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Modeling and Analysis of Perishable Inventory System with Retrial demands in Supply Chain

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Abstract

In this article, we consider a continuous review perishable inventory system with poisson demands. The maximum storage capacity at lower echelon (retailer) is S and the upper Echelon (Distribution Center) is $M (= nQ)$. The life time of each item is assumed to be exponential. The operating policy is (s, S) policy, that is, whenever the inventory level drops to s, an order for $Q = (S - s > s)$ item is placed. The ordered items are received after a random time which is distributed as exponential. We assume that demands occurring during the stock-out period enter into the orbit. These orbiting demands send out signal to complete for their demand which is distributed as exponential. The joint probability distribution of the inventory level at retailer, inventory level at DC and the number of demands in the orbit are obtained in the steady state case. Various system performance measures are derived and the results are illustrated numerically.

Keywords: Two-echelon; Perishable inventory; Retrial demand; Supply Chain.

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1 Introduction

The analysis of perishable inventory systems has been the theme of many articles due to its potential applications in sectors like food, chemicals, pharmaceuticals, photography and blood bank management. The often quoted review articles [12,28] and the recent review articles [29,23] provide excellent summaries of many of these modeling efforts.

Most of these models deal with either the periodic review systems with fixed life times or continuous review systems with instantaneous supply of reorders. In the case of continuous review perishable inventory models with random life times for the items, most of the models assume instantaneous supply of order [26,27,24]. The assumption of positive lead times further increases the complexity of the analysis of these models and hence there are only a limited number of papers dealing with positive lead times. Moreover they are mostly devoted to the systems with base stock policy [25] or fixed reorder level [21].

In all these models, authors assumed that the demands that occurred during stock-out is either backlogged or lost and the number of sources that generate demands are infinite. In this paper we relax these assumptions. We assume that the demands that occurred during stock- out enter into the orbit and retry for their demands after a random time. The concept of retrial demands in inventory was introduced in [19] and only few papers [31,30] have appeared in this area. However, considerable interest is shown in the study of Queuing models with retrial customers [20,17-19,22].

In this article, we consider a continuous review perishable inventory system with poisson demands. The maximum storage capacity at lower echelon (retailer) is S and the upper Echelon (Distribution Center) is $M(= nQ)$. The life time of each item is assumed to be exponential. The operating policy is (s, S) policy, that is, whenever the inventory level drops to s, an order for $Q = (S - s > s)$ item is placed. The ordered items are received after a random time which is distributed as exponential. We assume that demands occurring during the stock-out period enter into the orbit. These orbiting demands send out signal to complete for their demand which is distributed as exponential.

The first quantitative analysis in inventory studies started with the work of Harris (1915) [9].Clark and Scarf (1960) [4] had put forward the multi-echelon inventory first. They analyzed a N-echelon pipelining system without considering a lot size, Recent developments in two-echelon models may be found in Q.M. He and E.M. Jewkes (2000)[13]. Sven Axaster (1990)[1] proposed an approximate model of inventory structure in SC. One of the oldest papers in the field of continuous review multi-echelon inventory system is a basic and seminal paper written by Sherbrooke [15] in 1968. He assumed (S-1, S) polices in the Deport-Base systems for repairable items in the American Air Force and could approximate the average inventory and stock out level in bases.

Continuous review models of multi-echelon inventory system in 1980's concentrated more on repairable items in a Depot-Base system than as consumable items (see Graves [6,7], Moinzadeh and Lee [11]). All these papers deal with repairable items with batch ordering. Jokar and Seifbarghy [14] analyzed a two echelon inventory system with one warehouse and multiple retailers controlled by continuous review (R, Q) policy. A Complete review was provided by Benita M. Beamon (1998)[2]. The supply chain concept grow largely out of two-stage multi-echelon inventory models, and it is important to note that considerable research in this area is based on the classic work of Clark and Scarf (1960)[4]. A continuous review perishable inventory system at Service Facilities was studied by Elango (2001) [5]. A continuous review (s, S) policy with positive lead times in two-echelon Supply Chain was considered by Krishnan. K and ElangoC.2005 [10].

The rest of the paper is organized as follows. In Section 2, we describe the mathematical model. The steady-state analysis of the model is presented in Section 3. In Section 4, some key system performance measures are derived. In section 5, we calculate the total expected cost rate. In Section 6, the results are illustrated numerically. The last section concludes the paper.

Notation:

- $[A]_{i,j}$: (i, j) th element/ block of the matrix A
- I_n : Identity matrix of order n
- e : column vector of ones with appropriate dimension
- S = The maximum inventory level at retailer nodes
- s = Reorder level at retailer nodes
- $Q = S - s$
- $E = \{(i, j, k) | i = 0, 1, \ldots, N,\ j = 0, 1, \ldots, S,\ k = Q, 2Q, \ldots, nQ\}$
- $\sum_{k=Q}^{nQ}(.)$ stands for $\sum_{k=Q}(.) + \sum_{k=2Q}(.) + \sum_{k=3Q}(.) +, \ldots, + \sum_{k=nQ}(.)$

2. Model Description

We consider a two level supply chain consisting one product, one manufacturing facility, one warehousing facility and one retailer. The demands initiated at retailer node follow Poisson process with parameter $\lambda\ (> 0)$ and the lead times are exponentially distributed with parameter $\mu\ (> 0)$. The retailer follows (s, S) policy for maintaining his inventory and the distributor follow $(0, nQ)$ policy for maintaining his inventory. The items are perishable in nature. The life time of an item is exponentially distributed with parameter $\gamma\ (> 0)$. The unsatisfied customers are treated as retrial customers and they are waiting in the orbit with finite capacity N. The repeated customers from the orbit (with capacity i) are entered into the system with rate $i\theta(> 0)$. Even though we have adopted two different policies in the Supply Chain, the distributors olicy is depends upon the retailers policy. The model minimizes the total cost incurred at all the locations subject to the service level constraints. The system performance measures and the total cost are computed in the steady state. The results are illustrated numerically.

3. Analysis

Let $X(t); Y(t)$ and $Z(t)$ respectively denote the number of demands in the orbit, the on hand inventory level in the retailer node and the number of items in the Distribution centre at time t. From the assumptions on the input and output processes, clearly $X^1(t) = \{(X(t), Y(t), Z(t)) : t > 0\}$ is a Markov process with state space E. The infinitesimal generator of this process $A = (a(i, k, m : j, l, n)),\ (i, j, m), (j, l, n) \in E$ can be obtained from the following arguments.

- The primary arrival of demand to the retailer node makes a transition in the Markov process from (i, j, k) to $(i-1, j, k)$ with intensity of transition λ.
- The arrival of a demand at retailer node from orbit transition in the Markov Process from (i, j, k) to $(i-1, j\ 1, k)$ with intensity of transition $i\theta$.
- The item expires makes a transition from (i, j, k) to $(i-1, j, k)$ with intensity of transition γ.
- Replenishment of inventory at retailer node makes a transition from (i, j, k) to $(i, j+Q, k-Q)$ with rate of transition μ.

Then, the infinitesimal generator has the following finite QBD structure:

$$
P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ \vdots \\ N-2 \\ N-1 \\ N \end{array}
\begin{array}{c} \begin{array}{ccccccc} 0 & 1 & 2 & \cdots & N-2 & N-1 & N \end{array} \\
\begin{pmatrix}
A_0 & C & & & & & \\
B_1 & A_1 & C & & & & \\
 & B_2 & A_2 & & & & \\
 & & \ddots & \ddots & \ddots & & \\
 & & & & A_{N-2} & C & \\
 & & & & B_{N-1} & A_{N-1} & C \\
 & & & & & B_N & A_N
\end{pmatrix} \end{array}
$$

Where,

$$
[A_i]_{k,l} = \begin{cases}
-(\lambda+\mu)I_n & l = k \quad k = 0, \\
-(\lambda+\mu+i\theta+k\gamma)I_n & l = k \quad k = 1,2,\cdots,s, \\
-(\lambda+i\theta+k\gamma)I_n & l = k \quad k = s+1, s+2,\cdots,S, \\
(\lambda+k_\gamma)I_n & l = k-1 \quad k = 1,2,\cdots,S, \\
D & l = k+Q \quad k = 0,1,\cdots,s, \\
0 & otherwise
\end{cases}
$$

$$[B_i]_{k,l} = \begin{cases} i\theta I_n & l = k-1 \quad k = 1,2,\cdots,S, \\ 0 & otherwise \end{cases}$$

$$[C]_{k,l} = \begin{cases} \lambda I_n & l = k \quad k = 0 \\ 0 & otherwise \end{cases}$$

$$D = \begin{array}{c} \\ Q \\ 2Q \\ 3Q \\ \vdots \\ nQ \end{array} \begin{array}{c} \begin{array}{ccccc} Q & 2Q & 3Q & \ldots & nQ \end{array} \\ \begin{pmatrix} & & & & \mu \\ \mu & & & & \\ & \mu & & & \\ & & \ddots & & \\ & & & \mu & \end{pmatrix} \end{array}$$

Here the matrices A_i, B_i and C are the square matrices of order $(S+1)n$ and note that the matrix D is the square matrix of order n.

3.1 Steady state analysis

Since the state space is finite and P is irreducible, the stationary probability vector Π for the generator P always exists and satisfies $\Pi P=0 \quad \Pi e=1$

The vector Π can be represented by

$$\Pi = (\Pi^{<0>}, \Pi^{<1>}, \Pi^{<2>}, \ldots\ldots, \Pi^{<N>}$$

Where, $0 \le i \le 1$

$$\Pi^{<i>} = (\pi^{<<i,0>>}, \pi^{<<i,1.>>}, \ldots\ldots, \pi^{<<i,S>>})$$

$$\Pi^{<<i,j>>} = (\pi^{<<<i,j,Q>>>}, \pi^{<<<i,j,2Q>>>}, \ldots\ldots, \pi^{<<<i,j,nQ>>>}), j = 0,1,2,\ldots\ldots, S$$

Now the structure of P shows, the model under study is a finite birth death model in the Markovian environment. Hence we use the Gaver algorithm for computing the limiting probability vector. For the sake of completeness we provide the algorithm here.

Algorithm:

1. Determine recursively the matrix D_n, $0 \le n \le N$ by using

$$D_0 = A_0 \tag{3.1}$$

$$D_n = A_n + B_n(-D_{n-1}^{-1})\, C, \quad n = 1,2,\ldots.K \tag{3.2}$$

2. Slove the system

$$\Pi^{<N>} D_N = 0\,. \tag{3.3}$$

3. Compute recursively the vector $\Pi^{<n>}$, $\quad n = N-1,\ldots..,0$ using

$$\Pi^{<n>} = \Pi^{<n+1>} B_{n+1}(-D_n^{-1}), n = n-1,\ldots\ldots,0\,. \tag{3.4}$$

4. Re-normalize the vector Π, using

$$\Pi e = 1. \tag{3.5}$$

4 Performance measures

Consider the event r_R of reorders at nodes and D. Observe that r_D event occur whenever the inventory level at DC node reaches 0 whereas the r_R event occurs whenever the inventory level at retailer node reaches reorder level s.

4.1 Mean reorder rate

Let I_R denote the expected inventory level in the steady state at retailer node and I_D denote the expected inventory level at distribution centre.

$$I_R = \sum_{i=0}^{N}\sum_{j=1}^{S}\sum_{k=Q}^{nQ} j\Pi^{<<<i,j,k>>>} \quad (4.1)$$

$$I_D = \sum_{i=0}^{N}\sum_{j=0}^{S}\sum_{k=Q}^{nQ} k\Pi^{<<<i,j,k>>>} \quad (4.2)$$

4.2 Expected number of demands in the orbit

Let $E(o)$ denote the expected number of customer in the orbit which is given by

$$E(o) = \sum_{i=1}^{N}\sum_{j=0}^{S}\sum_{k=Q}^{nQ} i\Pi^{<<<i,j,k>>>} \quad (4.3)$$

4.3 Mean reorder rate

The mean reorder rate at retailer node is given by

$$r_R = \sum_{i=0}^{N}\sum_{k=Q}^{nQ}(\lambda + i\theta + (s+1)\gamma)\Pi^{<<<i,s+1,k>>>} \quad (4.4)$$

4.4 Shortage rate

Shortage occurs only at retailer node and the shortage rate for the retailer is denoted by α_R and which is given by

$$\alpha_R = \sum_{i=0}^{N}\sum_{k=Q}^{nQ}\lambda\Pi^{<<<i,0,k>>>} + \sum_{i=1}^{N}\sum_{k=Q}^{nQ} i\theta\,\Pi^{<<<i,0,k>>>} \quad (4.5)$$

5 Cost Analysis

In this section we analyze the cost structure for the proposed models by considering the minimization of the steady state total expected cost per time.

The long run expected cost rate for the model is defined to be

$$TC(S,s,n) = h_R\ I_R + h_D\ I_D + k_R\ r_R + g_R\ \alpha_R + c_0\ E(0)$$

h_R- denote the inventory holding cost/ unit / unit time at retailer node

h_D- denote the inventory at distribution centre

k_R- denote the setup cost/ order at retailer node

g_R- denote the shortage cost/ unit shortage at retailer node

c_o- denote the back ordering of a demand in the orbit / unit time.

6 Numerical Illustration

Example: We analyzed the following in the numerical section.

1. Table 1 and Table 2 give the effect of total cost function by varying the set-up cost, holding cost, shortage cost and the back ordering cost.
2. Figure 1 shows that the effect of the demand rate λ and lead time μ on long run expected cost.

3. Figure 2 shows that the effect of the primary demand rate λ and orbiting demand rate θ on long run expected cost.
4. Figure 3 shows that the effect of the lead time μ and orbiting demand rate θ on long run expected cost
5. Figure 4 shows that the effect of the demand rate λ and perishable demand γ rate on long run expected cost.
6. Figure 5 shows that the effect of the orbiting demand rate θ and perishable demand rate γ on long run expected cost.
7. Figure 6 shows that the effect of the lead time μ and perishable demand rate γ on long run expected cost.
8. Finally Figure 7 shows that the effect of maximum number in the orbit on the total cost rate.

	h_D = 0.04	h_D = 0.08	h_D = 0.12	h_D = 0.16	h_D = 0.20
h_R = 0.002000	220.112907	220.395536	220.678164	220.960793	221.243421
h_R = 0.004000	220.117181	220.399809	220.682438	220.965066	221.247694
h_R = 0.006000	220.121454	220.404082	220.686711	220.969339	221.251968
h_R = 0.008000	220.125727	220.408356	220.690984	220.973613	221.256241
h_R = 0.010000	220.130001	220.412629	220.695258	220.977886	221.260514

Table 1 : h_R vs h_D on TC (18, 4, 6)

	g_R = 0.2	g_R = 0.4	g_R = 0.6	g_R = 0.8	g_R = 0.10
k_D = 10	193.290714	199.355262	205.419811	211.484359	217.548907
k_D = 15	204.361368	210.425917	216.490465	222.555013	228.619561
k_D = 20	215.432023	221.496571	227.561119	233.625667	239.690215
k_D = 25	226.502677	232.567225	238.631773	244..696321	250.760869
k_D = 30	237.573331	243.637879	249.702427	255.766975	261.831523

Table 2 : k_R vs g_R on TC (18, 4, 6)

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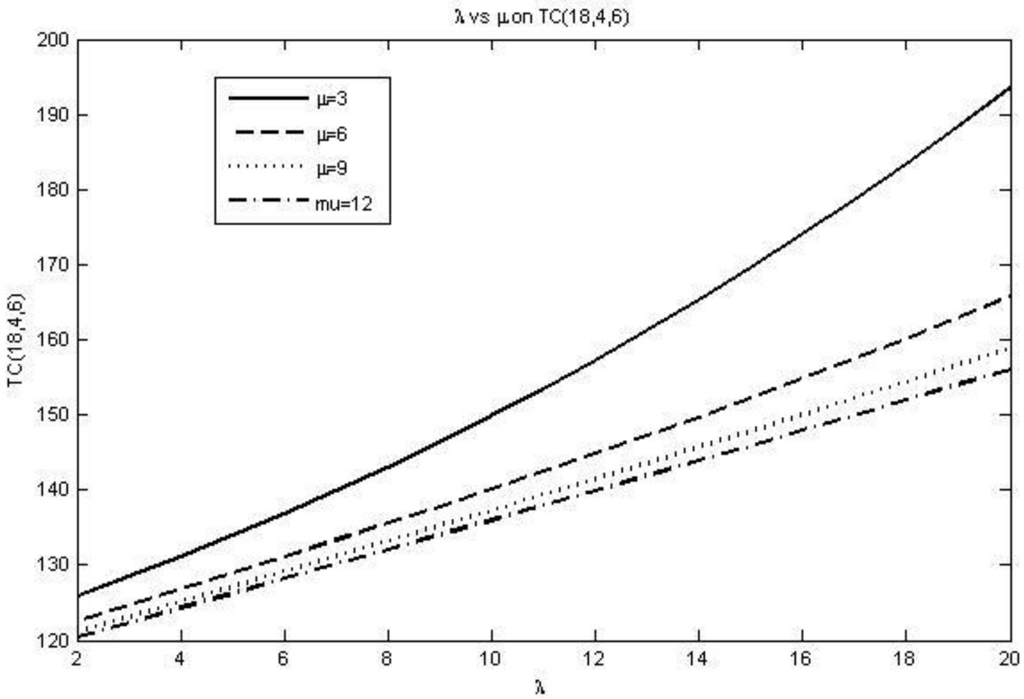

Figure 1: λ vs μ on TC (18, 4, 6)

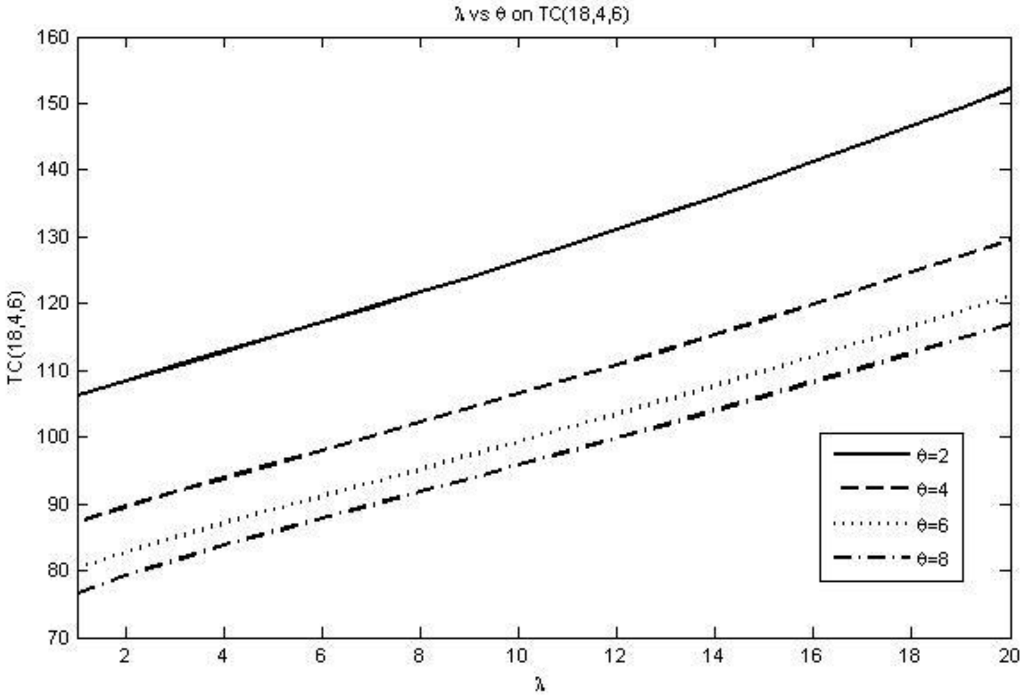

Figure 2 : λ vs θ on TC (18,4,6)

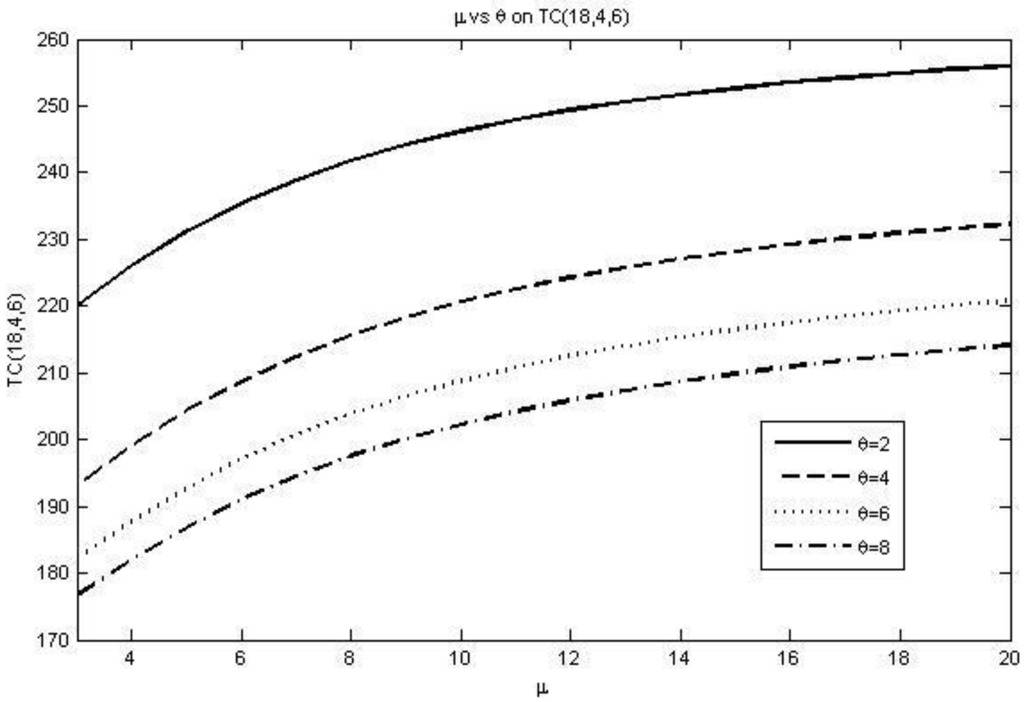

Figure 3 : μ vs θ on $\mathrm{TC}\,(18, 4, 6)$

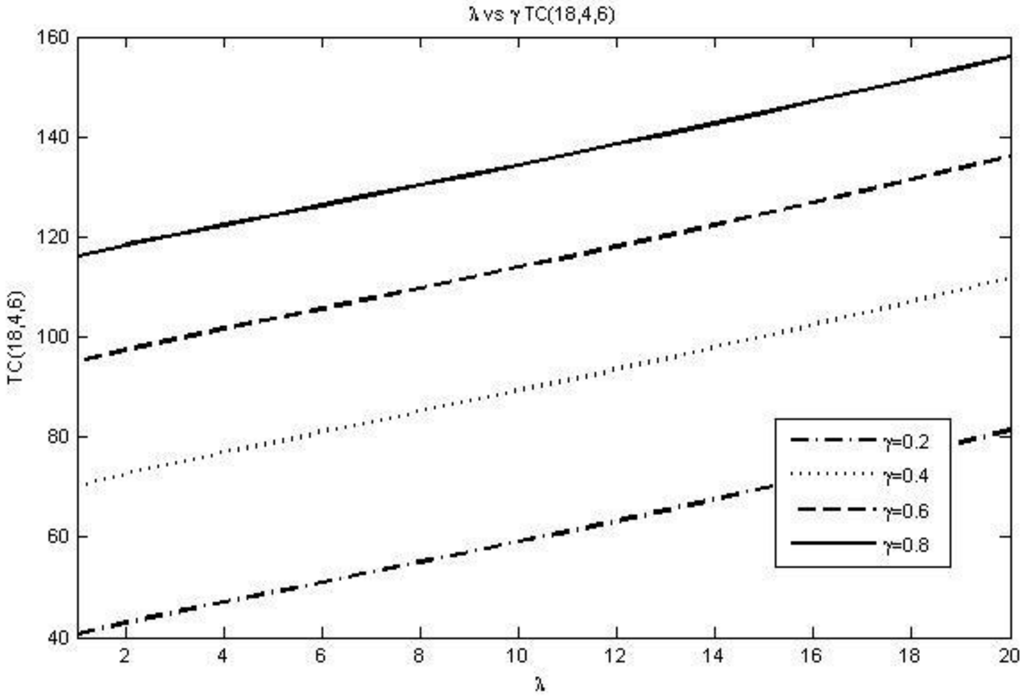

Figure 4 : λ vs γ on $\mathrm{TC}\,(18,4,6)$

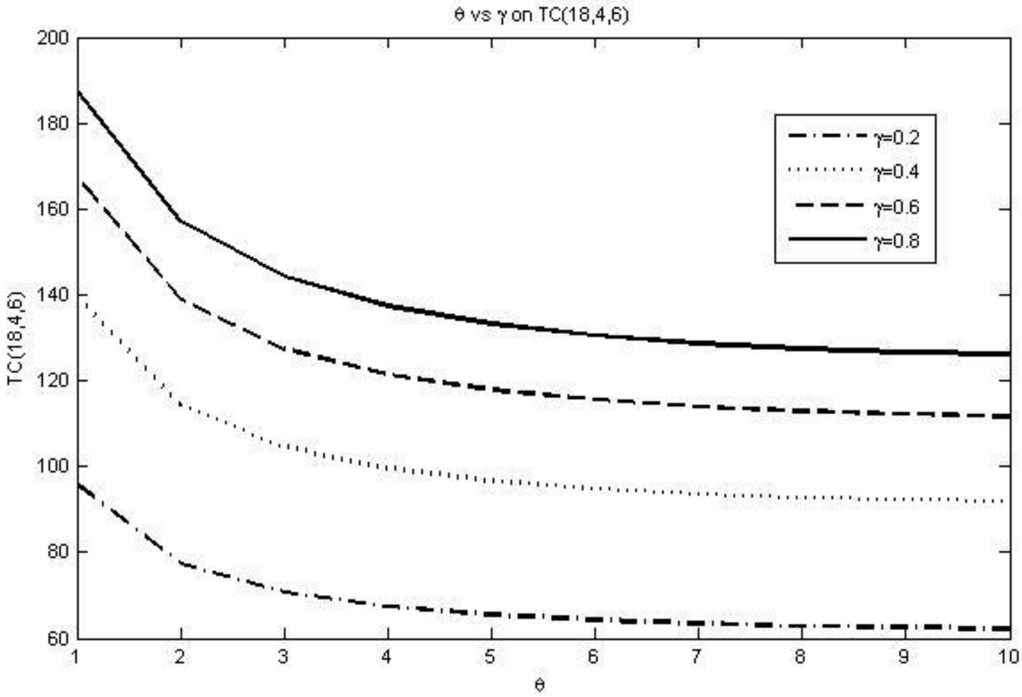

Figure 5 : θ vs γ on $TC\ (18, 4, 6)$

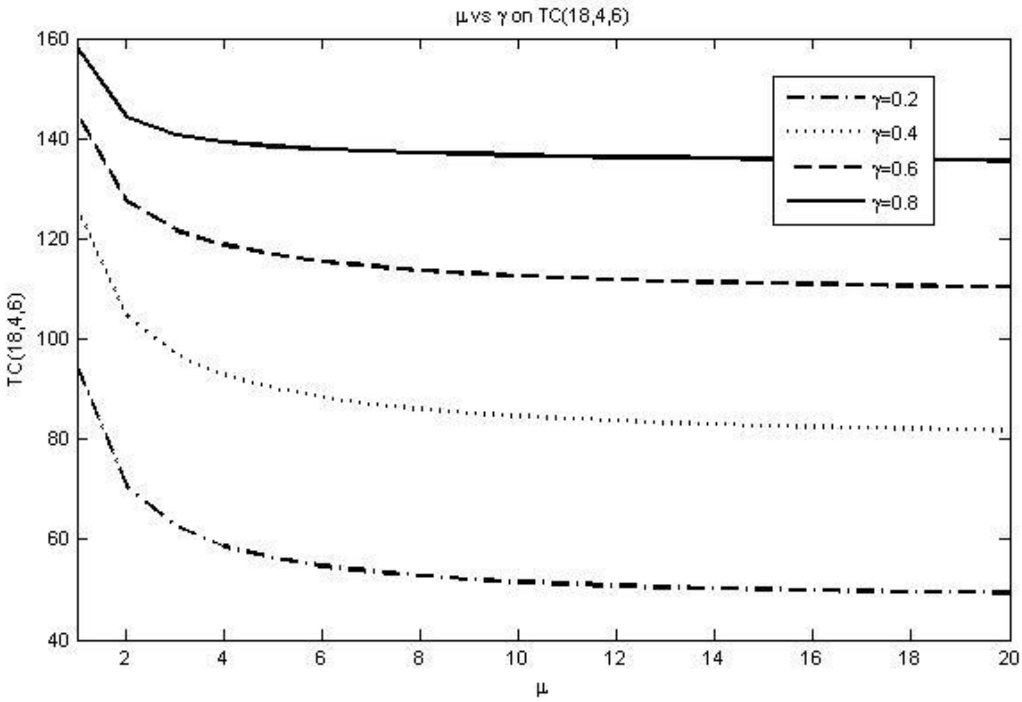

Figure 6 : μ vs γ on $TC\ (18, 4, 6)$

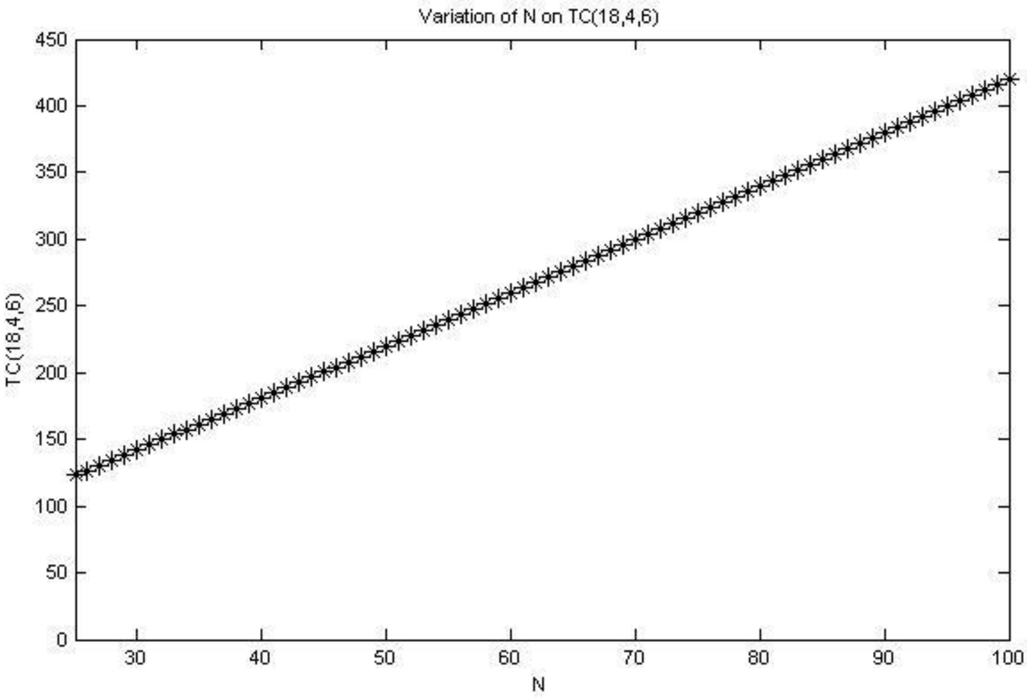

Figure 7 : N vs TC (18, 4, 6)

From the numerical work, we conclude the following:

1. As is to be expected λ increases total cost increases, μ and θ increases total cost decreases.
2. Total cost increases when the costs h_D, h_R, g_R and k_R increases.

6 Conclusion

In this article, we analyzed a continuous review stochastic perishable inventory system with retrial demands. The arrival of demands form a Poisson distribution. The life time of each items, lead times of reorder and the retrial demand time points forms independent exponential distributions. The model is analyzed within the framework of Markov processes. Joint probability distribution of inventory level at retailer, distribution and the number of customers in the orbit is obtained in the steady state. Various system performance measures are derived and the long-run expected cost rate is calculated. By assuming a suitable cost structure on the inventory system, we have presented extensive numerical illustrations to show the effect of change of values for constants on the total expected cost rate. It would be interesting to analyze the problem discussed in this article where the life time of items are constant. Naturally, with the inclusion of constant life time of each items, the problem will be more challenging. Another important extension could be made by relaxing the assumption of exponentially distributed lead times to a class of arbitrarily distributed lead times using techniques from renewal theory and semi-regenerative processes. Once this is done, the general model can be used to generate various special cases. For example, three different lead time distributions one with coefficient of variation greater than one, one with coefficient of variation less than one and another with coefficient of variation equal to one (this model) can be compared. Cost analysis can then be carried out for (s, Q), (s, S) and lot-for-lot models using each of the three different lead time distributions to determine which policy is optimal for any given lead time distribution. The author is currently working on the above extensions, and these will be reported in future publications.

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