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# (g*p)**- CLOSED SETS IN TOPOLOGICAL SPACES

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**Abstract:**

In this paper, we have introduced a new class of sets called $(g*p)^{**}$ -closed sets which is properly placed in between the class of closed sets and the class of (g*p)*-closed sets. As an application, we introduce three new spaces namely, ${}_gT^{**}{}_p$ , ${}_{\alpha g}T^{**}{}_p$ and ${}_{gs}T^{**}{}_p$ -spaces.

We have also introduced $(g*p)^{**}$-continuous and $(g*p)^{**}$-irresolute maps and their properties are investigated.

**Keywords:** $(g*p)^{**}$-closed sets, $(g*p)^{**}$-continuous maps, $(g*p)^{**}$-irresolute maps and, ${}_gT^{**}{}_p$ , ${}_{\alpha g}T^{**}{}_p$ and ${}_{gs}T^{**}{}_p$ -spaces..

## 1. INTRODUCTION

Levine [10] introduced the class of g-closed sets in 1970.Maki.et.al [12] defined $\alpha g$ -closed sets and $g\alpha$ -closed sets in 1994...Arya and Tour [3] defined $gs$ -closed sets in 1990.Dontchev [8] introduced $gsp$ -closed set in 1995. Veerakumar [23] introduced and studied the concepts of g*-pre closed sets and g*-pre continuity in topological spaces in 1991. Pauline Mary Helen and Anitha [20] introduced $(g*p)*$-closed sets in 2014.

The purpose of this paper is to introduce the concept of $(g*p)**$-closed sets, gT**p, $_{\alpha g}$ T**p and $_{gs}$ T**p spaces. Further we have introduced $(g*p)**$-continuous and $(g*p)**$-irresolute maps.

## 2. PRELIMINARIES

Throughout this paper $(X,\tau)$ and $(Y,\sigma)$ represents non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a $(X,\tau)$ space, cl (A) and int(A) denote the closure and the interior of A respectively.

The class of all closed subsets of a space of a space $(X,\tau)$ is denoted by C$(X,\tau)$ .

**Definition 2.1:** A subset A of a topological space $(X,\tau)$ is called

(1) a $pre-open$ [15] if $A\subseteq \text{int}\,(cl\,(A))$ and a $pre-closed$ set if $cl\,(\text{int}\,(A))\subseteq A$.

(2) a $semi-open$ [11] if $A\subseteq cl\,(\text{int}\,(A))$ and a $semi-closed$ if $\text{int}\,(cl\,(A))\subseteq A$.

(3) a $semi-preopen$ [1] if $A\subseteq cl\,(\text{int}\,(cl\,(A)))$ and a $semi-preclosed$ if $\text{int}\,(cl\,(\text{int}\,(A)))\subseteq A$.

(4) an $\alpha-open$ [18] if $A\subseteq \text{int}\,(cl\,(\text{int}\,(A)))$ and $\alpha-closed$ [16] if $cl\,(\text{int}\,(cl\,(A)))\subseteq A$.

**Definition 2.2:** A subset A of a topological space $(X,\tau)$ is called

**(1)** a generalized closed set(briefly g-closed) [10] if $cl\,(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is open in $(X,\tau)$.

**(2)** a generalized semi-closed set (briefly $gs-closed$ ) [3] if $scl\,(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is open in $(X,\tau)$.

**(3)** a semi-generalized closed set(briefly $sg-closed$ ) [5] if $scl\,(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is semi-open in $(X,\tau)$.

**(4)** an $\alpha-$generalized closed set(briefly $\alpha g-closed$ ) [12] if $\alpha cl\,(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is open in $(X,\tau)$.

**(5)** an generalized $\alpha-$ closed set(briefly $g\alpha-closed$ ) [13] if $\alpha cl\,(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is $\alpha$ - open in $(X,\tau)$.

**(6)** a $\alpha-doublestar\ closed$ set (briefly $\alpha**-closed$ ) [25] if $cl\,(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is $\alpha*$-open in $(X,\tau)$.

**(7)** a $\alpha-star\ closed$ set (briefly $\alpha*-closed$ ) [24] if $cl\,(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is $\alpha$ -open in $(X,\tau)$.

**(8)** a $wg-$closed set[17] if $cl\,(\text{int}\,(A))\subseteq U$ whenever $A\subseteq U$ and $U$ is open in $(X,\tau)$.

**(9)** a generalized semi –pre closed set(briefly $gsp-closed$ ) [8] if $spcl\,(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is open in $(X,\tau)$.

**(10)** a generalized semi –pre closed star set (briefly $(gsp)*-closed$ ) [21] if $cl\,(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is gsp-open in $(X,\tau)$.

**(11)** a generalized –pre closed set(briefly $gp-closed$ ) [14] if $pcl\,(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is open in $(X,\tau)$.

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**(12)** a $g*p$-pre closed set (briefly $g*p$-closed)[23] if $pcl(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is $g$ - open in $(X,\tau)$

**(13)** a $g*$-closed set (briefly $g*$-closed)[22] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is $g$ - open in $(X,\tau)$

**(14)** a strongly $g*$-closed set (briefly $strongly\ g*-closed$ -closed)[19] if $cl(\text{int}(A))\subseteq U$ whenever $A\subseteq U$ and $U$ is g-open in $(X,\tau)$.

**(15)** a $(g*p)*$- closed set (briefly $(g*p)*$-closed) [20] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is $g*p$- open in $(X,\tau)$.

**Definition 2.3:** A function $f:(X,\tau)\to(Y,\sigma)$ is called

1. $\alpha g$-continuous [9] if $f^{-1}(V)$ is an $\alpha g$-closed set of $(X,\tau)$ for every closed set V of $(Y,\sigma)$
2. $gs$ - continuous [7] if $f^{-1}(V)$ is a $gs-$ closed set of $(X,\tau)$ for every closed set V of $(Y,\sigma)$.
3. $gp$ - continuous [2] if $f^{-1}(V)$ is a $gp-$ closed set of $(X,\tau)$ for every closed set V of $(Y,\sigma)$
4. $wg$ - continuous [17] if $f^{-1}(V)$ is a $wg-$ closed set of $(X,\tau)$ for every closed set V of $(Y,\sigma)$.
5. $gsp$- continuous [8] if $f^{-1}(V)$ is a $gsp-$ closed set of $(X,\tau)$ for every closed set V of $(Y,\sigma)$.

**Definition: 2.4:** A topological space $(X,\tau)$ is said to be

1. a $T_{1/2}*$- space [24] if every $g*-$closed set in it is closed.
2. a $T_b$ - space [6] if every $gs-$closed set in it is closed
3. a ${}_{\alpha}T_b$ - space [4] if every $\alpha g-$closed set in it is closed.

## 3. BASIC PROPERTIES OF $(g*p)**$-CLOSED SETS

We now introduce the following definition.

**Definition 3.1: A** subset A of a topological space $(X,\tau)$ is called a $(g*p)**$-closed set, if $cl(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is $(g*p)*-open$ .

**Proposition 3.2:** Every closed set is $(g*p)**$-closed.

Proof follows from the definition but not conversely.

**Example 3.3:** Let $X=\{a,b,c\}$ and $\tau=\{\phi,X,\{c\},\{a,c\}\}$ and let $A=\{b,c\}$. Then *A* is not closed but $(g*p)**$ - closed.

**Proposition 3.4:** Every $(g*p)**$-closed set is (1) $\alpha g$-closed (2) $gs$-closed (3) $gp$-closed (4) $wg$ - closed (5) $gsp$-closed but not conversely.

**Proof:** Let $A$ be a $(g*p)**$-closed set. Let $A\subseteq U$ and $U$ be open .Then $U$ is $(g*p)*$–open.

Since $A$ is $(g*p)**$-closed,

(1) $\alpha cl(A)\subseteq cl(A)\subseteq U$ and hence $A$ is $\alpha g$–closed.

(2) $scl(A)\subseteq cl(A)\subseteq U$ and hence $A$ is $gs$–closed.

(3) $pcl(A)\subseteq cl(A)\subseteq U$ and hence $A$ is $gp$–closed.

(4) $cl\subseteq U$ and which implies $cl(\text{int}(A)\subseteq cl(A)\subseteq U)$ hence $A$ is $wg$–closed.

(5) $cl(A)\subseteq U$ and hence $spcl(A)\subseteq U$ therefore $A$ is $gsp$–closed.

**Example 3.5:** Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{c\}, \{a,c\}\}$ and let $A = \{a\}$. Then $A$ is $\alpha g$-closed, gs-closed, gp-closed, wg-closed, gsp-closed but it is not $(g*p)**$- closed.

**Proposition 3.6:** Every $\alpha**$- closed set is $(g*p)**$-closed set but not conversely.

**Proof:** Let $A$ be a $\alpha**$-closed set. Let $A \subseteq U$ and $U$ be $(g*p)*$- open .Then $U$ is $\alpha*$ –open.

Since $A$ is $\alpha**$-closed, $cl(A) \subseteq U$ therefore $A$ is $(g*p)**$-closed.

**Example 3.7:** Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{b\}, \{a,c\}\}$ and let $A = \{a,b\}$. Then $A$ is

$(g*p)**$-closed but it is not $\alpha**$- closed.

**Proposition 3.8:** Every $(gsp)*$- closed set is $(g*p)**$ -closed set but not conversely.

**Proof:** Let $A$ be a $(gsp)*$-closed set. Let $A \subseteq U$ and $U$ be $(g*p)*$-open .Then $U$ is gsp–open.

Since $A$ is $(gsp)*$-closed, $cl(A) \subseteq U$ therefore $A$ is $(g*p)**$-closed.

**Example 3.9:** Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{b\}, \{a,c\}\}$ and let $A = \{c\}$. Then $A$ is

$(g*p)**$- closed but it is not $(gsp)*$- closed.

**Proposition 3.10:** Every $g*$- closed set is $(g*p)**$ -closed set.

**Proof:** Let $A$ be a $g*$-closed set. Let $A \subseteq U$ and $U$ be $(g*p)*$-open .Then $U$ is g–open.

Since $A$ is $g*$-closed, $cl(A) \subseteq U$ therefore $A$ is $(g*p)**$-closed.

**Remark 3.11:** $g\alpha$ -closedness is independent of $(g*p)**$-closedness.

**Example 3.12:** Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{a\}\}$ and let $A = \{a,b\}$. Then $A$ is $(g*p)**$- closed but it is not $g\alpha$ - closed.

**Example 3.13:** Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{c\}, \{a,c\}\}$ and let $A = \{c\}$. Then $A$ is $g\alpha$ - closed but it is not
$(g*p)**$- closed.

**Remark 3.14:** $sg$ -closedness is independent of $(g*p)**$-closedness.

**Example 3.15:** Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{c\}, \{a,c\}\}$ and let $A = \{a\}$. Then $A$ is $sg$ - closed but it is not
$(g*p)**$- closed.

**Example 3.16:** Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{c\}\}$ and let $A = \{a,c\}$. Then $A$ is $(g*p)**$- closed but it is not $sg$ - closed.

**Remark 3.17:** strongly $g*$-closedness is independent of $(g*p)**$-closedness.

**Example 3.18:** Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{c\}, \{a,c\}\}$ and let $A = \{a\}$. Then $A$ is strongly $g*$-closed but it is not $(g*p)**$- closed.

**Example 3.19:** Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{a\}\}$ and let $A = \{a,b\}$. Then $A$ is $(g*p)**$- closed but it is not strongly
$g*$-closed.

**Remark 3.20:** $g*p$ -closedness is independent of $(g*p)**$-closedness.

**Example 3.21:** Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{c\}, \{a,c\}\}$ and let $A = \{a\}$. Then $A$ is

$g*p$ - closed but it is not $(g*p)**$- closed.

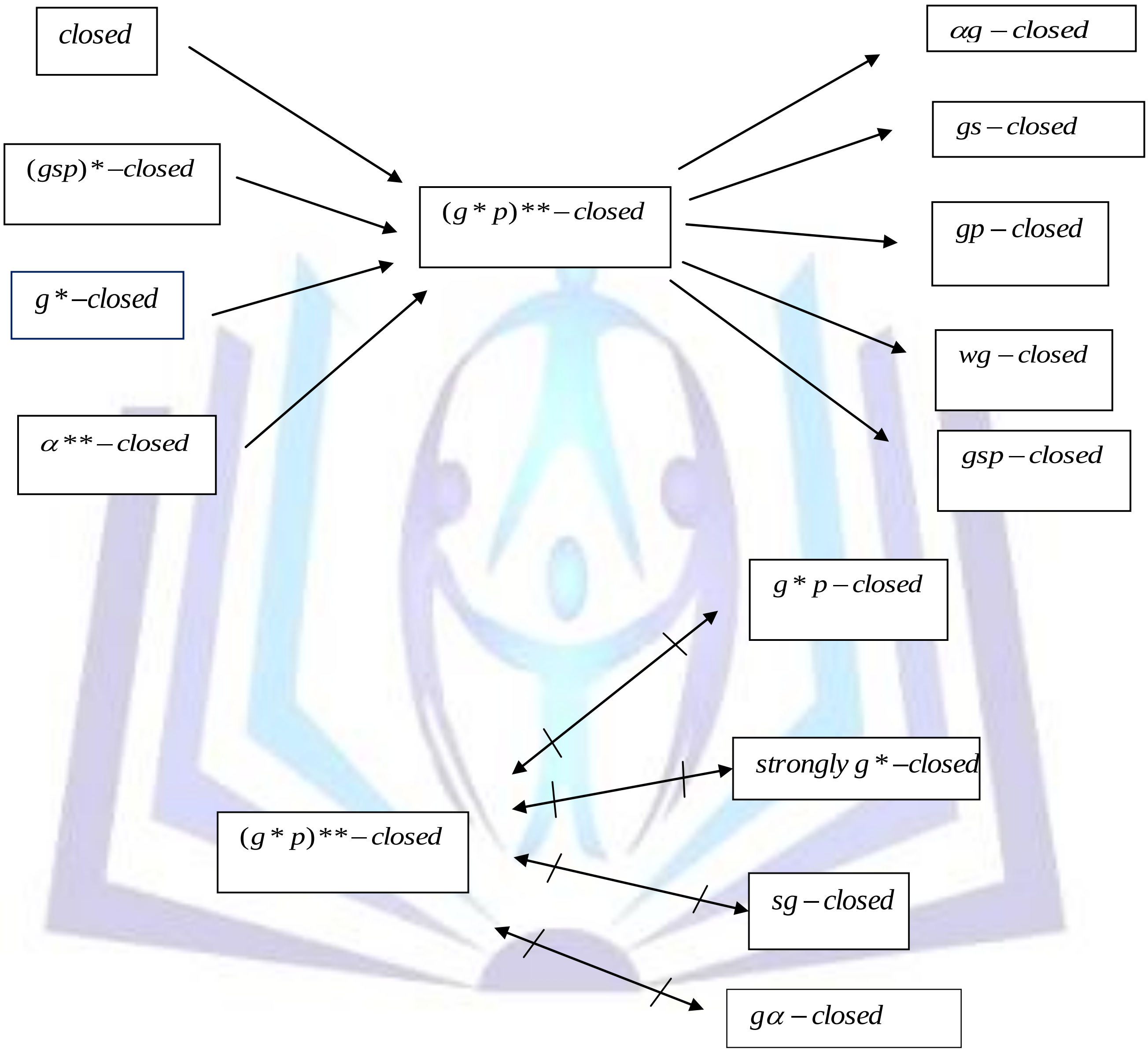
**Example 3.22:** Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{a\}\}$ and let $A = \{a,c\}$. Then $A$ is

$(g*p)**$- closed but it is not $g*p$ - closed.

**Proposition 3.23:** If $A$ and $B$ are $(g*p)**$-closed sets, then $A \cup B$ is also a $(g*p)**$ - closed set.

Proof follows from the fact that $cl\ (A \cup B) = cl\ (A)\ \cup\ cl\ (B)$.

The above results can be represented in the following figure.




Where A ⟶ B (resp A ⟷ B) represents A implies B and B need not imply A (resp. A and B independent)

## 4. $(g*p)**$-CONTINUOUS MAPS AND $(g*p)**$-IRRESOLUTE MAPS

We introduce the following definitions.

**Definition: 4.1:** A map $f:(X,\tau)\to(Y,\sigma)$ is called $(g*p)**$- continuous if the inverse image of every closed set in $(Y,\sigma)$ is $(g*p)**$- closed in $(X,\tau)$.

**Definition: 4.2:** A map $f:(X,\tau)\to(Y,\sigma)$ is said to be a $(g*p)**$- irresolute map if $f^{-1}(V)$ is a $(g*p)**$- closed set in $(X,\tau)$ for every $(g*p)**$- closed set V of $(Y,\sigma)$.

**Theorem 4.3:** Every continuous map is $(g*p)^{**}$- continuous.

**Proof:** Let $f:(X,\tau)\to(Y,\sigma)$ be a continuous map and let $F$ be a closed set in $(Y,\sigma)$. Then $f^{-1}(F)$ is closed in $(X,\tau)$. Since every closed set is $(g*p)^{**}$- closed, $f^{-1}(F)$ is $(g*p)^{**}$- closed. Then $f$ is $(g*p)^{**}$-continuous.

**Example 4.4:** Let $X=Y=\{a,b,c\}, \tau=\{\phi,X,\{c\},\{a,c\}\}$, $\sigma=\{\phi,Y,\{a\}\}$ $f:(X,\tau)\to(Y,\sigma)$ be the identity map. The inverse image of all closed sets of $(Y,\sigma)$ are $(g*p)^{**}$-closed in $(X,\tau)$. Therefore $f$ is $(g*p)^{**}$-continuous but not continuous.

**Theorem 4.5:** Every $(g*p)^{**}$ -continuous map is $\alpha g$ –continuous, $gs$-continuous, $gp$-continuous,

$wg$ -continuous and $gsp$ –continuous but not conversely.

**Proof:** Let $f:(X,\tau)\to(Y,\sigma)$ be a $(g*p)^{**}$- continuous map. Let $V$ be a closed set in $(Y,\sigma)$. Since $f$ is $(g*p)^{**}$-continuous, $f^{-1}(V)$ is $(g*p)^{**}$ - closed in $(X,\tau)$. Then $f^{-1}(V)$ is $\alpha g$ -closed,
$gs$-closed, $gp$-closed, $wg$-closed and $gsp$–closed set of $(X,\tau)$.

**Example 4.6:** Let $X=Y=\{a,b,c\}, \tau=\{\phi,X,\{c\},\{a,c\}\}$, $\sigma=\{\phi,Y,\{b,c\}\}$ $f:(X,\tau)\to(Y,\sigma)$ be the identity map. Then $f^{-1}(\{a\})=\{a\}$ is not $(g*p)^{**}$ -closed in $(X,\tau)$. But $\{a\}$ is, $\alpha g$ -closed set, gs-closed set. Then $f$ is $\alpha g$ - continuous,
$gs$ - continuous but not $(g*p)^{**}$-continuous.

**Example 4.7:** Let $X=Y=\{a,b,c\}, \tau=\{\phi,X,\{c\},\{a,c\}\}$, $\sigma=\{\phi,Y,\{a,c\}\}$. is defined as
$f(a)=b, f(b)=c, f(c)=a$. Then $f^{-1}(\{b\})=\{a\}$ is $gp$-closed but not $(g*p)^{**}$ -closed. Then $f$ is $gp$ - continuous but not
$(g*p)^{**}$-continuous.

**Example 4.8:** Let $X=Y=\{a,b,c\}, \tau=\{\phi,X,\{c\},\{a,c\}\}$, $\sigma=\{\phi,Y,\{a,b\}\}$. $f:(X,\tau)\to(Y,\sigma)$ is defined as
$f(a)=c, f(b)=a, f(c)=b$. Then $f^{-1}(\{c\})=\{a\}$ is $wg$ -closed but not $(g*p)^{**}$-closed in $(X,\tau)$. Hence $f$ is
$wg$ - continuous but not $(g*p)^{**}$ -continuous.

**Example 4.9:** Let $X=Y=\{a,b,c\}, \tau=\{\phi,X,\{c\},\{a,c\}\}$, $\sigma=\{\phi,Y,\{b,c\}\}$.

$f:(X,\tau)\to(Y,\sigma)$ is defined as $f(a)=a, f(b)=c, f(c)=b$. Then $f^{-1}(\{a\})=\{a\}$ is not

$(g*p)^{**}$-closed in $(X,\tau)$, but it is $gsp$-closed .Hence $f$ is $gsp$ - continuous but not
$(g*p)^{**}$ -continuous.

**Theorem 4.10:** Every $(g*p)^{**}$- irresolute map is $(g*p)^{**}$-continuous**.**

Proof follows from the definitions of $(g*p)^{**}$- irresolute map and $(g*p)^{**}$-continuous.

**Theorem 4.11:** Every $(g*p)^{**}$ -irresolute map is $\alpha g$ - continuous, $gs$ -continuous, $gp$ - continuous,

$wg$ - continuous and $gsp$ - continuous .

Proof follows from theorems (4.4) and (4.11).

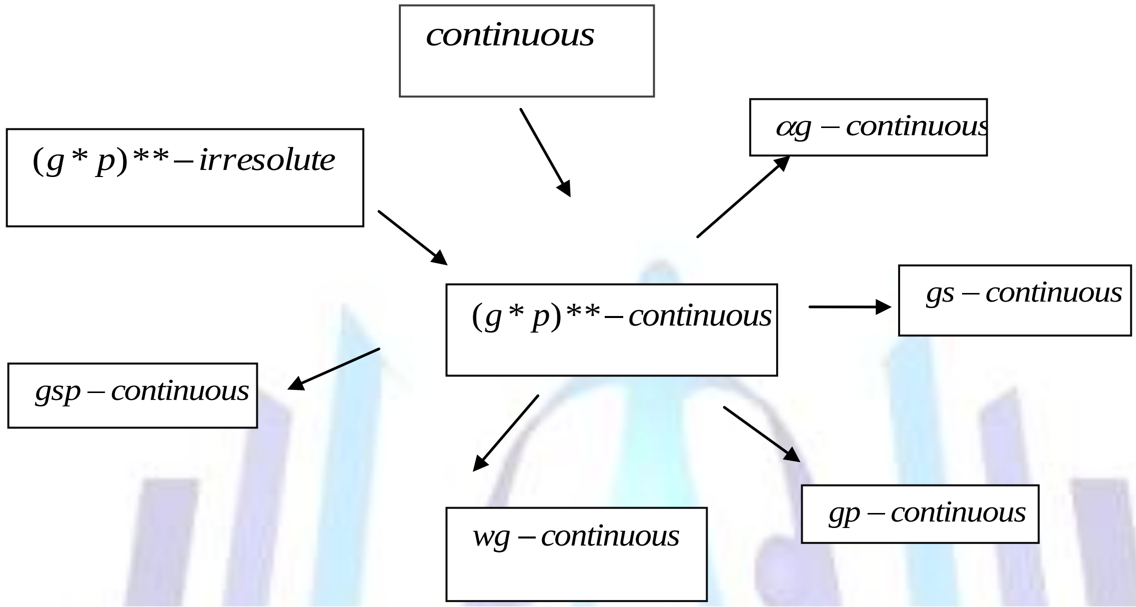
The converse of the above theorem need not be true in general as seen in the following examples.

**Example 4.12:** Let $X=Y=\{a,b,c\}$ $\tau=\{\phi,X,\{c\},\{a,c\}\}$ and $\sigma=\{\phi,Y,\{c\}\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be the identity map. $\phi$, $Y$, $\{a,b\}$ are closed sets of Y. $f^{-1}(\{a,b\})=\{a,b\}$ is
gs-closed, gp-closed, wg-closed,gsp-closed. Hence $f$ is $gs$ -continuous,
$gp$ - continuous, $wg$ - continuous and $gsp$ - continuous. $(g*p)^{**}$-closed sets of
Y are $\phi, Y, \{a\}, \{b\}, \{a,b\}$ $\{a,c\}, \{b,c\}$. $f^{-1}(\{a\})=\{a\}$ is not $(g*p)^{**}$-closed in $(X,\tau)$.
Hence $f$ is not a $(g*p)^{**}$-irresolute.

**Example 4.13:** Let $X=Y=\{a,b,c\}$ $\tau=\{\phi,X,\{a\}\}$ and $\sigma=\{\phi,Y,\{c\}\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be the identity map. $\phi$, $Y$, $\{a,b\}$ are closed sets of Y. $f^{-1}(\{a,b\})=\{a,b\}$ is

$\alpha g$ -closed. Hence $f$ is $\alpha g$ -continuous. $(g*p)**$-closed sets of Y are $\phi, Y, \{a\}, \{b\}, \{a,b\} \{a,c\}, \{b,c\}$. $f^{-1}(\{a\})=\{a\}$ is not $(g*p)**$-closed in $(X,\tau)$. Hence $f$ is $\alpha g$ -continuous but not a $(g*p)**$-irresolute.

The above results can be represented in the following figure.




where A ⟶ B represents A implies B and B need not imply A.

## 5. APPLICATION OF (g*p)**-CLOSED SETS

We introduce the following definitions.

**Definition: 5.1:** A space $(X,\tau)$ is called a $_gT**_p$ -space if every set $(g*p)**$-closed set is closed.

**Definition: 5.2:** A space $(X,\tau)$ is called a $_{\alpha g}T**_p$ -space if every $\alpha g$ - closed set is $(g*p)**$-closed.

**Definition: 5.3:** A space $(X,\tau)$ is called a $_{gs}T**_p$ -space if every $gs$ - closed set is $(g*p)**$-closed.

**Theorem 5.4:** Every $_gT**_p$ -space is a $T_{1/2}*$ -space .

**Proof:** Let $(X,\tau)$ be a $_gT**_p$ -space. Let $A$ be a $g*$-closed set. Since every $g*$-closed set is $(g*p)**$ -closed, A is $(g*p)**$-closed. Since $(X,\tau)$ is $_gT**_p$ -space, A is closed. $\therefore$ $(X,\tau)$ is a $T_{1/2}*$ -space .

**Theorem 5.5:** Every $T_b$ space is a $_gT**_p$ -space but not conversely.

Proof follows from the definitions of $T_b$ space and $_gT**_p$ -space.

**Example 5.6:** Let $X=\{a,b,c\}$ and $\tau=\{\phi, X, \{c\}\}$ . Here $(g*p)**$-closed sets are $\phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}$ and the $gs$ - closed sets are $\phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}$. Since every $(g*p)**$-closed set is closed, the space $(X,\tau)$ is a $_gT**_p$ - space. A = {a,c} is $gs$ -closed but not closed. Therefore the space $(X,\tau)$ is not a $T_b$ - space.

**Theorem 5.7:** Every ${}_{\alpha}T_b$ -space is a ${}_{g}T^{**}{}_{p}$ –space

Proof follows from the definitions of ${}_{\alpha}T_b$ -space and ${}_{g}T^{**}{}_{p}$ –space. The converse is not true.

**Example 5.8:** Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{c\}, \{a,c\}\}$ $(g*p)^{**}$-closed sets are $\phi, X, \{b\}, \{a,b\}, \{b,c\}$ and $\alpha g$ - closed sets are $\phi, X, \{a\}, \{b\}, \{a,b\}, \{b,c\}$. Since every $(g*p)^{**}$-closed set is $\alpha g$ -closed, the space $(X,\tau)$ is a ${}_{g}T^{**}{}_{p}$ - space. A = {a} is $\alpha g$ -closed but not closed. Therefore the space $(X,\tau)$ is not a ${}_{\alpha}T_b$ - space.

**Theorem 5.9:** Every ${}_{g}T^{**}{}_{p}$ - space is a $T_{\alpha}^{**}$ - space.

Proof follows from the definitions of ${}_{g}T^{**}{}_{p}$ - space and $T_{\alpha}^{**}$ - space. The converse is not true.

**Example 5.10:** Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{b\}, \{a,c\}\}$ $(g*p)^{**}$-closed sets are all the subsets of X and $\alpha g$ - closed sets are $\phi, X, \{b\}, \{a,c\}$. Since every $\alpha^{**}$-closed set is closed, the space $(X,\tau)$ is a $T_{\alpha}^{**}$ - space. A = {b,c} is $(g*p)^{**}$-closed but not closed. Therefore the space $(X,\tau)$ is not a ${}_{g}T^{**}{}_{p}$ - space.

**Theorem 5.11:** Every ${}_{\alpha}T_b$ -space is a ${}_{\alpha g}T^{**}{}_{p}$ - space.

**Proof:** Let $(X,\tau)$ be a ${}_{\alpha}T_b$ – space. Let $A$ be $\alpha g$ -closed. Then A is $\alpha g$ -closed. Since the space is ${}_{\alpha}T_b$ -space, $A$ is closed and hence $A$ is $(g*p)^{**}$–closed. Therefore the space $(X,\tau)$ is a ${}_{\alpha g}T^{**}{}_{p}$ –space.

**Example 5.12:** Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{a\}\}$ $(g*p)^{**}$-closed sets are $\phi, X, \{b\}, \{c\}, \{a,b\}, \{a,c\}\{b,c\}$ and $\alpha g$ - closed sets are $\phi, X, \{b\}, \{c\}, \{a,b\}, \{a,c\}\{b,c\}$. A = {b} is $(g*p)^{**}$–closed and also $\alpha g$ -closed. Therefore the space $(X,\tau)$ is a ${}_{\alpha g}T^{**}{}_{p}$ - space. A = {c} is $\alpha g$ -closed but not closed. Therefore the space $(X,\tau)$ is not a ${}_{\alpha}T_b$ - space.

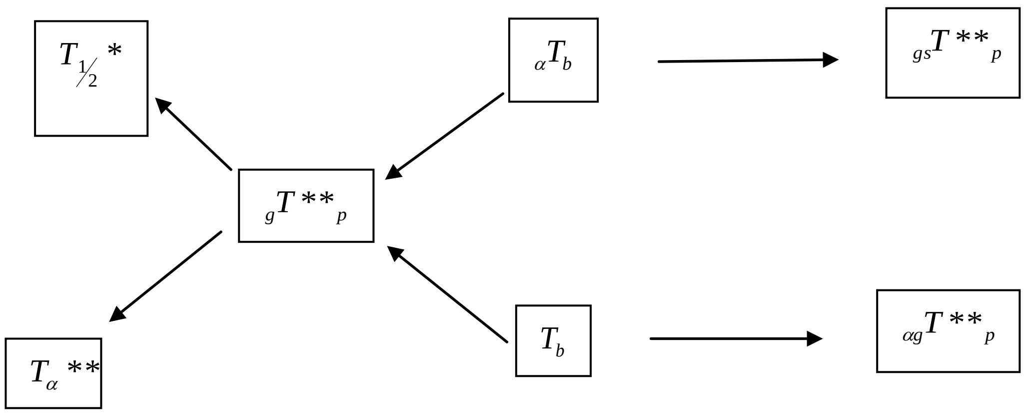
**Theorem 5.13:** Every $T_b$ - space is a ${}_{gs}T^{**}{}_{p}$ - space but not conversely.

Proof follows from the definitions of $T_b$ - space and ${}_{gs}T^{**}{}_{p}$ - space.

**Example 5.14:** Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{c\}, \{a,c\}\}$. Here $(g*p)^{**}$-closed sets are $\phi, X, \{b,c\}, \{a,b\}, \{b\}$ , gs- closed sets are $\phi, X, \{a\}, \{b\}, \{a,b\}, \{b,c\}$. A = {b,c} is $(g*p)^{**}$–closed and also $gs$ -closed. Therefore the space $(X,\tau)$ is a ${}_{gs}T^{**}{}_{p}$ - space. A = {a} is $gs$ -closed but not closed.

Therefore the space $(X,\tau)$ is not a $T_b$ - space.

The above results can be represented in the following figure




where A ⟶ B represents A implies B and B need not imply A.

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