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AL-Tememe transformation for solving some types PDEs with using initial and boundary conditions

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ABSTRACT

The aim of this paper is to use Al-Tememe transformation for solving some types of PDEs which have the general form

$$At^2u_{tt} + Btu_t + Ch_1(x)u_{xx} + Eh_2(x)u_x + Gth_3(x)u_{tx} + Pu = f(x,t)$$

Where A, B, C, E, G and P are constants and $f(x,t)$ is a function of x and t.

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1. INTRODUCTION

Laplace transformation is considered as one of the important transformation which is known to solve some LPDE with constant coefficients and which has the general form

$$\sum_{i=0}^{n}\sum_{j=0}^{m} a_{ij}\,\frac{\partial^{i+j}u(x,t)}{\partial t^{i}\partial x^{j}}$$

Where a_{ij} are constants and with one condition namely that Laplace transformation of the function $f(x,t)$ is known.

In this paper , we will use Al-Tememe transformation to solve LPDE with variable coefficients which has the general form

$$At^2u_{tt}+Btu_t+Ch_1(x)u_{xx}+Eh_2(x)u_x+Gth_3(x)u_{tx}+Pu=f(x,t)$$

Where $\mathrm{A},\mathrm{B},\mathrm{C},\mathrm{E},\mathrm{G}$ and P are constants and $f(x,t)$ is a function of x and t.This transformation is defined for some function for example constant function, logarthim functions and polynomial functions ,as well as other functions.

2. Preliminaries

Definition 1: [1]

Let f is defind function at period (a,b)then the integral transformation for$f$ whose it's symbol $F(p)$ is defined as :

$$F(p)=\int_{a}^{b}k(p,x)f(x)dx$$

Where k is a fixed function of two variables, called the kernel of the transformation, and a,b are real numbers or $\mp\infty$, such that the integral above converges.

Definition 2:[2]

The Al-Tememe transformation for the function $f(x)$where $(x>1)$ is defined by the following integral :

$$\mathcal{T}[f(x)]=\int_{1}^{\infty}x^{-s}f(x)dx=F(s)$$

Such that this integral is convergent ,s is positive constant. From the above definition we can write

$$\mathcal{T}[u(x,t)]=\int_{1}^{\infty}t^{-s}u(x,t)dt=v(x,s)$$

Such that $u(x,t)$ is a function of $x$and t.

Property 1:[2]

This transformation is characterized by the linear property ,that is

$\mathcal{T}[Au_1(x,t)+Bu_2(x,t)]=A\mathcal{T}[u_1(x,t)]+B\mathcal{T}[u_2(x,t)]$,

Where A , B are constants ,the functions$u_1(x,t)$, $u_2(x,t)$ are defined when $t>1$.

The Al-Tememe transform of some fundamental functions are given in table(1)[2]

ID	Function ,$f(x)$		Region of convergence
1	k	s	
2	x	s	
3	l	(	
4	x	[	
5	s	(	
6	c	(	
7	s	(	
8	c	(	
9	$(ln\,x)^n\,, n \in N,$	(	
10	x n	[	

Table (1).

Definition 3: [2]

Let $u(x,t)$ be a function where $(\,t\ > 1)$ and $\mathcal{T}[u(x,t)] = v(x,s), u(x,t)$ is said to be an inverse for the Al-Tememe transformation and written as $\mathcal{T}^{-1}[v(x,s)] = u(x,t)$, where $\mathcal{T}^{-1}$returns the transformation to the original function.

5-Solving the linear partial differential equations with variable coefficients:

One of the most important applications of the T-transform is solving the linear partial differential equations with variable coefficients.This transformation change the LPDE to LODE with variable coefficients.After that take T^{-1} from the result equation then we get the solution of LPDE with variable coefficients.

Now let us consider the LPDE

Where $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{E}, \mathrm{G}$ and P are constants and $\mathrm{f(x,t)}$ is a function of x and t such that the $(\mathrm{T}.\mathrm{T}^{-1})$ for the $\mathrm{f(x,t)}$ is known.

Poperties:

1- $\mathrm{T}\{\mathrm{tu_t(x,t)}\} = -\mathrm{u(x,1)} + (\mathrm{s}-1)\mathrm{v(x,s)}$

Proof:

$\mathrm{T}\{\mathrm{tu_t(x,t)}\} =$

$= \mathrm{t^{-s+1}u(x,1)}$

$= -\mathrm{u(x,1)} + ($

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2-$T\{t^2u_{tt}(x,t)\} = -u_t(x,1) - (s-2)u(x,1) + (s-2)(s-1)v(x,$

Proof:-

$T\{t^2u_{tt}(x,t)\}$

-$T\{h(x)u_x(x,t)\} = x\frac{d}{dx}v(x,s)$**3**

Proof:-

$T\{h(x)u_x(x,t$

$T\{h(x)u_{xx}(x,t)\} = h(x)\frac{d^2}{dx^2}v(x,s)$**-4**

Proof:-

$T\{h(x)u_{xx}(x,$

$T\{h(x)tu_{tx}(x,t)\} = h(x)\frac{d}{dx}[-u(x,1) + (s-1)v(x,s)]$

Proof:

$T\{h(x)tu_{tx}(x,t$

$= h(x)\frac{d}{dx}[t^{-s}$

$= h(x)\frac{d}{dx}[-u$

6-Solving some LPDEs by using Al-Tememe transform with using initial an boundary conditions

Example:

To solve the following PDE

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By using Al-Tememe transformation where $u(x,1) = u(0,t) = 0$.

solution*:*

We take T-transformation to both sides of the equation, so we get

Now to solve the above ordinary differential equation

Firstly to find the homogenous solution

Then the homogenous solution is

Since we have $u(0,t) = 0 \rightarrow T[u(0,t)] = v(0,t) = 0$

Now to find the particular solution

To find A_1, A_2, A_3 and A_4

Then $A_1 = 0, A_2 = 0, A_3 = x$and $A_4 = -1$.

Then we get T^{-1} to both sides , so we get

Example:

To solve the following PDE

By using Al-Tememe transformation where $u(x,1) = u(0,t) = u_t(x,1) = 0$.

solution*:*

We take T-transformation to both sides of the equation, so we ge

$-u_t(x,1$

Now to solve the above ordinary differential equation we assume $z = lnx$,thus$D_1 = xD$.

After substitute the above assumption in the differential equation we get

Firstly to find the homogenous solution

Then the homogenous solution is

Since we have $u(1,t) = 0 \to T[u(1,t)] = v(1,t) = 0$

Now to find the particular solution

To find the form of v_p , we substitute (1) instead of D_1 , then

Then

Then

Then we get T^{-1} to both sides , so we get

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