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LITERAL ANALYTICAL SOLUTION OF THREE DIMENSIONAL PHOTOGRAVITATIONAL CIRCULAR RESTRICTED THREE BODY PROBLEM

M.A. Sharaf[1], N. S. Abd El Motelp[2, 3]
[1]Department of Astronomy, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia
Sharaf_adel@hotmail.com
[2]Department of Astronomy, National Research Institute of Astronomy and Geophysics, Cairo, Egypt
[3]Department of Mathematics, Preparatory Year, Hail University Girls branch, Saudi Arabia
nihad_planet@hotmail.com

ABSTRACT

This paper is devoted to construct the literal analytical solutions in power series form for photogravitational circular restricted three body problem in three dimensional space, and taking into account that both primaries are light energy source (RT3D2R).The importance of these analytical solutions is that, they are invariant under many operations such, addition, multiplication, exponentiation, integration, differentiation, etc. Actually power series provides us an excellent flexibility in obtaining simple analytical expressions which related to the motion of the third body. Besides power series enables us of obtaining full numerical solutions of restricted three body problem at any given set of initial values. As a typical example, literal analytical expressions for the coefficients of the power series were developed in terms of the initial values for N=3.

Indexing terms/Keywords

Restricted three body problem; symbolic solution; recurrent algorithm; radiation pressure.

Academic Discipline And Sub-Disciplines

Perturbation theory; Celestial Mechanics; Dynamical Astronomy.

SUBJECT CLASSIFICATION

Celestial Mechanics

TYPE (METHOD/APPROACH)

Analytical solutions in power series form for photogravitational circular restricted three body problem

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INTRODUCTION

Three dimensional solution of the restricted three body problem has been studied by a number of authors. The photogravitational circular restricted three body problem was first studied by Radzievskyii (1950, 1953). He formulated the problem when one of the interacting masses is an intense emitter of radiation. He discussed it for 3 specific bodies: Sun, a planet and dust particle. Simmons et al (1985) gave a complete solution of the restricted 3 body problem; they also discussed the existence and linear stability for Lagrangian points. Jagadish (1999) studied the stability of the triangular points in the generalized photogravitational restricted three body problem, when the primaries are radiated and oblate spheroids. In the present paper, literal analytical solution in power series forms are established for photogravitational circular restricted three body problem in three dimensional space, when both primaries emit light energy. (RT3D2R).

2. Equations of motion

Equations of motion when the two primaries are radiated are given as:

$$\ddot{X} - 2\dot{Y} = \frac{\partial\Omega}{\partial X} \tag{1.1}$$

$$\ddot{Y} + 2\dot{X} = \frac{\partial\Omega}{\partial Y} \tag{1.2}$$

$$\ddot{Z} = \frac{\partial\Omega}{\partial z} \tag{1.3}$$

$$\frac{\partial\Omega}{\partial X} = X - \frac{q_1\nu(-1+X+\nu)}{{R_1}^3} - \frac{q_0(1-\nu)(X+\nu)}{{R_0}^3} \tag{1.4}$$

$$\frac{\partial\Omega}{\partial Y} = Y - \frac{q_1 Y\nu}{{R_1}^3} - \frac{q_0(1-\nu)Y}{{R_0}^3} \tag{1.5}$$

$$\frac{\partial\Omega}{\partial z} = -\frac{q_1 Z\nu}{{R_1}^3} - \frac{q_0 Z(1-\nu)}{{R_0}^3} \tag{1.6}$$

$$\Omega = \frac{1}{2}(X^2+Y^2) + \frac{q_0(1-\nu)}{R_0} + \frac{q_1\nu}{R_1},$$

$$R_0 = \sqrt{(x+\nu)^2 + y^2 + z^2},$$

$$R_1 = \sqrt{(1-x-\nu)^2 + y^2 + z^2},$$

$\nu = \frac{m_1}{m_0+m_1}$ $\quad 0 \le \nu \le 1$ (Placing the larger primary to the left)

q_i , i=0, 1 Represent the effects of radiation pressure from the two primaries.

$q_0 = 1,\ q_1 = 1$ Represent the classical problem means no radiation pressure.

3. Linearization of equation of motion

First of all, let us replace $1/R_i^3$ such that:

$$S_i = \frac{1}{R_i^3}, i = 0,1$$

The equations of motion will be written in linear form as:

$$\dot{X} = U, \tag{2.1}$$

$$\dot{Y} = V, \tag{2.2}$$

$$\dot{Z} = W, \tag{2.3}$$

Substitute from Equations (2.1), (2.2), (2.3) into Equations (1) we get

$$\dot{U} = X - q_0(1-\nu)(X+\nu)S_0 - q_1\nu(X+\nu-1)S_1 + 2V \tag{2.4}$$

$$\dot{V} = Y(1 - q_0 S_0(1-\nu) - q_1 S_1 \nu) - 2U \tag{2.5}$$

$$\dot{W} = -Z(q_0 S_0(1-\nu) + q_1 S_1 \nu) \tag{2.6}$$

$$R_0\dot{R}_0 = (X+\nu)\dot{X} + Y\dot{Y} + Z\dot{Z} \tag{3.1}$$

$$R_1\dot{R}_1 = (X+\nu-1)\dot{X} + Y\dot{Y} + Z\dot{Z} \tag{3.2}$$

$$R_0\dot{S}_0 = -3S_0\dot{R}_0 \tag{3.3}$$

$$R_1\dot{S}_1 = -3S_1\dot{R}_1 \tag{3.4}$$

4. Power Series Solution of (RT3D2R) Problem

Assume that the power series of the variables as:

$$X = \sum_{n=1}^{\infty} x_n t^{n-1} \qquad Y = \sum_{n=1}^{\infty} y_n t^{n-1} \qquad Z = \sum_{n=1}^{\infty} z_n t^{n-1}, \tag{4.1}$$

$$U = \sum_{n=1}^{\infty} u_n t^{n-1} \qquad V = \sum_{n=1}^{\infty} v_n t^{n-1} \qquad W = \sum_{n=1}^{\infty} w_n t^{n-1}, \tag{4.2}$$

$$S_0 = \sum_{n=1}^{\infty} s_n^{(0)}\, t^{n-1} \qquad S_1 = \sum_{n=1}^{\infty} s_n^{(1)}\, t^{n-1} \tag{4.3}$$

$$R_0 = \sum_{n=1}^{\infty} r_n^{(0)}\, t^{n-1} \qquad R_1 = \sum_{n=1}^{\infty} r_n^{(1)}\, t^{n-1} \tag{4.4}$$

$$\dot{X} = \sum_{n=1}^{\infty} n\, x_{n+1} t^{n-1} \qquad \dot{Y} = \sum_{n=1}^{\infty} n\, y_{n+1} t^{n-1} \qquad \dot{Z} = \sum_{n=1}^{\infty} n\, z_{n+1} t^{n-1} \tag{5.1}$$

$$\dot{U} = \sum_{n=1}^{\infty} n\, u_{n+1}\, t^{n-1} \qquad \dot{V} = \sum_{n=1}^{\infty} n\, v_{n+1} t^{n-1} \qquad \dot{W} = \sum_{n=1}^{\infty} n w_{n+1} t^{n-1} \tag{5.2}$$

$$\dot{S}_0 = \sum_{n=1}^{\infty} n\, s_{n+1}^{(0)}\, t^{n-1} \qquad \dot{S}_1 = \sum_{n=1}^{\infty} s_{n+1}^{(1)}\, t^{n-1} \tag{5.3}$$

$$\dot{R}_0 = \sum_{n=1}^{\infty} n\, r_{n+1}^{(0)}\, t^{n-1} \qquad \dot{R}_1 = \sum_{n=1}^{\infty} n\, r_{n+1}^{(1)}\, t^{n-1} \tag{5.3}$$

The initial values of power series are:

$x_1 = X_0,$ $\qquad y_1 = Y_0,$ $\qquad z_1 = Z_0,$

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$u_1 = U_0, \qquad v_1 = V_0, \qquad w_1 = W_0,$

$r_n^{(0)} = \sqrt{(X+\nu)^2 + Y^2 + Z^2}, \quad r_n^{(1)} = \sqrt{(X+\nu-1)^2 + Y^2 + Z^2},$

$s_1^{(0)} = \left(r_1^{(0)}\right)^{-3}, \qquad s_1^{(1)} = \left(r_1^{(1)}\right)^{-3}$

Substitute by both Equations (4) and (5) into Equations (2), and then equate the coefficients of t^{n-1} we deduce,

$x_{n+1} = \frac{u_n}{n}, \qquad y_{n+1} = \frac{v_n}{n}, \qquad z_{n+1} = \frac{w_n}{n},$

$$U_{n+1} = \frac{1}{n}\left(x_n - q_1 \nu \sum_{p=1}^{n} h_p\, s_{n-p+1}^{(1)} - q_0(1-\nu)\sum_{p=1}^{n} g_p s_{n-p+1}^{(0)} + 2v_n\right),$$

$$v_{n+1} = \frac{1}{n}\left(y_n - q_0(1-\nu)\sum_{p=1}^{n} y_p\, s_{n-p+1}^{(0)} - q_1 \nu \sum_{p=1}^{n} y_p\, s_{n-p+1}^{(1)} - 2u_n\right),$$

$$w_{n+1} = -q_0(1-\nu)\sum_{p=1}^{n} z_p\, s_{n-p+1}^{(0)} - q_1 \nu \sum_{p=1}^{n} z_p\, s_{n-p+1}^{(1)},$$

$$r_{n+1}^{(0)} = \frac{1}{n r_1^{(0)}}\left(\epsilon_n \sum_{p=2}^{n}(n-p+1) r_p^{(0)}\, r_{n-p+2}^{(0)} + \sum_{p=2}^{n}(n-p+1) g_p x_{n-p+2} + \right.$$

$$\left. \sum_{p=1}^{n}(n-p+1) y_p\, y_{n-p+2} + \sum_{p=1}^{n}(n-p+1) z_p\, z_{n-p+2}\right),$$

$$r_{n+1}^{(1)} = \frac{1}{n r_1^{1}}\left(\epsilon_n \sum_{p=2}^{n}(n-p+1) r_p^{(1)}\, r_{n-p+2}^{(1)} + \sum_{p=2}^{n}(n-p+1) h_p\, x_{n-p+2} + \right.$$

$$\left. \sum_{p=1}^{n}(n-p+1) y_p\, y_{n-p+2} + \sum_{p=1}^{n}(n-p+1) z_p\, z_{n-p+2}\right),$$

$$s_{n+1}^{(0)} = \frac{1}{n r_1^{0}}\left(\epsilon_n \sum_{p=2}^{n}(n-p+1) s_{n-p+2}^{(0)}\, r_p^{(0)} - 3\sum_{p=1}^{n}(n-p+1) s_p^{(0)}\, r_{n-p+2}^{(0)}\right),$$

$$s_{n+1}^{(1)} = \frac{1}{n r_1^{(0)}}\left(\epsilon_n \sum_{p=2}^{n}(n-p+1) s_{n-p+2}^{(1)}\, r_n^{(1)} - 3\sum_{p=1}^{n}(n-p+1) s_p^{(1)}\, r_{n-p+2}^{(1)}\right),$$

Where:

$$g_p = \begin{cases} x_1 + \nu & if\ p = 1 \\ x_p & if\ p \geq 2 \end{cases},$$

$$h_p = \begin{cases} -1 + x_1 + \nu & if\ p = 1 \\ x_p & if\ p \geq 2 \end{cases},$$

$$\epsilon_n = \begin{cases} 0 & if\ \ n = 1 \\ -1 & if\ \ n\ \geq 2 \end{cases}$$

These equations are what we required to set up coefficients of power series solution of (RT3D2R) Problem.

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5. Examples of symbolic expressions for (RT3D2R) coefficients

The symbolic expressions of the coefficients of power series representation (**(RT3D2R)**) are listed in Appendix A, when N=3.In concluding the present paper, literal analytical solutions in power series forms are developed for the solution of three dimensional photogravitational restricted three body problem. These solutions are characterized that obtain them in form of recurrent power series in term of time t, which enable to deduce solution at any value of time t^* very simply. On the other hand, the solution of equations by any numerical differential equation solver gives us solutions at definite value of time t, which is belonging to the set S, where: $S = \{0, \ h, \ 2h \ , 3h, \cdots \}$

"h" the step size uses in numerical differential equation solver.

The interpolation formula is applied if the values of solutions $t^* \notin S$ .A process which needs more execution time, it will be less accuracy usual associated with the usage of interpolation formula but

if the time is divided into smaller interval, power series will be convergent to known value.

Appendix A

Symbolic expressions for the power series coefficients of (RT3D2R) for N=3

$$X_1 = x_0$$

$$X_2 = u_0$$

$$X_3 = \frac{1}{2}\left(2\ v_0 + x_0 - \frac{\nu q_1(-1+\nu+x_0)}{\left(\sqrt{(-1+\nu+x_0)^2+{y_0}^2+{z_0}^2}\right)^3} + \frac{(-1+\nu)\ q_0\,(\nu+x_0)}{\left(\sqrt{(\nu+x_0)^2+{y_0}^2+{z_0}^2}\right)^3}\right)$$

$$Y_1 = y_0$$

$$Y_2 = v_0$$

$$Y_3 = \frac{1}{2}\left(-2u_0 + y_0 - \frac{\nu q_1 y_0}{\left(\sqrt{(-1+\nu+x)^2+{y_0}^2}\right)^3} + \frac{(-1+\nu)\ q_0\, y_0}{\left(\sqrt{(\nu+x_0)^2+{y_0}^2}\right)^3}\right)$$

$$Z_1 = z_0$$

$$Z_2 = w_0$$

$$Z_3 = \frac{1}{2} z_0\left(-\frac{\nu q_1}{\left(\sqrt{(-1+\nu+x_0)^2+{y_0}^2+{z_0}^2}\right)^3} + \frac{(-1+\nu)\ q_0}{\left(\sqrt{(\nu+x_0)^2+{y_0}^2+{z_0}^2}\right)^3}\right)$$

$$U_1 = u_0$$

$$U_2 = 2v_0 + x_0 - \frac{\nu q_1(-1+\nu+x_0)}{\left(\sqrt{(-1+\nu+x_0)^2+{y_0}^2+{z_0}^2}\right)^3} + \frac{(-1+\nu) q_0(\nu+x_0)^2}{\sqrt{((\nu+x_0)^2+{y_0}^2+{z_0}^2)^3}}$$

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$$
\begin{aligned}
U_3 = \frac{1}{2}\Bigg(& u_0 - \nu q_1 \left(-\frac{3\,(-1+\nu+x_0)\,(u_0(-1+\nu+x_0)+v_0 y_0+w_0 z_0)}{\left(\sqrt{(-1+\nu+x_0)^2+{y_0}^2+z_0^2}\right)^5} + \frac{u_0}{\left(\sqrt{(\nu+x_0)^2+{y_0}^2+{z_0}^2}\right)^3} \right) - \\
& (1-\nu)\,q_0 \left(\frac{3(\nu+x_0)(u_0(\nu+x_0)+v_0 y_0+w_0 z_0)}{\left(\sqrt{(\nu+x_0)^2+{y_0}^2+z_0^2}\right)^5} + \frac{u_0}{\left(\sqrt{(\nu+x_0)^2+{y_0}^2+z_0^2}\right)^3} \right) \\
& + 2\left(-2u_0 + y_0 - \frac{\nu q_1 y_0}{\left(\sqrt{(-1+\nu+x_0)^2+{y_0}^2+{z_0}^2}\right)^3} + \frac{(-1+\nu) q_1 y_0}{\left(\sqrt{(\nu+x_0)^2+{y_0}^2+{z_0}^2}\right)^3} \right)\Bigg)
\end{aligned}
$$

$$V_1 = v_0$$

$$V_2 = -2u_0 + y_0 - \frac{\nu q_1 y_0}{\left(\sqrt{(-1+\nu+x_0)^2+{y_0}^2+{z_0}^2}\right)^3} + \frac{(-1+\nu) q_0 y_0}{\left(\sqrt{(\nu+x_0)^2+{y_0}^2+{z_0}^2}\right)^3}$$

$$
\begin{aligned}
V_3 = \frac{1}{2}\Bigg(& v_0 - \nu q_1 \left(-\frac{3y_0(u_0(-1+\nu+x_0)+v_0 y_0+w_0 z_0)}{\left(\sqrt{(-1+\nu+x_0)^2+{y_0}^2+{z_0}^2}\right)^3} + \frac{v_0}{\left(\sqrt{(-1+\nu+x_0)^2+{y_0}^2+{z_0}^2}\right)^3} \right) - \\
& (1-\nu) q_0 \left(-\frac{3y_0(u_0(\nu+x_0)+v_0 y_0+w_0 z_0)}{\left(\sqrt{(\nu+x_0)^2+{y_0}^2+{z_0}^2}\right)^3} + \frac{v_0}{\left(\sqrt{(\nu+x_0)^2+{y_0}^2+{z_0}^2}\right)^3} \right) - \\
& 2\left(2v_0 + x_0 - \frac{\nu q_1(-1+\nu+x_0)+y_0}{\left(\sqrt{(-1+\nu+x_0)^2+{y_0}^2+{z_0}^2}\right)^3} \right) + \frac{(-1+v_0) q_0 (\nu+x_0)}{\left(\sqrt{(\nu+x_0)^2+{y_0}^2+{z_0}^2}\right)^3} \Bigg)
\end{aligned}
$$

REFERENCES

[1] Ian W. Roxburgh and Lynne M. Stockman, 1999. Power series solutions of the polytrope equation, Mon. Not. R. Astron. Soc. 303, 466-470

[2] Jagadish Singh, Ashetu Umar. 2012. On stability of triangular points in the elliptic R3BP under radiating and oblate primaries. Astron.J.143, 109

[3] Radzievsky,V.V. 1950. The restricted problem of three bodies taking into account of light pressure, Astron. Zh, Vol. 27,pp. 250.

[4] Radzievsky,V.V. 1953. Astron. Zh.30,3 , 265-273 (In Russian)

[5] Sharaf, M. A., Hassan, I. A. Ghoneim, R. and Alshaery. A. A, 2012. Symbolic Solution of the Three Dimensional Restricted Three-Body Problem. Al-Azhar Bull. Sci. Vol.23, No. 1 (June 2012), p. 17-28

[6] Sharaf, A. A. and Sharaf, M. A. 1994. Power Series Solutions of the Lane-Emden Equation, Bull. Fac. Sci., Cairo Univ., 62, 121

[7] Simmons, J.F.L. McDonald,A.J.C & Brown,J.C. 1985. The restricted three body problem with radiation. J. Celes. Mech., Vol.35, (Feb1985), p.145-187

[8] Szebehely 1967. Theory of Orbits-The Restricted Problem of Three Bodies, Academic Press New York, London.