqrt{z} = \mp (\sqrt{c}i \mp \sqrt{a+c}\,)$ (6)

From the rule of the completed square:

B^2=4AC (7)

Then substituting of A,B and C in eq.(4), the result is:

b^2 = 4c (a+c) (8)

Then, b^2 = 4ac +4c^2 (9)

Rearrange, $4c^2+4ac-b^2=0$ (10)

Then, the constant (c) can be found by solving the equation (10)

$$c = \frac{-4a}{8} \mp \frac{\sqrt{16a^2+4\times4\times b^2}}{8} \quad (11)$$

$$c = \frac{-a}{2} \mp \sqrt{\frac{a^2+b^2}{4}} \quad (12)$$

By neglecting negative root then

$$\text{Then, } \sqrt{z} = \mp \left(\sqrt{a - \frac{a}{2} + \sqrt{\frac{a^2+b^2}{4}}} \mp i \sqrt{\frac{-a}{2} + \sqrt{\frac{a^2+b^2}{4}}} \right) \quad (13)$$

$$\text{Hence, } \sqrt{z} = \mp \left(\sqrt{\frac{a}{2} + \sqrt{\frac{a^2+b^2}{4}}} \mp i \sqrt{\frac{-a}{2} + \sqrt{\frac{a^2+b^2}{4}}} \right) \quad (14)$$

If the value of (b) positive, then the equation becomes:

$$\sqrt{z} = \mp \left(\sqrt{\frac{a}{2} + \sqrt{\frac{a^2+b^2}{4}}} + i \sqrt{\frac{-a}{2} + \sqrt{\frac{a^2+b^2}{4}}} \right) \quad (15)$$

While, if (b) is negative, then:

$$\sqrt{z} = \mp \left(\sqrt{\frac{a}{2} + \sqrt{\frac{a^2+b^2}{4}}} - i \sqrt{\frac{-a}{2} + \sqrt{\frac{a^2+b^2}{4}}} \right) \quad (16)$$

3. Examples on The Used Technique

Find the square root of the following complex numbers:

1. z = 3+i4

solution:

a=3, b=4

$$\sqrt{z} = \mp\left(\sqrt{\frac{a}{2} + \sqrt{\frac{a^2+b^2}{4}}} + i \sqrt{\frac{-a}{2} + \sqrt{\frac{a^2+b^2}{4}}} \right)$$

$$\sqrt{z} = \mp\left(\sqrt{\frac{3}{2} + \sqrt{\frac{3^2+4^2}{4}}} + i \sqrt{\frac{-3}{2} + \sqrt{\frac{3^2+4^2}{4}}} \right)$$

$$\sqrt{z} = \mp(\sqrt{1.5+2.5} + i\sqrt{-1.5+2.5})$$

$$\sqrt{z} = \mp(2 + i1)$$

For checking,

$z = (2^2-1^2)+i\times 2 \times 2 \times 1$

z=3+i4

2. z = 5+i7

solution:

a=5, b=7

$$\sqrt{z} = \mp\left(\sqrt{\frac{a}{2} + \sqrt{\frac{a^2+b^2}{4}}} + i \sqrt{\frac{-a}{2} + \sqrt{\frac{a^2+b^2}{4}}} \right)$$

$$\sqrt{\frac{a^2+b^2}{4}} = \sqrt{\frac{5^2+7^2}{4}} = 4.3011$$

$$\sqrt{z} = \mp\left(\sqrt{\frac{5}{2} + 4.3011} + i \sqrt{\frac{-5}{2} + 4.3011} \right)$$

$\sqrt{z}$ =∓(2.6079+i1.342)

z = 5+i6.9996≈5+i7

3. z = i6
solution:
a=0, b=6

$$\sqrt{z} = \mp\left(\sqrt{\frac{a}{2}+\sqrt{\frac{a^2+b^2}{4}}} + i\sqrt{\frac{-a}{2}+\sqrt{\frac{a^2+b^2}{4}}}\right)$$

$$\sqrt{\frac{a^2+b^2}{4}} = \sqrt{\frac{0^2+7^2}{4}} = \frac{7}{2} = 3.5$$

$$\sqrt{z} = \mp\left(\sqrt{0+\frac{7}{2}} + i\sqrt{0+\frac{7}{2}}\right)$$

$$\sqrt{z} = \mp\left(\sqrt{\frac{7}{2}} + i\sqrt{\frac{7}{2}}\right)$$

4. z = 6+i8
solution:
a=6, b=8

$$\sqrt{z} = \mp\left(\sqrt{\frac{a}{2}+\sqrt{\frac{a^2+b^2}{4}}} + i\sqrt{\frac{-a}{2}+\sqrt{\frac{a^2+b^2}{4}}}\right)$$

$$\sqrt{\frac{a^2+b^2}{4}} = \sqrt{\frac{6^2+8^2}{4}} = 5$$

$$\sqrt{z} = \mp\left(\sqrt{3+5} + i\sqrt{-3+5}\right)$$

$$\sqrt{z} = \mp\left(\sqrt{8} + i\sqrt{2}\right)$$

5. z = 3-i4
solution:
a=3, b=-4

$$\sqrt{z} = \mp\left(\sqrt{\frac{a}{2}+\sqrt{\frac{a^2+b^2}{4}}} - i\sqrt{\frac{-a}{2}+\sqrt{\frac{a^2+b^2}{4}}}\right)$$

$$\sqrt{z} = \mp\left(\sqrt{\frac{3}{2}+\sqrt{\frac{3^2+4^2}{4}}} - i\sqrt{\frac{-3}{2}+\sqrt{\frac{3^2+4^2}{4}}}\right)$$

$$\sqrt{z} = \mp\left(\sqrt{1.5+2.5} - i\sqrt{-1.5+2.5}\right)$$

$$\sqrt{z} = \mp(2 - i1)$$

4. References

[1] Burton, David M., *The History of Mathematics*. New York. USA.1995; 3, p.294. McGraw-Hill, ISBN 978-0-07-009465-9.