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REGIONAL GRADIENT STRATEGIC SENSORS CHARACTERIZATIONS

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ABSTRACT

In the present paper, the characterizations of regional gradient strategic sensors notions have been given for different cases of regional gradient observability. The results obtained are applied to two dimensional linear infinite distributed system in Hilbert space where the dynamic is governed by strongly continuous semi-group. Various cases of regional strategic sensors are considered and analyzed in connection with regional gradient strategic sensors concepts. Also, we show that there is a various sensors which are not gradient strategic in usual sense for the considered systems, but may be regionally gradient strategic of this system.

Keywords: ω_G-strategic sensors; exactly ω_G-observability; weakly ω_G-observability; diffusion systems.

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1. NTRODUCTION

The analysis of distributed parameter systems refers to a set of concepts such as controllability, observability, detectability [13-14, 18]. The study of these concepts can be made via actuators and sensors structures see [14-17], these concepts give an important link between a system and it's environment [15-18], so that the concepts of actuators and strategic sensors for a class of distributed parameter systems are introduced in order that controllability and observability can be achieved [14-18]. The regional analysis is one of the most important notion of system theory [20-22], it consist to reconstruction the state observation on a sub-region ω of spatial domain Ω in finite time [19-23, 25-28], this concepts introduced and developed by El-Jai *et al.* An important extended to the asymptotic case for infinite time by El-Jai and Al-Saphory in several works [1-7]. The study of regional gradient observability for a diffusion system has been given in [27-28] where one is interested in knowledge of the state gradient only in a critical sub-region of the system domain without the knowledge of the state itself. Moreover, the applications are motivated by many real world see [10-12, 22]. Commercial buildings are responsible for a significant fraction of the energy consumption and greenhouse gas emissions in the U.S. and worldwide. Consequently, the design, optimization and control of energy efficient buildings can have a tremendous impact on energy cost and greenhouse gas emission. Mathematically, building models are complex, multi-scale, multi-physics, highly uncertain dynamical systems with wide varieties of disturbances [10].

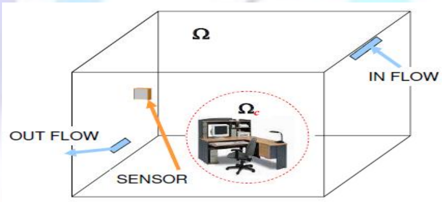

Fig. 1: Room control model with sensor, in flow and out flow

In this paper we use a model problem to illustrate that distributed parameter control based on PDEs, combined with high performance computing can be used to provide practical insight into important issues such as optimal sensor/actuator placement (may be best or strategic sensors/ actuators) and optimal supervisory building control. In order to illustrate some of the ideas, we consider the problem illustrated by a single room shown in (Figure 1). This model one can reformulated [11] as spatial case of more general model of distributed parameter systems and represented in the next section (see Figure 2). In addition, the characterization of regional strategic sensors have been given for various types of regional observability in [7].

The purpose of this paper is to extended these results in [7] to the case of regional gradient sensors. Thus, we give a characterization of regional gradient strategic sensors for different cases of regional gradient observation. Therefore, we study and analyze the relationship between the regional gradient strategic sensors and the regional exactly gradient observability. So, the outline of this paper is organized as follows:

Section 2 is present problem statement and basic definitions with characterization of the regional gradient observability. The mathematical concepts of regional gradient strategic sensors in a various situations are studied and developed in section 3. In the last section we gives an application about different sensors locations.

2. REIONAL GRADIENT OBSERVABILITY

In this section, we are interested to recall the notion of regional gradient observability and give original results related to particular systems as in [27-28].

2.1 Problem Statement

Let Ω be a regular bounded open subset of R^n, with a smooth boundary $\partial\Omega$ and ω be a non-empty given sub-region of Ω. Let $[0,T], T > 0$ be a time of measurement interval. We denoted $Q = \Omega \times]0,T[$ and $\Sigma = \partial\Omega \times]0,\mathrm{T}[$. Consider the following distributed parabolic defined by

$$\begin{cases} \frac{\partial x}{\partial t}(\xi,t) = Ax(\xi,t) + Bu(t) & \text{in } Q \\ x(\xi,0) = x_0(\xi) & \text{in } \Omega \\ x(\eta,t) = 0 & \text{in } \Sigma \end{cases} \tag{1}$$

with the measurements given by the output function

$$y(\cdot,t) = Cx(\,,t) \tag{2}$$

We have

$A = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_j}(a_{ij}\frac{\partial}{\partial x_j})$, with $a_{ij} \in \mathcal{D}(\bar{Q})$.

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Suppose that $-A$ is elliptic, *i.e.*, there exists $\alpha > 0$ such that

$$\sum_{i,j=1}^{n} a_{ij}\,\xi_i\xi_j \geq \alpha \sum_{j=1}^{n}\left|\xi_j\right|^2, \text{ almost everywhere (a.e) on } Q, \forall \xi = (\xi_1, \dots, \xi_n) \in R^n.$$

This operator is a second order linear differential operator, which generator a strongly continuous semi-group $(S_A(t))_{t\geq 0}$ on the Hilbert space $X = H^1(\Omega)$ and is self-adjoint with compact resolvent. The operator $B \in L(R^p, X)$ and $C \in L(X, R^q)$, depend on the structure of actuators and sensors [18]. The space X, U and $\mathcal{O}$ be separable Hilbert spaces where X is the state space, $U = L^2(0, T, R^p)$ is the control space and $\mathcal{O} = L^2(0, T, R^q)$ is the observation space where p and q are the numbers of actuators and sensors (see Figure 2).

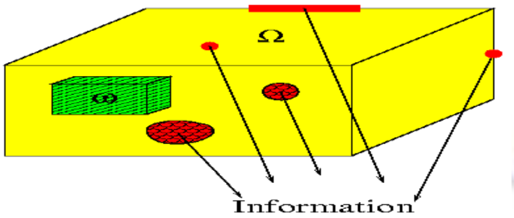

Fig. 2: The domain of Ω, the sub-region ω, various sensors locations

Under the given assumption, the system (1) has a unique solution [24]:

$$x(\xi, t) = S_A(t)x_0(\xi) + \int_0^t S_A(t-\tau)Bu(\tau)d\tau \tag{3}$$

The problem is to provide sufficient conditions to ensure that, how to extend the results in [7], so that to give a sufficient conditions of regional gradient strategic sensors which achieve the observability in sub-region ω using various regional gradient sensors.

2.2 Definitions And Characterizations

The regional gradient observability concept and reconstruction gradient state have been introduced by Zerrik E. *et al.* as in ref.s [27-28] and recently this concept is developed to the regional asymptotic case by Al-Saphory R [1-7]. Consider the autonomous system to (1) given by

$$\begin{cases} \frac{\partial x}{\partial t}(\xi, t) = Ax(\xi, t) & \text{in } Q \\ x(\xi, 0) = x_0(\xi) & \text{in } \Omega \\ x(\eta, t) = 0 & \text{in } \Sigma \end{cases} \tag{4}$$

The solution of (4) is given by the following form,

$$x(\xi\,, t) = S_A(t)x_0(\xi), \quad \forall t \in [0, T] \tag{5}$$

The measurements are obtained through the output function

$$y(.\,, t) = Cx(\xi\,, t)$$

- We first recall a sensors is defined by any couple (*D, f*), where *D* is spatial support represented by a nonempty part of $\bar{\Omega}$ and *f* represents the distribution of the sensing measurements on *D*.

Depending on the nature of *D* and *f*, we could have various type of sensors. A sensor may be pointwise if *D* = {*b*} with $b \in \bar{\Omega}$ and $f = \delta(.-b)$, where δ is the Dirac mass concentrated at *b*. In this case the operator *C* is unbounded and the output function (2) can be written in the form [13-14]

$$y(t) = x(b, t)$$

- It may be zonal when $D \subset \bar{\Omega}$ and $f \in L^2(D)$. The output function (2) can be written in the form

$$y(t) = \int_D x(\xi, t)f(\xi)$$

- Now, we define the operator

$$K: x \in X \to Kx = CS_A(.)\, x \in \mathcal{O} \tag{6}$$

Thus, we get that

$$y(\cdot\,, t) = K(t)x(\cdot\,,0)$$

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where K is bounded linear operator [8].

- We note that $K^*: \mathcal{O} \to X$ is the adjoint operator of K defined by

$$K^* y^* = \int_0^t S_A^*(s) C^* y^*(s) ds \tag{7}$$

- Consider the operator

$$\nabla: \begin{cases} H^1(\Omega) & \to (L^2(\Omega))^n \\ x \to \nabla x = (\frac{\partial x}{\partial \xi_1}, \dots, \frac{\partial x}{\partial \xi_n}) \end{cases} \tag{8}$$

It's adjoint ∇^* is given by

$$\nabla^*: \begin{cases} (L^2(\Omega))^n \to H^1(\Omega) \\ x \quad\quad \to \nabla^* x = v \end{cases} \tag{9}$$

where v is a solution of the Dirichlet problem

$$\begin{cases} \Delta v = -\mathrm{div}(x) & \text{in } \Omega \\ v = 0 & \text{in } \partial\Omega \end{cases}$$

- For a nonempty subset ω of Ω, we consider the operators

$$\chi_\omega: \begin{cases} (L^2(\Omega))^n \to (L^2(\omega))^n \\ x \quad \to \quad \chi_\omega x = x\,|_\omega \end{cases} \tag{10}$$

and

$$\tilde{\chi}_\omega: \begin{cases} L^2(\Omega) \to L^2(\omega) \\ x \to \tilde{\chi}_\omega x = x\,|_\omega \end{cases} \tag{11}$$

where $x\,|_\omega$ is the restriction of x to ω [9].

- Their adjoints are respectively denoted by χ_ω^* and $\tilde{\chi}_\omega^*$ are defined by

$$\chi_\omega^*: \begin{cases} (L^2(\omega))^n \quad \to \quad (L^2(\Omega))^n \\ x \to \chi_\omega^* x = \begin{cases} x\,|_\omega & \text{in } \omega \\ 0 & \text{in } \Omega \setminus \omega \end{cases} \end{cases} \tag{12}$$

and

$$\tilde{\chi}_\omega^*: \begin{cases} L^2(\omega) \to L^2(\Omega) \\ x \to \tilde{\chi}_\omega^* x = \begin{cases} x\,|_\omega & \text{in } \omega \\ 0 & \text{in } \Omega \setminus \omega \end{cases} \end{cases} \tag{13}$$

- The idea of gradient observability is based on the existence of an operator $H: \mathcal{O} \to (L^2(\omega))^n$ such that $Hy = \nabla x_0$. This is a natural extension of the observability concept [8]. Then we defined the operator $H = \chi_\omega \nabla K^*$ from $\mathcal{O}$ into $(L^2(\omega))^n$ as in [27]. Now, let us denoted the system (4) together with the output (2) by (4)-(2).

Definition 2.1: The system (4)-(2) is said to be regionally exactly observable on a sub-region ω (*exactly* ω *- observable*), if

$$Im\tilde{\chi}_\omega K^* = L^2(\omega)$$

Definition 2.2: The system (4)-(2) is said to be regionally weakly observable on ω (*weakly* ω*- observable*), if

$$\overline{Im\tilde{\chi}_\omega K^*(.)} = L^2(\omega)$$

Definition 2.3: The system (4)-(2) is said to be regionally exactly gradient observable on ω (*exactly* ω_G *- observable*), if

$$Im\chi_\omega \nabla K^* = (L^2(\omega))^n$$

Definition 2.4: The system (4)-(2) is said to be regionally weakly gradient observable on ω (*weakly* ω_G*- observable*), if

$$\overline{Im\chi_\omega \nabla K^*(.)} = (L^2(\omega))^n$$

We see that if a system is weakly ω_G– observable then there is one to one relationship between the output and the initial gradient, viz., if y is given and x_0 satisfies $y = CS(.)x_0$, then ∇x_0 is a unique.

Remark 2.5: We can deduced that, the definition 2.4 is equivalent to say that the system (4)-(2) is weakly ω_G- observable, if

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$$ker\ K(t)\nabla^*\chi_\omega^* = \{0\}$$

Then, the following characterization can extend to the regional gradient case as in ref. [26].

Proposition 2.6: The system (4)-(2) is exactly ω_G-observable if and only if there exist $c > 0$ such that for all $x^* \in (L^2(\omega))^n$, such that,

$$\| \chi_\omega x^* \|_{(L^2(\omega))^n} \le c \|K(t)\nabla^*\chi_\omega^* x^*\|_{\mathcal{O}} \tag{14}$$

Proof: The proof of this property is deduced from the usual results on observability considering $\chi_\omega \nabla K^*$ [13]. Let V, W and X be a reflexive Banach space and let $F \in L(V, X), G \in L(W, X)$, then the following conditions are equivalent

1. $Im\ F \subset Im\ G$.
2. There exist $c > 0$ such that $\|F^* x^*\|_{V^*} \le c\|G^* x^*\|_{W^*}$ $\quad \forall x^* \in G^*$

Now, by applying the above result we obtain the equivalent condition for exactly ω_G-observable as:

Let $V = X = (L^2(\omega))^n$, $W = \mathcal{O}$, $F = Id_{(L^2(\omega))^n}$ and $G = \chi_\omega \nabla K^*$.

Now, since the system is exactly ω_G-observable we have $Im\ F \subset Im\ G$, which is equivalent to that fact there exist $c > 0$, such that

$$\|F^* x^*\|_{L^2(\omega))^n} \le c\|G^* x^*\|_{w^*} \qquad \forall x^* \in G^*. \square$$

Remark 2.7: We have:

(1) The regional state reconstruction will be more precise than the whole domain if we estimate the state in the whole the domain.

(2) From (14) there exists a reconstruction error operator that gives the estimation $\tilde{x}_0$ of the initial state x_0 in ω, and then, If we put $e = x_0 - \tilde{x}_0$, we have

$$\|e\|_{(L^2(\omega))^n} \le \|e\|_{(L^2(\Omega))^n}$$

$$\Rightarrow \| x_0 - \tilde{x}_0\|_{(L^2(\omega))^n} \le \|x_0 - \tilde{x}_0\|_{(L^2(\Omega))^n}$$

Where x_0 is the exact state of the system and $\tilde{x}_0$ is the estimated state of the system.

Proposition 2.8: If the system is exactly ω-observable then it is exactly ω_G-observable.

Proof: Since the system is exactly ω-observable there exist $\gamma > 0$ such that $\forall x_0 \in L^2(\omega)$, we have

$$\|x_0\|_{L^2(\omega)} \le \gamma \|K\tilde{\chi}_\omega^* x_0\|_{L^2(0,T,\mathcal{O})} \qquad , \ \forall \gamma > 0$$

since $(L^2(\omega))^n \subset L^2(\omega)$, then

$$\|\nabla x_0\|_{(L^2(\Omega))^n} = \|x_0\|_{(L^2(\omega))^n} \le \|x_0\|_{L^2(\omega)}, \forall\ x_0 \in L^2(\omega), \text{ where}$$

$$L^2(\omega) = \{x_0 : \int_\omega |x_0|^2 < \infty\}, \tag{15}$$

and then

$$(L^2(\omega))^n = \left\{\nabla x_0 = g_i : \int_\omega |g_i|^2 < \infty,\ g_i = \frac{\partial x_0}{\partial \xi_i}\ \forall i = 1,2,\dots\right\}.$$

To prove $\|x_0\|_{(L^2(\omega))^n} \le c\|K\nabla^*\chi_\omega^* x_0\|_{L^2(0,T,\mathcal{O})}$ then from (15) and since a system exactly ω-observable, then there exist $\gamma > 0$ and $c > 0$ such that $\gamma = \frac{1}{c}$. By choosing

$$c = \frac{\|K\tilde{\chi}_\omega^* x_0\|_{\mathcal{O}}}{\|K\nabla^*\chi_\omega^* x_0\|_{\mathcal{O}}} \tag{16}$$

Then, we can get

$$\|x_0\|_{(L^2(\omega))^n} \le \|x_0\|_{L^2(\omega)} \le \gamma\|K\tilde{\chi}_\omega^* x_0\|_{\mathcal{O}} \tag{17}$$

And by substituting (16) in (17), we obtain

$$\|x_0\|_{(L^2(\omega))^n} \le \|K\nabla^*\chi_\omega^* x_0\|_{\mathcal{O}}$$

Therefore this system is exactly ω_G-observable with $\gamma = 1$.$\square$

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Remark 2.9: From the above proposition we can get the following result:
If the system is exactly ω-observable then it is exactly G-observable in ω^1 for all $\omega^1 \subset \omega$ (*exactly ω_G^1-observable*).

3. REGIONAL GRADIENT STRATEGIC SENSORS

The purpose of this section is to give the characterization for sensors in order that the system (4)-(2) which is observable in ω.

3.1 ω_G-Strategic Sensors

Definition 3.1: A sensor (D, f) is regional gradient strategic on ω *(ω_G-strategic)* if the observed system is weakly ω_G-observable.

Definition 3.2: A sensor $(D_i, f_i)_{1\le i\le q}$is ω_G-strategic if there exist at least one sensor (D_1, f_1) which ω_G-strategic.

We can deduce that the following result:

Corollary 3.3: A sensor is ω_G-strategic if the observed system is exactly ω_G-observable.

Proof: Let the system exactly ω_G-observable, then , we have

$$Im\, \chi_\omega \nabla K^* = (L^2(\,\omega))^n$$

From decomposition sub-space of direct sum in Hilbert space, we represent $(L^2(\,\Omega))^n$ by the unique form [13]

$$ker\chi_\omega + Im\chi_\omega^* \chi_\omega \nabla K^* = (L^2(\,\Omega))^n$$

We obtain

$$ker\; K(t)\nabla^* \chi_\omega^* = \{0\}$$

This is equivalent to [9]

$$\overline{Im\, \chi_\omega \nabla K^*(.)} = (L^2(\,\omega))^n$$

Finally, we can deduce this system is weakly ω_G- observable and therefore this sensor is ω_G-strategic.□

Corollary 3.4: A sensor is ω_G-strategic if and only if the operator $N_\omega = HH^*$ is positive definite.

Proof: Since a sensor is ω_G-strategic this mean that the system is weakly ω_G- observable,

let $x^* \in (L^2(\omega))^n$such that

$$< N_\omega x^*, x^* >_{(L^2(\omega))^n} = 0 \quad \text{then } \|H^* x^*\|_O = 0$$

and therefore $x^* \in kerH^*$, thus, $x^* = 0$, i.e., N_ω is positive definite.

Conversely, let $x^* \in (L^2(\omega))^n$ such that

$$H^* x^* = 0, \text{ then } < H^* x^*, H^* x^* >_O = 0$$

and thus,

$$< N_\omega x^*, x^* >_{(L^2(\omega))^n} = 0$$

Hence $x^* = 0$ thus the system is weakly ω_G- observable and therefore a sensor is ω_G-strategic.□

Remark 3.5: From the previous results, we obtain that:

(1) If the system is exactly ω_G-observable then the system is weakly ω_G-observable and therefore this sensor is ω_G-strategic.

(2) A sensor which is regional gradient strategic sensor in ω^1 (ω_G^1-*strategic*) for a system where $\omega^1 \subset \Omega$, is regional gradient strategic sensor in ω^2 (ω_G^2-strategic) for any $\omega^2 \subset \omega^1$.

(3) The concept of exact ω_G-observability is more restrictive than weak ω_G-observability.

Now, assume that the operator A has a complete set of eigenfunction in $H^1(\Omega)$, denoted by φ_{nj}, which is orthonormal in $L^2(\omega)$ and the associated with the eigenvalue λ_n of multiplicities r_n, then the concept of regional gradient strategic on ω can be characterized by the following result:

Theorem 3.6: Assume that $\sup r_n = r < \infty$, then the suite of sensors $(D_i, f_i)_{1\le i\le q}$, ω_G-strategic if

(1) $q \geq r$

(2) $rank\ G_n = r_n \quad \forall n \geq 1, where$

Where $G_n = (G_n)_{ij}$ for $1 \leq i \leq q, 1 \leq j \leq r_n$, and

$$(G_n)_{ij} = \begin{cases} \sum_{k=1}^{m} \frac{\partial \varphi_{nj}}{\partial \xi_k}(b_i) & \text{in the pointwise case} \\ \sum_{k=1}^{n} < \frac{\partial \varphi_{nj}}{\partial \xi_k}, f_i >_{L^2(D_i)} & \text{in the zonal case} \end{cases}$$

Proof: We will discussed the case where the sensors are of pointwise type and located inside the domain Ω. The suite of sensors $(b_i, \delta_{b_i})_{1 \leq i \leq q}$ is ω_G-strategicif and only if

$$\left\{x^* \in (L^2(\omega))^n \middle| < Hy\,, x^* >_{(L^2(\omega))^n} = 0, \forall y \in \mathcal{O}\right\} \Longrightarrow x^* = 0.$$

Suppose that the suite of sensors $(b_i, \delta_{b_i})_{1 \leq i \leq q}$ is ω_G-strategicbut for a certain $n \in N, rank\ G_n \neq r_n$, then there exists a vector $x_n = (x_{n1}, x_{n2}, \dots, x_{nr_n})^{tr} \neq 0$, such that $G_n x_n = 0$. So, we can construct a nonzero $x_0 \in L^2(\omega)$ considering $< x_0, \varphi_{pj} >_{L^2(\omega)} = 0\ if\ p \neq n$ and

$$< x_0, \varphi_{nj} >_{L^2(\omega)} = x_{nj}\ , 1 \leq j \leq r_n.$$

Let $x_0 = \sum_{j=1}^{r_n} x_{nj}\ \varphi_{nj}$, $x_0 = (x_0, x_0, \dots, x_0)$, then

$$< Hy, x_0 >_{(L^2(\omega))^n} = \sum_{k=1}^{n} < \chi_\omega \frac{\partial}{\partial \xi_k}(K^* y), x_0 >_{L^2(\omega)}$$

$$= \sum_{k=1}^{n} < \frac{\partial}{\partial \xi_k}(\tilde{x}(T)),\ \chi_\omega^* x_0 >_{L^2(\Omega)}$$

where $\tilde{X}$ is the solution of the following system:

$$\begin{cases} \frac{\partial \tilde{x}}{\partial t}(\xi, t) = A^* \tilde{x}(\xi, t) + \sum_{i=1}^{q} \delta_{b_i} y_i(T - t) & \text{in } Q \\ \tilde{x}(\xi, 0) = 0 & \text{in } \Omega \\ \tilde{x}(\eta, t) = 0 & \text{in } \Sigma \end{cases} \tag{18}$$

Consider the system:

$$\begin{cases} \frac{\partial \varphi}{\partial t}(\xi, t) = -A\varphi(\xi, t) & \text{in } Q \\ \varphi(\xi, 0) = \chi_\omega^* x_0 & \text{in } \Omega \\ \varphi(\eta, t) = 0 & \text{in } \Sigma \end{cases} \tag{19}$$

Multiply (18) by $\frac{\partial \varphi}{\partial \xi_k}$ and integrate on Q, we obtain

$$\int_Q \frac{\partial \varphi}{\partial \xi_k}(\xi, t) \frac{\partial \tilde{x}}{\partial t}(\xi, t) d\xi\, dt = \int_Q A^* \tilde{x}(\xi, t) \frac{\partial \varphi}{\partial \xi_k}(\xi, t) d\xi\, dt +$$

$$\int_Q \left(\sum_{i=1}^{q} \delta_{b_i} y_i(T - t)\right) \frac{\partial \varphi}{\partial \xi_k}(\xi, t) d\xi dt.$$

But we have

$$\int_Q \frac{\partial \varphi}{\partial \xi_k}(\xi, t) \frac{\partial \tilde{x}}{\partial t}(\xi, t) d\xi\, dt = \int_\Omega \left[\frac{\partial \varphi}{\partial \xi_k}(\xi, t) \tilde{x}(\xi, t) d\xi\right]_0^T + \int_Q A \frac{\partial \varphi}{\partial \xi_k}(\xi, t) \tilde{x}(\xi, t) d\xi\, dt$$

$$= \int_\Omega \frac{\partial \varphi}{\partial \xi_k}(\xi, t) \tilde{x}(\xi, t) d\xi + \int_Q A \frac{\partial \varphi}{\partial \xi_k}(\xi, t) \tilde{x}(\xi, t) d\xi\, dt$$

then:

$$\int_\Omega \frac{\partial \varphi}{\partial \xi_k}(\xi, t) \tilde{x}(\xi, t) d\xi = -\int_Q A \frac{\partial \varphi}{\partial \xi_k}(\xi, t) \tilde{x}(\xi, t) d\xi + \int_Q A^* \tilde{x}(\xi, t) \frac{\partial \varphi}{\partial \xi_k}(\xi, t) d\xi\, dt$$

$$+ \int_Q \left(\sum_{i=1}^{q} \delta_{b_i} y_i(T - t)\right) \frac{\partial \varphi}{\xi_k}(\xi, t) d\xi dt$$

integrating by parts we obtain

$$\int_{\Omega} \frac{\partial \varphi}{\partial \xi_k}(\xi,t)\tilde{x}(\xi,t)d\xi = -\int_{\pi} \frac{\partial \tilde{x}(\eta,t)}{\partial v_{A^*}} \frac{\partial \varphi}{\partial \xi_k}(\eta,t)d\eta dt$$

$$\int_{\pi} \frac{\partial}{\partial v_{A^*}}\left(\frac{\partial \varphi}{\partial \xi_k}(\eta,t)d\eta dt\right)\tilde{x}(\eta,t)d\eta dt + \int_{Q} \left(\sum_{i=1}^{q} \delta_{b_i} y_i(T-t)\right)\frac{\partial \varphi}{\partial \xi_k}(\xi,t)d\xi dt$$

the boundary conditions give

$$\int_{\Omega} \frac{\partial \varphi}{\partial \xi_k}(\xi,t)\tilde{x}(\xi,t)d\xi = \int_{Q} \left(\sum_{i=1}^{q} \delta_{b_i} y_i(T-t)\right)\frac{\partial \varphi}{\partial \xi_k}(\xi,t)d\xi dt.$$

Thus

$$\int_{\Omega} \varphi(\xi,t)\frac{\partial \tilde{x}}{\partial \xi_k}(\xi,T)d\xi = -\sum_{i=1}^{q}\int_0^T \frac{\partial \varphi}{\partial \xi_k}(b_i,t)\, y_i(T-t)dt$$

and we have

$$< \chi_\omega \nabla K^* y, x_0 >_{(L^2(\omega))^n} = \sum_{k=1}^{n}\int_{\Omega} \frac{\partial \tilde{x}}{\partial \xi_k}(\xi,t)\varphi(\xi,t)d\xi$$

$$= -\sum_{K=1}^{q}\int_0^T \sum_{k=1}^{n} \frac{\partial \varphi}{\partial \xi_k}(b_i,t)\, y_i(T-t)dt.$$

But

$$\varphi(\xi,t) = \sum_{p=1}^{\infty} e^{-\lambda_p(T-t)} \sum_{j=1}^{r_p} < x_\circ, \varphi_{pj} >_{L^2(\omega)} \varphi_{pj}\,,$$

Then

$$\sum_{k=1}^{n} \frac{\partial \varphi}{\partial \xi_k}(b_i,t) = \sum_{p=1}^{\infty} e^{-\lambda_p(T-t)} \sum_{j=1}^{r_p} < x_0, \varphi_{pj} >_{L^2(\omega)} \sum_{k=1}^{n} \frac{\partial \varphi}{\partial \xi_k}(b_i)$$

$$= \sum_{p=1}^{\infty} e^{\lambda_p(T-t)}(G_p x_p)_i$$

therefore

$$< \chi_\omega \nabla K^* y, x_0 >_{(L^2(\omega))^n} = -\sum_{i=1}^{q}\int_0^T \sum_{p=1}^{\infty} e^{\lambda_p(T-t)}(G_p x_p)\, y_i(T-t)dt \tag{20}$$

Thus

$$< \chi_\omega \nabla K^* y, x_0 >_{(L^2(\omega))^n} = -\sum_{i=1}^{q}\int_0^T e^{\lambda_n(T-t)}(G_n x_n)_i\, y_i(T-t)dt = 0$$

This is true for all $y \in L^2\,(0,T;\,R^q)$, then $x_0 \in Ker\,H^*$ which contradicts the assumption that the suite of sensors is ω_G-strategic.□

We can deduced the following result:

Corollary 3.7: In the one dimension case, a sensor is ω_G-strategic if and only if $q \geq r = sup\, r_n$ and $rank\; G_n = r_n, \forall n \geq 1$, where G_n is given in theorem 3.6.

Remark 3.8: From the previous results, we can get

(1) The Theorem 3.6 implies that the required number of sensors is greater than or equal to the largest multiplicity of the eigenvalues.

(2) By infinitesimally deforming the domain, the multiplicity can be reduced to one [19]. Consequently, ω_G-strategic sensors can be achieved using only one sensor.

Now, we can deduced that various sensors which are not strategic in usual sense for systems, but may be ω_G-strategic and achieve the ω_G-observability. This is illustrated in the following counter- example.

3.2 A Counter- Example

Consider the system described by the parabolic equation

$$\begin{cases} \frac{\partial x(\xi,t)}{\partial t} = \frac{\partial^2 x}{\partial \xi^2}(\xi,t) & \text{in }]0,1[\times]0,T[\\ x(0,1) = x(1,t) = 0 & \text{in }]0,T[\\ x(\xi,0) = x_0(\xi) & \text{in }]0,1[\end{cases} \tag{21}$$

Suppose that the measurement is given by pointwise sensor located in $b \in]0,1[$ which is given by the following output function

$$y(.,t) = \int_{\Omega} x(\xi,t)\delta(\xi - t)d\,\xi = x(b,t),\ t \in (0,T)$$

(22)

Where $\varphi_n = \sqrt{2}\sin(n\pi\xi)$ and $\lambda_n = -n^2\pi^2$. First, we must prove that the system (21)-(22) is not weakly observable in Ω, that means the sensors (δ_b, b) is not strategic. For this purpose, we can write the system (21) as a state space one dimensional system

$$\dot{x}(\xi,t) = Ax(\xi,t)$$

$$x(\xi\,,0) = x_0(\xi)$$

Where $A = \frac{\partial^2}{\partial\xi^2}$ generate the continuous semigroup $(S(t))_{t\geq 0}$ given by [17].

$$S(t)x_0 = \sum_{i=1}^{\infty} e^{\lambda_i t} < x_0, \varphi_i >_{L^2(\Omega)} \varphi_i$$

Where, $\varphi_n = \sqrt{2}\sin(n\pi\xi)\,, \lambda_n = -n^2\pi^2$ are the eigenfunctions associated with the eigenvalues of A. Then from solution of (21), we have

$$y(\xi,t) = \sum_{i=1}^{\infty} e^{\lambda_i t} < x_0, \varphi_i >_{L^2(\Omega)} \varphi_i(b) = CS(t)x_0 = K(t)x_0$$

The system (21)-(22) is weakly observable if $ker\, K(t) = \{0\}$.

As proved in [27], if $b \in Q$ then system (21)-(22) is not weakly observable on Ω=(0,1) and a sensor (δ_b, b) is not strategic.

A sensor is ω-strategic on (0,1) $\Leftrightarrow$ $:b \notin S = \cup_{n=1}^{\infty}\left\{\frac{k}{n} | k \in [1, n-1] \cap N\right\}$. Since $\sin(n\pi b) = 0 \Leftrightarrow nb = k \Rightarrow b = \frac{k}{n}$. Consequently, the system is weakly observable on (0,1). And then, it is G-strategic on (0,1) $\Leftrightarrow$ $b \notin S_G = \cup_{n=1}^{\infty}\left\{\frac{2k+1}{2n} | k \in\right.$ $[0,n-1] \cap N$. Since $\cos n\pi b=0 \Leftrightarrow nb=2k+12 \Rightarrow b=2k+12n$. Consequently, the system is weakly G-observable on (0,1).□

Corollary 3.9: If the system (21)-(22) is exactly ω_G-observable, rank condition in theorem (3.6) is satisfied and a sensor is ω_G-strategic.

Now, assume that a sensor is not gradient strategic in whole the domain Ω and let $(\bar{\varphi}_i)_{i\in N^n}$ be a basis in $(L^2(\Omega))^n$. Let $I \subset N^n$ be such that $ker\, K\nabla^* = span\,\{(\bar{\varphi}_i)_{i\in I}\}$ and $J = N^n \backslash I$.

Proposition 3.10: The following properties are equivalent:

1. A sensor is ω_G-strategic.
2. $\overline{span\,\{(\chi_\omega \bar{\varphi}_i)_{i\in J}\}} = (L^2(\omega))^n$
3. If $x \in (L^2(\omega))^n$ is such that $< x, \chi_\omega \bar{\varphi}_i >_{(L^2(\omega))^n} = 0$ for all $i \in J$, then $x = 0$.
4. If $\sum_{i\in I} a_i \bar{\varphi}_i = 0$ in $\Omega\backslash\omega$, then $a_i = 0$ for all $i \in I$.

Proof: 1⟹2 Since sensors are ω_G-strategic this mean that the system is weakly ω_G-observable.

Let $x \in (L^2(\omega))^n$. Then for $\varepsilon > 0\ \exists\ y \in \mathcal{O}$ such that

$$\|x - \chi_\omega \nabla K^*\|_{(L^2(\omega))^n} \leq \varepsilon, \text{ but}$$

$$\nabla K^* y = \sum_{i\in N^n} < \nabla K^* y, \bar{\varphi}_i >_{(L^2(\Omega))^n} \bar{\varphi}_i = \sum_{i\in J} < y, K\nabla^* \bar{\varphi}_i >_{\mathcal{O}} \bar{\varphi}_i, \text{ and thus}$$

$$\chi_\omega \nabla K^* y = \sum_{i\in J} < y, K\nabla^* \bar{\varphi}_i >_{\mathcal{O}} \chi_\omega \bar{\varphi}_i. \text{ Then}$$

$$\left\|x - \sum_{i\in J} < y, K\nabla^* \bar{\varphi}_i >_{\mathcal{O}} \chi_\omega \bar{\varphi}_i\right\|_{(L^2(\omega))^n} < \varepsilon$$

and hence $x \in \overline{\{\chi_\omega \bar{\varphi}_i\}_{i\in J}}$.

2⟹3 Let $x \in (L^2(\omega))^n$. For any $\varepsilon > 0\ \exists\ \alpha_j\,(j \in J)$ such that

$$\left\|x - \sum_{i\in J} \alpha_j\, \chi_\omega \bar{\varphi}_j\right\|^2_{(L^2(\omega))^n} < \varepsilon, \text{ with}$$

$$< x, \chi_\omega \bar{\varphi}_j >_{(L^2(\omega))^n} = 0, \forall j \in J$$

we deduced that

$$\|x\|^2_{(L^2(\omega))^n} < \varepsilon\ . \text{ Thus, } x = 0.$$

3⟹4 Let $\sum_{i\in I} a_i \bar{\varphi}_i = 0$ in $\Omega\backslash\omega$.

Now Consider $x = \chi_\omega(\sum_{i\in I} a_i \bar{\varphi}_i)$. For $j \in J$, we have

$$< x, \chi_\omega \bar{\varphi}_i >_{(L^2(\omega))^n} = \sum_{i\in I} a_i < \bar{\varphi}_i, \bar{\varphi}_j >_{(L^2(\Omega))^n} = 0.$$

Since $x = 0$, we get

$$\sum_{i\in I} a_i \bar{\varphi}_i = 0 \text{ in } \Omega \text{ and } a_i = 0, \forall i \in \mathrm{I}.$$

4⟹1 Consider $x \in (L^2(\omega))^n$ such that

$K\nabla^* \chi_\omega^* x = 0$. We have $\chi_\omega^* x \in (L^2(\Omega))^n$

then

$$K\nabla^* \chi_\omega^* x = K\nabla^* \left(\sum_{i\in N^n} < x, \chi_\omega \bar{\varphi}_i >_{(L^2(\omega))^n} \bar{\varphi}_i\right) = K\nabla^* (\sum_{i\in j} < x, \chi_\omega \bar{\varphi}_i >_{(L^2(\omega))^n} \bar{\varphi}_i) = 0.$$ Therefore,

$$\sum_{i\in j} < x, \chi_\omega \bar{\varphi}_i >_{(L^2(\omega))^n} \bar{\varphi}_i \in span\{(\bar{\varphi}_i)_{i\in I}\}$$

and then

$$\sum_{i\in I} < x, \chi_\omega \bar{\varphi}_i >_{(L^2(\omega))^n} = 0, \forall j \in J.$$

Therefore

$$\chi_\omega^* x = \sum_{i\in I} < x, \chi_\omega \bar{\varphi}_i >_{(L^2(\omega))^n} \bar{\varphi}_i = 0 \text{ in } \Omega \backslash \omega.$$

From the assumption we have $< x, \chi_\omega \bar{\varphi}_i >_{(L^2(\omega))^n} = 0, \forall i \in I$. Hence $x = 0$.□

We can deduced the following result:

Corollary 3.11: Under the hypotheses of Proposition 3.10, a sensors is ω_G-strategic in all $\omega \subset \Omega$ such that $< \bar{\varphi}_i, \bar{\varphi}_j >_{(L^2(\omega))^n} = 0, \ \forall i,j \in I, i \neq j$.

Proof: To deduce the result from previous Proposition 3.10, we take $\sum_{i\in I} a_i \bar{\varphi}_i = 0$ in $\Omega \backslash \omega$. Then we only need to show that $a_i = 0, \forall i \in I$. Let $x = \sum_{i\in I} a_i \bar{\varphi}_i$ in Ω and $i_0 \in \mathrm{I}$. Then

$$< x, \bar{\varphi}_{i0} >_{(L^2(\Omega))^n} = < \sum_{i\in I} a_i \bar{\varphi}_i , \bar{\varphi}_{i0} >_{(L^2(\Omega))^n} = \sum_{i\in I} a_i < \bar{\varphi}_i, \bar{\varphi}_{i0} >_{(L^2(\Omega))^n} = a_{i0} \tag{23}$$

Since $x = 0$ in $\Omega \backslash \omega$, under the assumption of Corollary 3.11 we have

$$< x, \bar{\varphi}_{i0} >_{(L^2(\Omega))^n} = \sum_{\in I} a_i < \bar{\varphi}_i, \bar{\varphi}_{i0} >_{(L^2(\omega))^n} = a_{i0} \|\bar{\varphi}_{i0}\|^2_{(L^2(\omega))^n} \tag{24}$$

From (23)-(24), we obtain $a_i = 0 , \forall i \in I$.

4. APPLICATION TO SENSORS LOCATIONS

In this section, we give specific results related to the different case presented in the above section. First we consider internal sensors (zonal, pointwise, filament in rectangular and disk domain) the presented result give information on the structure of ω. Consider the system

$$\begin{cases} \frac{\partial x}{\partial t}(\xi_1, \xi_2, t) = \Delta x(\xi_1, \xi_2, t) & \text{in } Q, \\ x(\xi_1, \xi_2, 0) = x_0(\xi_1, \xi_2) & \text{in } \Omega, \\ x(\eta_1, \eta_2, t) = 0 & \text{in } \Sigma \end{cases} \tag{25}$$

Let $\Omega = (0,1) \times (0,1)$ and let $\omega = (\alpha_1, \beta_1) \times (\alpha_2, \beta_2)$ be the considered region is subset of Ω, the eigenfunctions and the eigenvalue of the system (25) are given by:

$$\varphi_{ij}(\xi_1, \xi_2) = \frac{2}{\sqrt{(\beta_1 - \alpha_1)(\beta_2 - \alpha_2)}} \sin i\pi \frac{(\xi_1 - \alpha_1)}{(\beta_1 - \alpha_1)} sinj\pi \frac{(\xi_2 - \alpha_2)}{(\beta_2 - \alpha_2)} \tag{26}$$

Associated with eigenvalue

$$\lambda_{ij} = -\frac{i^2}{(\beta_1 - \alpha_1)^2} + \frac{j^2}{(\beta_2 - \alpha_2)^2} \tag{27}$$

4.1 Internal Zone Sensor

Consider the system (25) together with output function (2) where the sensor supports D are located in Ω. The output (2) can by written by the form

$$y(t) = \int_D x(\xi_1, \xi_2, t) f(\xi_1, \xi_2) d\xi_1 d\xi_2 \tag{28}$$

Where $D \subset \Omega$ is location of zone sensor and $f \in L^2(D)$. In this case of (see Figure 3), the eigenfunctions and the eigenvalues (26) and (27).

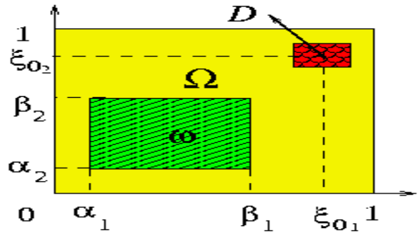

Fig. 3: Domain Ω, sub-region ω and location D of internal zone sensor

However , if we suppose that

$$\frac{(\beta_1-\alpha_1)^2}{(\beta_2-\alpha_2)^2} \notin Q$$

Then multiplicity of λ_{ij} is $r_{ij} = 1$ and then one sensor (D, f) my be sufficient to achieve ω_G-observable of the systems (25) and (28) [19]. Let the measurement support is rectangular with

$$D = [\xi_{01} - l_1, \xi_{01} + l_1] \times [\xi_{02} - l_2, \xi_{02} + l_2] \in \Omega$$

Then, we have the following result

Corollary 4.1: If f_1 is symmetric about $\xi_1 = \xi_{01}$ and f_2 is symmetric about $\xi_2 = \xi_{02}$, then the sensor (D, f) is ω_G-strategic if

$$\frac{i(\xi_{01}-\alpha_1)}{(\beta_1-\alpha_1)} \text{and} \frac{j(\xi_{02}-\alpha_1)}{(\beta_2-\alpha_2)} \notin N \text{ for some } i, j.$$

4.2 Internal Pointwise Sensor

In this case the out put function is given by:

$$y(t) = \int_D x(\xi_1, \xi_2, t)\delta(\xi_1 - b_1, \xi_2 - b_2)d\xi_1 d\xi_2 \tag{29}$$

With $b = (b_1, b_2)$ is location of pointwise sensor as defined in (see Figure 4)

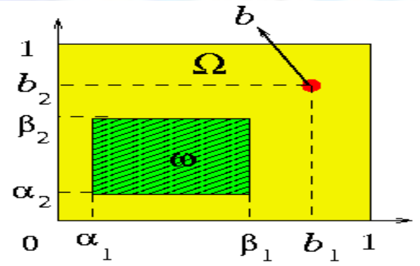

Fig. 4: Rectangular domain, and location b of internal pointwise sensor

If $\frac{(\beta_1-\alpha_1)}{(\beta_2-\alpha_2)} \notin Q$, then $r_{ij} = 1$ and one sensor (b, δ_b) may be sufficient for ω_G-observability of the systems (25)-(29)

Corollary 4.2: The sensor (b, δ_b) is ω_G-strategic if

$$\frac{i(b_1-\alpha_1)}{(\beta_1-\alpha_1)} \text{and} \frac{j(b_2-\alpha_2)}{(\beta_2-\alpha_2)} \notin N, \text{ for some } i, j.$$

4.3 Internal Filament Sensor

Consider the case where the observation is given on the curve $\sigma = Im(\gamma)$ with $\gamma \in C^1(0,1)$ (see Figure 5)

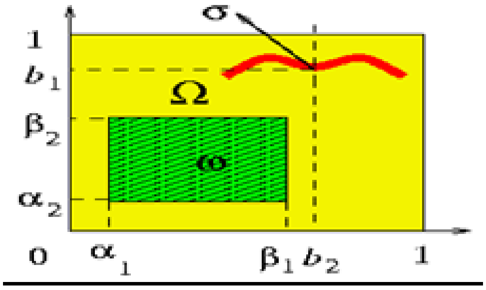

Fig. 5: Rectangular domain, and location σ of internal filament sensors

Corollary 4.3: If the measurements recovered by filament sensor (σ, δ_σ) such that is symmetric with respect to the line $\xi = \xi_0$. Then the sensor (σ, δ_σ) is ω_G-strategic if

$$\frac{i(\xi_{01}-\alpha_1)}{(\beta_1-\alpha_1)} \text{ and } \frac{j(\xi_{02}-\alpha_2)}{(\beta_2-\alpha_2)} \notin N, \text{ for } i, j = 1, \ldots, J.$$

Remark 4.4: These results can be extended to the following:

1. Case of Neumann or mixed boundary conditions [4-5].
2. Case of disc domain $\Omega = (D,1)$ and $\omega = (0, r_\omega)$ where $\omega \subset \Omega$ and $0 < r_\omega < 1$ [1-3] .
3. Case of boundary sensors where $C \notin L(X,\ R^q)$, we refer to see [13-14].
4. We can show that the observation error decreases when the number and support of sensors increases [23, 25].

5. CONCLUSION

We have been introduced a sufficient condition of regional gradient strategic sensors in order to achieves regional gradient observability. Many interesting results concerning the choice of sensors structure are given and illustrated in specific situations. Various questions still opened under consideration. For example, these result can be extended to the boundary case with parabolic and hyperbolic systems [8].

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REFERENCES

[1] Al-Saphory, R. and El Jai, A. 2001. Sensor structures and regional detectability of parabolic distributes systems. International Journal of Sensors and Actuators. Vol. 90.163-171.

[2] Al-Saphory, R. and El Jai, A. 2001. Sensors characterizations for regional boundary detectability in distributed parameter systems. International Journal of Sensors and Actuators. Vol. 94.1-10.

[3] Al-Saphory, R. and El Jai A. 2002. Regional asymptotic state reconstruction. International Journal of Systems Science. Vol. 33. 1025-1037.

[4] Al-Saphory, R. 2010. Strategic sensors and regional exponential observation in Neumann boundary conditions. Zanco Journal of Pure and Applied Science. Vol. 22, 1-11.

[5] Al-Saphory, R., Al-Jawari, N. and Al-Qaisi, A . 2010. Regional gradient detectability for infinite dimensional systems. Tikrit Journal of Pure Science. Vol. 15, No. 2. 1-6.

[6] Al-Saphory, R. 2011. Strategic sensors and regional exponential observability. ISRN Applied Mathematics. Vol. 2011. Article ID 673052. pp. 1-13.

[7] Al-Saphory, R., Al-Joubory, M. and Jasim, M. 2013. Regional strategic sensors characterizations. Journal of Mathematical and Computational Science. 3, 401-418.

[8] Boutoulout, A., Bourray, H. and Khazari, A. 2014. Flux observability for hyperbolic systems. Applied Mathematics and Information Sciences Letters. Vol. 2 , No. 1, 13-24.

[9] Brezis, H. 1987. Analyse fonctionnalle; Theorie et applications. 2em tirage. Masson. Paris, France.

[10] Burns, J., Jorggaard, J., Cliff, M. and Zietsman, L. 2009. A PDE approach to optimization and control of high performance buildings, Workshop in Numerical Techniques for Optimization Problems with PDE Constraints, No. 04/2009, DOI: 10.4171/OWR/2009/04. 25-31 January. Houston, Denmark.

[11] Burns, J., Jorggaard, J., Cliff, M. and Zietsman, L. 2010. Optimal sensor design for estimation and optimization of PDE Systems, 2010 American Control Conference. Marriott Waterfront, Baltimore, MD, USA. June 30-July 02, 2010.

[12] Chen Y., Moore K., and Song, Z. 2004. Diffusion boundary determination and zone control via, mobile actuators-sensors networks (MAS-net)-challenge and opportunities. SCOIS. Utah state university, Logan, UT84322, USA.

[13] Curtain, R. F. and Pritchard, A. J. 1978. Infinite dimensional linear theory systems; Lecture Note in Control and Information Sciences. Spring – Verlag, New York.

[14] Curtain, R. F. and Zwart, H. 1995. An introduction to infinite dimensional linear system theory. Springer-Verlag, New York.

[15] El Jai, A. and Pritchard, A. j. 1987. Sensors and actuators in distributed system. International Journal of Control. Vol. 46, 1139-1153.

[16] El Jai, A. and Amouroux, M. 1988. Sensors and observers in distributed parameter systems. International Journal of Control. Vol. 47, 333-347.

[17] El Jai, A. 1991. Distributed systems analysis via sensors and actuators, International Journal of Sensors and Actuators.Vol. 29, 1-11.

[18] El Jai, A.. and Pritchard A. j. 1988. Sensors and controls in analysis of distributed system, Ellis Harwood series in Mathematics and Applications, Wiley, New York.

[19] El Jai, A. and El Yacoubi, S. 1993. On the number of actuators in a parabolic system, Applied Mathematics and Computer Science. Vol.3, No.4, 139-150.

[20] El Jai, A., Simon, M.C. and Zerrik, E. 1993. Regional observability and sensor structures, International Journal of Sensor and Actuator. Vol. 39, No. 2, 95-102.

[21] El Jai, A., Amouroux, M. and Zerrik, E. 1994. Regional observability of distributed systems. International Journal of Systems Science. 1994, Vol. 25, No.2, 301-313.

[22] El Jai, A., Simon, M.C., Zerrik, E. and Amouroux, M. 1995. Regional observability of a thermal proces. IEEE Transaction on Automatic Control. Vol. 40, 518-521.

[23] El Jai A. and Hamzauoi, H. 2009, Regional observation and sensors. International Journal of Applied Mathematics and Computer Sciences. Vol. 19, No. 1, 5-14.

[24] Gressang, R. and Lamont, G.1975. Observers for systems characterized by semi-group. IEEE on Automatic and Control. 20, 523-528.

[25] Hamzauoi, H., 2008, On choice of actuators-sensors structures in the systems, Ph. D. Thesis, Perpignan, France.

[26] Zerrik, E. 1993. Regional analysis of distributed parameter systems, Ph.D. Thesis. University of Rabat, Morocco.

[27] Zerrik, E., Bourray H. and Badraoui L. 2000. How to reconstruct a gradient for parabolic systems. Conference of MTNS 2000. Perignan, France, June-2000, 19-23.

[28] Zerrik, E. and Bourray, H. 2003. Gradient observability for diffusion systems. International Journal of Applied Mathematics and Computer Sciences. Vol. 2, 139-150.

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