ISSN 2347-1921

# Physics, Mechanics, Mathematics

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## ABSTRACT

Dynamics features movement and stable means. Continuous

Stable dynamics thus means continuous movement or motion. That is a moving object which enjoys continuous movement. For example, the electron continuous revolution round the nucleus, the revolution of the moon round the earth and that of the earth round the sun. In this formula, the continuous movement of the moving object round the origin of coordinates in space is studied.

Regarding the importance of masses movement in space, the necessity is felt that in order to design and optimize dynamic systems (dynamic mechanics) and all relevant subsets, a reasonable relation should be presented (Some scientists believe that a charged particle is the 5$^{th}$ dimension in which case a particle enjoying continuous movement can be named the 5$^{th}$ dimension.

Key words: stable dynamics, materials

ISSN 2347-1921

## Introduction

In dynamic mechanics literature, the issue of movement relevant to a moving object is discussed and proved.

As an example, the throwing of a particle in space, a moving object range, force driving apogee, mass and acceleration of the moving object are discussed and have documented and compiled formulas.

In this formula, a moving object continuous movement is studied, and as detailed in the abstract, the formula of the moving object continuous movement, that is, the same stable dynamics, has been established.

The formula in question is initially proved in plane xoy and is then extended to planes xoz and yoz.

In the end, we have 3 images of the moving object in the above mentioned planes to obtain the moving object general formula in space.

$m.V_A.(\overline{OH}_A) = m.V_B(\overline{OH}_B) = m.V_C.(\overline{OH}_C) = m.V_D(\overline{OH}_D)$

m= moving mass

at point (A) oxyz (movingspeedinspace=$V_A$

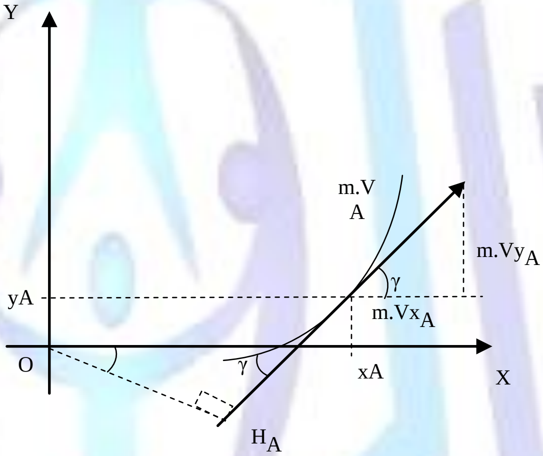
$(\overline{OH})$ A=A. Vertical distance from point to velocity vector $V_A$ at point

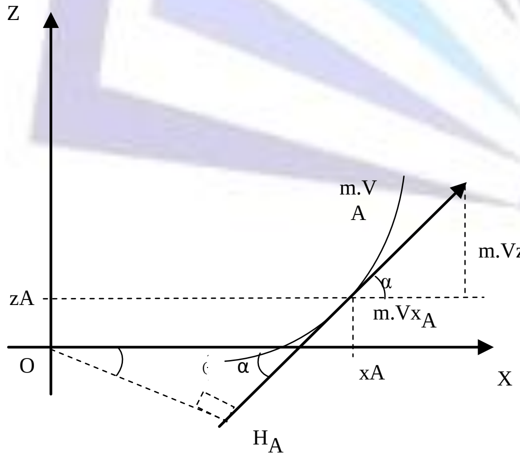
Given a moving object with mass m and velocity $V_A$ so that the moving object continuously moves round the origin of coordinates , the following relation can be written :

$A\begin{cases} m.Vx_A \\ m.Vy_A \end{cases}$

$V_{(x)}A$ : Velocity image on the x axis

$V_{(y)}A$ : Velocity image on the y axis




$\tan\gamma=\frac{Vy}{Vx}$

$\tan\alpha=\frac{Vz}{Vx}$

$\tan\beta=\frac{Vz}{Vy}$

Using momentum m.v (x , y) A, we will calculate the momentum relative to point (O):

$m.V_{(x,y)}\ _A.\overline{OH}_{(x,y)}\ _A = m.V_{x_A}.y_A+m.Vy_A.x_A$

ISSN 2347-1921

Both sides of the relation is divided by m to give:

If the moving object moves from point A to point B , C , D or any other point in its orbit relation

1)$V_{(x,y)A} . \overline{OH}_{(x,y)A} = V_{xA}.y_A + V_{yA}.x_A$

Will thus apply at points B,C,D , …

Relation (1) can thus be written as :

2) $V_{(x,y)B}. \overline{OH}_{(x,y)B} = V_{xB}.y_B + V_{yB}.x_B$

And it can be concluded that all the relations at points A , B , C , D , … are equal .

3) )$Vx_A.y_A + Vy_A.X_A = Vx_B.y_B + Vy_B.X_B = Vx_C.y_C + Vy_C.X_C = Vx_D.y_D + Vy_D.x_D$

Regarding the above mentioned relations 2 points are significant.

1. The movement of a moving object round point O in an orbit depends on its velocity and distance from the origin of coordinates at the point in question (A,B,C,D, …)

2. The movement of a moving object round point O does not depend on its mass (m) .

Formulas (1) to (3) apply to plane (x,y) , and the above mentioned formulas in planes xoz and yoz will equal ….

At point A and in plane xoz.

4 )$V_{(x,z)A} . \overline{OH}_{(x,z)A} = Vx_A.z_A + Vz_A.x_A$

5 )$V_{(x,z)B} . \overline{OH}_{(x,z)B} = Vx_B.z_B + Vz_B.x_B$

6 )$Vx_A.z_A + Vz_A.x_A = Vx_B.z_B + Vz_B.x_B = Vx_C.z_C + Vz_C.x_C = Vx_D.x_D + Vz_D.x_D$

and in plane yoz

7 )$V_{(y,z)A} . \overline{OH}_{(y,z)A} = Vy_A.z_A + Vz_A.y_A$

8 )$V_{(y,z)B} . \overline{OH}_{(y,z)B} = Vy_B.z_B + Vz_B.y_B$

9 ) $Vy_A.z_A + Vz_A.y_A = Vy_B.z_B + Vz_B.y_B = Vy_C.z_C + Vz_C.y_C = Vy_D.z_D + Vz_D.y_D$

If moving object m is continuously moving at velocity V round point O , formula (10) will apply.

10 ) $m.\overline{OH}_A.V_A \Rightarrow V_A = \sqrt{V_{xA}{}^2 + V_{yA}{}^2 + VzA^2} \Rightarrow \overline{OH}_A = \sqrt{(\overline{OH}x_A)^2 + (\overline{OH}y_A)^2 + (\overline{OH}z_A)^2}$

$\Rightarrow \overline{OH}x_A = \overline{OH}(x,y)_A.\mathrm{Cos}\,(\frac{\pi}{2} - \gamma) \Rightarrow \overline{OH}y_A = \overline{OH}(x,y)_A.\mathrm{Sin}(\frac{\pi}{2} - \gamma) \Rightarrow \overline{OH}z_A = \overline{OH}(x,z)_A.\mathrm{Sin}\,(\frac{\pi}{2} - \alpha)$

$m.\overline{OH}_A.V_A = m.\overline{OH}_B.V_B = m.\overline{OH}_C.V_C = m.\overline{OH}_D.V_D = \ldots\ldots\ldots = m.\overline{OH}_ʒ.V_ʒ$

$V_A$ = Velocity of the moving object in space

$\overline{OH}$= Vertical distance from point O to the vector of velocity V

Now regarding what preceded the proof of non-Euclidean trigonometric circle is dealt with and then, non-Euclidean trigonometric proved formula will be compared with Euclidean trigonometric circle.

In Euclidean trigonometric circle ,

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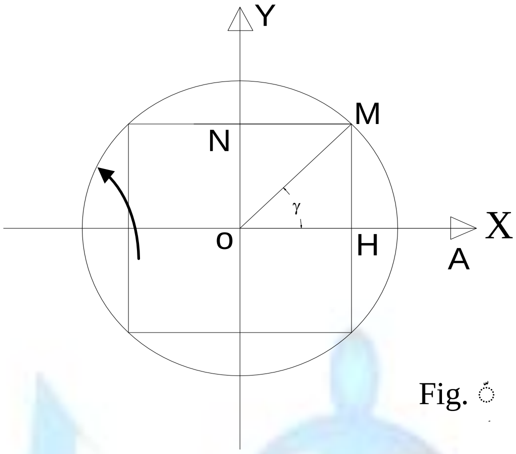



Fig.

We have :

$$\begin{cases} \overline{OM} = 1 \\ \overline{OH} = \text{Cos}\ \gamma \\ \overline{ON} = \text{Sin}\ \gamma \end{cases}$$

$$(\overline{OH})^2 + (\overline{ON})^2 = (\overline{OM})^2$$

$\text{Cos}^2\gamma + \text{Sin}^2\gamma = 1$

Non-Euclidean trigonometric circle is the same as the above Euclidean trigonometric circle (Fig.1) , the difference being that in the above trigonometric circle , the movement of the moving object round axes oy, ox and on the 4 quarters of the trigonometric circle region is studied , so that the movement of the moving object is in the direction of trigonometric movement , that is :

$$\left.\begin{matrix} \overline{OM} = 1 \\ \overline{OH} = (-Vx) \\ \overline{ON} = (Vy) \end{matrix}\right\} \Rightarrow \overline{OH} + \overline{ON} = \overline{OM} \Rightarrow \begin{cases} (-Vx)^2 + (Vy)^2 = 1 \\ Vx^2 + Vy^2 = 1 \end{cases}$$

If $1 \leq Vy$ and $1 \leq Vx$ , the following relation can be used: $C_1 = cte, (C^2{}_1 . V^2x + C^2{}_1 . V^2y = C^2{}_1)$

In Euclidean trigonometric circle , the location of each point M is specified on the trigonometric circle.

Euclidean trigonometry can thus be named local trigonometry.

In non-Euclidean trigonometric circle , since the movement of the moving object is studied , and at any moment in time , the moving object has a specific location , non-Euclidian trigonometry can be termed temporal trigonometry.

These 2 local and temporal trigonometric circles are now compared with each other.

In the comparison , it can be concluded that in local trigonometry , only the local situation is followed .

In temporal trigonometry however , both local and temporal situations are followed , and for test and control in 3 planes for local trigonometry, ($\tan\alpha = \tan\beta . \tan\gamma$) can be used , and in any type of movement , if temporal trigonometry is applied there will be no need for local trigonometry application . Problems 1 , 2 , … are only practices . If there are applied problems .

ISSN 2347-1921

| Firstquarter | | Secondquarter | | Thirdquarter | | Fourthquarter | |
|---|---|---|---|---|---|---|---|
| Cos γ | Vx | Cos γ | Vx | Cos γ | Vx | Cos γ | Vx |
| + | - | - | - | - | + | + | + |
| Sin γ | Vy | Sin γ | Vy | Sin γ | Vy | Sin γ | Vy |
| + | + | + | - | - | - | - | + |
| tan γ | $^{Vy}/_{Vx}$ | tan γ | $^{Vy}/_{Vx}$ | tan γ | $^{Vy}/_{Vx}$ | tan γ | $^{Vy}/_{Vx}$ |
| + | - | - | + | + | - | - | + |
| Cotan γ | $^{Vx}/_{Vy}$ | Cotan γ | $^{Vx}/_{Vy}$ | Cotan γ | $^{Vx}/_{Vy}$ | Cotan γ | $^{Vx}/_{Vy}$ |
| + | - | - | + | + | - | - | + |

No given relations z'=$\frac{Vz}{Vx}$andy'=$\frac{Vy}{Vx}$ In planes ozy and oxy , the following relations in 3 planes of oxy , oyz and oxz can be written as follows :

1 )$z' = \frac{Vz}{Vx}$= tan ∝(In plane oxz )

2 )$y' = \frac{Vy}{Vx}$ tan γ  (In plane oxy )

By dividing relation (1) by relation (2) , relation (3) can be obtained in plane oyz :

3 )$\frac{z'}{y'} = \frac{\frac{Vz}{Vx}}{\frac{Vy}{Vx}} = \frac{\tan\propto}{\tan\gamma} = \frac{Vz}{Vy} = \tan\beta \Rightarrow \frac{Vz}{Vy} = \tan\beta$

No , given relations 1 , 2 & 3 , relation 4 is obtained :

1) $\frac{Vz}{Vx} = \tan\propto$

2) $\frac{Vy}{Vx} = \tan\gamma \, \frac{Vz}{Vx} = \frac{Vz}{Vy} * \frac{Vy}{Vx} \Rightarrow 4\,)\ \tan\propto = \tan\beta * \tan\gamma$

3) $\frac{Vz}{Vy} = \tan\beta$

Relation (4) can be reckoned the basis for the original formula (dynamic trigonometry) . The following formulas will thus be the subset of formula (4) .

As an example :

Cos ∝=$\frac{1}{\sqrt{1+\tan^2\propto}} \Rightarrow 5)\ \text{Cos}\ \propto = \frac{1}{\sqrt{1+\tan^2\beta\,.\tan^2\gamma}}$

Sin ∝ =$\frac{\tan\propto}{\sqrt{1+\tan^2\propto}} = \frac{\tan\beta\,.\tan\gamma}{\sqrt{1+\tan^2\propto.\tan^2\gamma}} \Rightarrow\ 6\,)\text{Sin}\ \propto = \frac{\tan\beta\,.\tan\gamma}{\sqrt{1+\tan^2\beta\,.\tan^2\gamma}}$

Sin 2∝ = 2 Sin ∝. Cos ∝ =2$\frac{\tan\beta.\tan\gamma}{\sqrt{1+\tan^2\beta.\tan^2\gamma}} . \frac{1}{\sqrt{1+\tan^2\beta.\tan^2\gamma}} = \frac{2\tan\beta\,.\tan\gamma}{1+\tan^2\beta\,.\tan^2\gamma}$

7) Sin 2∝ =$\frac{2\tan\beta\,.\tan\gamma}{1+\tan^2\beta\,.\tan^2\gamma}$

Cos 2∝= $\text{Cos}^2\propto - \text{Sin}^2\propto = \frac{1}{1+\tan^2\beta\,.\tan^2\gamma} - \frac{\tan^2\beta\,.\tan^2\gamma}{1+\tan^2\beta\,.\tan^2\gamma}$

8) Cos 2∝ =$\frac{1-\tan^2\beta\,.\tan^2\gamma}{1+\tan^2\beta\,.\tan^2\gamma}$

Tan 2∝=$\frac{\sin 2\propto}{\cos 2\propto} = \frac{2\tan\beta\,.\tan\gamma / 1+\tan^2\beta\,.\tan^2\gamma}{1-\tan^2\beta\,.\tan^2\gamma / 1+\tan^2\beta\,.\tan^2\gamma} = \frac{2.\tan\beta\,.\tan\gamma}{1-\tan^2\beta\,.\tan^2\gamma}$

9 ) Tan 2∝ =$\frac{2\tan\beta\,.\tan\gamma}{1-\tan^2\beta\,.\tan^2\gamma}$

## Practice No 1.

Relation (4) is tan ∝ = tan β .tan ɣ Given

$\frac{\sin\propto}{\cos\propto} = \frac{\sin\beta.\sin\gamma}{\cos\beta.\cos\gamma} \Rightarrow \begin{cases}\sin\propto = \sin\beta.\sin\gamma \\ \cos\propto = \cos\beta.\cos\gamma\end{cases}$ , obtain the relation between tan β , tan ɣ, and in the end , obtain the relations between Vx , Vy and Vz.

$$\left.\begin{array}{l}\sin^2\propto = \sin^2\beta.\sin^2\gamma \\ \cos^2\propto = \cos^2\beta\cos^2\gamma\end{array}\right\} \Rightarrow \sin^2\propto + \cos^2\propto = 1 = \sin^2\beta.\sin^2\gamma + \cos^2\beta.\cos^2\gamma$$

cos β = $\frac{1}{\sqrt{1+\tan^2\beta}}$ cos ɣ = $\frac{1}{\sqrt{1+\tan^2\gamma}}$

sin β = $\frac{\tan\beta}{\sqrt{1+\tan^2\beta}}$ sin ɣ = $\frac{\tan\gamma}{\sqrt{1+\tan^2\gamma}}$

$$1 = \sin^2\beta.\sin^2\gamma + \cos^2\beta.\cos^2\gamma$$

If we substitute values $\sin\beta, \cos\beta, \sin\gamma, \cos\gamma$ in terms of tan ɣ and tan β in the above relation we will have :

$$\frac{1}{1} = \frac{\tan^2\beta}{(1+\tan^2\beta)}.\frac{\tan^2\gamma}{(1+\tan^2\gamma)} + \frac{1}{(1+\tan^2\beta).(1+\tan^2\gamma)} \Rightarrow (1+\tan^2\beta.\tan^2\gamma) = (1+\tan^2\beta).(1+\tan^2\gamma)$$

$$1+\tan^2\gamma + \tan^2\beta + \tan^2\beta * \tan^2\gamma = 1+\tan^2\beta.\tan^2\gamma$$

$$\tan^2\gamma + \tan^2\beta = 0 \Rightarrow \frac{V^2{}_y}{V^2{}_x} + \frac{V^2{}_z}{V^2y} = 0$$

## Practice No 2.

Given tan α = tan β .tan γ⇨tan γ = $\frac{\tan\propto}{\tan\beta}$ , calculate the relations between $V_x$ , $V_y$&$V_z$

$$\left.\begin{array}{l}\tan\propto = \frac{Vz}{Vx} \\ \tan\beta = \frac{Vz}{Vy} \\ \tan\gamma = \frac{Vy}{Vx}\end{array}\right\} \Rightarrow \tan\gamma = \frac{\tan\propto}{\tan\beta} \Rightarrow \frac{\sin\gamma}{\cos\gamma} = \frac{\sin\propto/\cos\propto}{\sin\beta/\cos\beta} = \frac{\sin\propto.\cos\beta}{\cos\propto.\sin\beta}$$

$$\begin{cases}\sin\gamma = \sin\alpha.\cos\beta \Rightarrow \sin^2\gamma = \sin^2\alpha.\cos^2\beta \\ \cos\gamma = \cos\alpha.\sin\beta \Rightarrow \cos^2\gamma = \cos^2\alpha.\sin^2\beta\end{cases}$$

sin² ɣ + cos² ɣ=1 ⇨ sin ² ɣ + cos² ɣ =sin² α . cos² β + cos² α . sin² β = 1

sin α = $\frac{\tan\propto}{\sqrt{1+\tan^2\propto}} = \frac{Vz/Vx}{\sqrt{1+(\frac{Vz}{Vx})^2}}$, sin β= $\frac{\tan\beta}{\sqrt{1+\tan^2\beta}} = \frac{Vz/Vy}{\sqrt{1+(\frac{Vz}{Vy})^2}}$

cos α= $\frac{1}{\sqrt{1+\tan^2\propto}} = \frac{1}{\sqrt{1+(\frac{Vz}{Vz})^2}}$ , cos β = $\frac{1}{\sqrt{1+\tan^2\beta}} = \frac{1}{\sqrt{1+(\frac{Vz}{Vy})^2}}$

$$\frac{(\frac{Vz}{Vx})^2}{1+(\frac{Vz}{Vx})^2} * \frac{1}{1+(\frac{Vz}{Vy})^2} + \frac{1}{1+(\frac{Vz}{Vx})^2} * \frac{(\frac{Vz}{Vy})^2}{1+(\frac{Vz}{Vy})^2} = \frac{1}{1}$$

$$\frac{(\frac{Vz}{Vx})^2 + (\frac{Vz}{Vy})^2}{[1+(\frac{Vz}{Vx})^2] * [1+(\frac{Vz}{Vy})^2]} = \frac{1}{1} \Rightarrow (\frac{Vz}{Vx})^2 + (\frac{Vz}{Vy})^2 = 1 + (\frac{Vz}{Vy})^2 + (\frac{Vz}{Vx})^2 + (\frac{Vz}{Vx})^2 * (\frac{Vz}{Vy})^2$$

$$1 + \frac{Vz^2}{Vx^2} * \frac{Vz^2}{Vy^2} = 0 \Rightarrow Vz4 = -Vx^2.Vy^2$$

ISSN 2347-1921

# Angular speed

## Abstract

In this paper , the angular speed formula has been established (proved) on any type of curve. Regarding the importance of the angular speed calculation in most of applied sciences such as dynamic mechanics , aerospace , dynamic systems and lock of a relation established in this connection , the need is felt that in order to design and optimize dynamic systems , a reasonable relation should be presented . This paper tries to prove such a relation in the easiest possible way.

Key Word : angular speed

## Introduction

Angular speed on any curve : In dynamic mechanics literature , "angular speed " has been defined like this : The angle covered by a moving object in time unit.

The unit of angular speed is radiant per second (Rad / sec) .

Angular speed formula : V = R.ω

Where V = linear speed on the curve , R = radius of rotation round the rotation axis and ω = the moving object angular speed.

N this paper angular speed is studied in any type of (closed / open) curve . It is initially studied in a closed curve in which there are 2 existing characteristic geometric forms which are common : circle and ellipse .

In the open – type curve , it includes any type of cure both ends of which are not linked to each other .

Now to begin with , a closed curve is studied , firstly a circle .

Angular speed in a circle consists of 2 parts :

A .Angular speed in a circle with constant linear speed linear velocity (V) is constant in a circle and the circle radius R is also constant . Angular speed is thus constant : V = R.ω

All the angles covered in equal time units will thus be ω = $V/_R$= cte ( with one $\overline{OA}$link)

B. Angular speed in the circle with noncontact (variable) linear speed.

C . Angular speed in the circle with arms length more than one , in different rotational motion directions and constant & variable angular speeds.

B . Since linear velocity (v) is not constant in a circle , the circle radius (R) is however constant , angular speed will thus be noncontact . We will thus have :

R = m (circle radius in meters)

$$\omega = \frac{V}{R} \mathrm{Rad}/_{\mathrm{sec}}$$

$$V = {}^{m}/_{sec} \text{ (linear velocity )}$$

$$V = R * \omega$$

$$dV = R * d\,\omega$$

Now regarding fig.1 it can be expressed that the moving object has moved from point A to point B in a time unit . That is , angle (α) , and hase moved from point (B) to point (C) in another time unit . That is angle (β)

Angle (α) will not equal angle (β) .

It can thus be concluded that all the angles will not be covered in equal time units, …. , in any time unit , an angle will be covered depending on the linear velocity variations .

## Angular speed in an ellipse:

Given the ellipse center as the rotation axis and the moving object moves so that it holds a constant angular speed , regarding the fact that rotation radius and linear velocity are variable formula V = R.ω can be investigated :

dV = R * dω + ω.dR = 0 + ω.dR

According to Fig.2, the angles covered are equal to each other. Yet, regarding the Fig.1 it can be concluded that the linear velocity variations are proportional to the rotation radius variations.

Now, angular speed is studied in an open curve to prove the angular speed formula.

ISSN 2347-1921

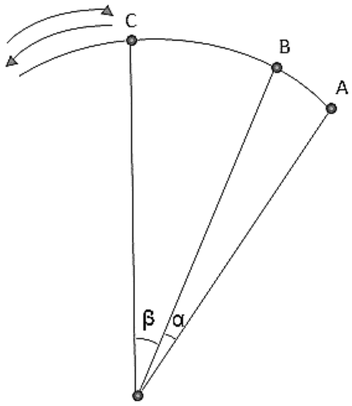
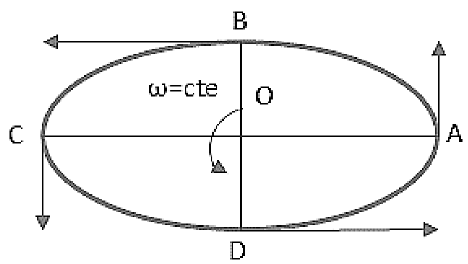



Fig. 1




ig. 2

In this paper, the angle the moving object covers in time unit is γ. That is, the moving object moves on the curve from point A to point B in time unit thus equals γ (Fig.3)

ω = (γ.t)

t = 1

ω = (γ.1) = (γ)

In triangle O'CH we have :

γ + β + π – α = π

γ + β = α

γ = α – β

N.B. : Angle γ may be γ = α ±β , depending on codirectional or counterdirectional speed .

$V_{xA} = {}^{m}/_{sec}$: Speed on the abscissa  at point A

$V_{yA} = {}^{m}/_{sec}$: Speed on the ordinate  at point B

$V_{xB} = {}^{m}/_{sec}$: Speed on the abscissa  at point B

$V_{yB} = {}^{m}/_{sec}$: Speed on the ordinate at point B

Its proved formula is discussion and proof .

$\tan\omega = \frac{V_{yA}.V_{xB} - V_{xA}.V_{yB}}{V_{xA}.V_{xB} + V_{yA}.V_{yB}}$discussion and proof:

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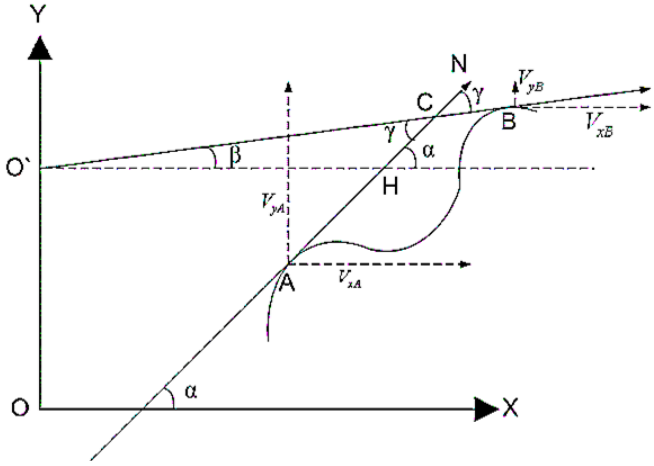


$$\tan\propto = \frac{V_{yA}}{V_{xA}}$$

$$\tan\beta = \frac{V_{yB}}{V_{xB}}$$

$$\tan\gamma = \tan(\propto - \beta)$$

$$\tan\gamma = \tan\omega = \frac{\tan\propto - \tan\beta}{1 + \tan\propto . \tan\beta} = \frac{\frac{V_{yA}}{V_{xA}} - \frac{V_{yB}}{V_{xB}}}{1 + \frac{V_{yA} . V_{yB}}{V_{xA} . V_{xB}}}$$

$$\tan\omega = \frac{V_{yA} . V_{xB} - V_{xA} . V_{yB}}{V_{xA} . V_{xB} + V_{yA} . V_{yB}}$$

In Fig.3 , if the moving object moves in the next second from point B to a hypothesized point like D , the moving object angular speed in the next second will be defined , and the formula of angular speed in the next second between 2 point D and B shall apply according to the formula presented between 2 point B and A . It is noteworthy that the above mentioned formula has been proved for each time unit. For example , the moving object is at second ($t_n$) at a point like A on the curve , and at second ($t_n$ + 1) at a point like B. The above mentioned proved formula thus applies between 2 point in time units ($t_n$) and ($t_n$ + 1). It accordingly applies between 2 points D and B at second ($t_n$ + 1) and ($t_n$ + 2) , etc.

Given an angular speed larger than 1, the angle covered has an average value and average angular speed is a separate option which will be described later.

Example : A moving object covers in the 1st second (α = $\frac{\pi}{4}$ ) and in the next second (β = $\frac{\pi}{6}$ ) .

For obtaining an average angular speed , angles (α) and (β) are added and then divided by 2

$$\frac{(\propto + \beta)}{2}$$

The average angular speed will thus be obtained.

$$\frac{(\frac{\pi}{4} + \frac{\pi}{6})}{2} = \frac{20\pi}{96} = \frac{5\pi}{24}$$

ISSN 2347-1921

It can now be concluded that angular speed is different than average angular speed , that nobody can obtain angular speed from average angular speed and that either of them is a separate option . Angular speed for 2 consecutive points , one second apart , has been proved . That is at the time between ($t_n$) and ($t_n$ + 1) , angular speed will be obtained. The formula proved on an open curve also applies to a closed curve . Using an example , the accuracy of the above formula on a closed curve can be proved.

C. Examples are solved, starting with the simplest ones at constant and variable angular speeds and constant and variable relations in circles to clarify angular speed different aspects in a curve.

Ex.1. The movement of 2 arms $\{\overline{OA} = \overline{AB}\}$ at 2 codirectionally equal angular speeds (ωOA = ωAB) are considered

Angular speed : $\begin{cases}\text{constant}\\ \text{variable}\end{cases}$

Length of arm : $\begin{cases}\text{equal}\\ \text{nonequal}\end{cases}$

Direction of movement : $\begin{cases}\text{codirectional}\\ \text{counter directional}\end{cases}$

Ex.2 . Solve the 1st EM . counterdirectional relative to each other

Ex.3 . Solve Ex.1 given the following assumptions codirectional angular speed ωOA = ωAB , $\overline{OA} = \overline{2AB}$

Ex.4 . Solve Ex.1 given the following assumptions counterdirectional angular speed ωOA = ωAB , $\overline{OA} = \overline{2AB}$

Ex.5 . Solve Ex.1 given the following assumptions codirectional angular speed 2. ωOA = ωAB , $\overline{OA} = \overline{AB}$

Ex.6 . Solve Ex.1 given the following assumptions counterdirectional angular speed 2. ωOA = ωAB , $\overline{OA} = \overline{AB}$

Ex.7 . Solve Ex.1 given the following assumptions codirectional angular speed 2. ωOA = ωAB , $\overline{OA} = \overline{2AB}$

Ex.8 . Solve Ex.1 given the following assumptions codirectional angular speed 2. ωOA = ωAB , $\overline{OA} = \overline{2AB}$

Calculation of angular speed at (t) seconds in such problems , the following method can be practiced :

In Ex.3 , for calculating angular speed at t = 2 sec , using triangle $\widehat{OAB}$, length $\overline{OB}$ can be calculated.

In triangle $\widehat{OAB}$, 2 sides and an angle are known .That is lengths $\overline{OA}$ and $\overline{AB}$ and angle (π-2α) are known.

Angles (γ and β) will thus also be obtained. By obtaining angle (γ) , it can be added to angle 2α. Total angles ϴ = (γ + 2α) will thus constitute angular speed covered by point B within 2 seconds . That is ωOB = (γ + 2α) . For calculating other angular speeds at different times t , the above mentioned method can be applied.

Ex.9 . Calculation of angular speed at t seconds :

In Ex.9 for calculating angular speed , we shall proceed as in Ex.3 : Firstly , using the above mentioned method in triangle $\widehat{OAB}$ applying 2 relations of $\overline{OA}$ and $\overline{AB}$, angle (γ) is obtained and then in triangle $\widehat{OBC}$, by obtaining length $\overline{OB}$ and having length $\overline{BC}$ and angle $\widehat{ABC}$, 2 angles and the other side of triangle $\widehat{OBC}$ can be obtained . By having an angle near point O (λ,θ), this angle can be added to its nearby angles , the total of which will constitute angular speed at time (t).

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**Ex 1**

Relations $\overline{OA} = \overline{AB}$are given. Point (A) has angular speed $\omega OA = \frac{\pi}{18}$relative to point (O) , and point B has angular speed $\omega AB = \frac{\pi}{18}$relative to point (A). 2 arms $\{\overline{OA} = \overline{AB}\}$moves towards the trigonometric circle. Please find :

1. Angular speed ωOB= ?trace its curve via drawing and calculating at different times.
2. Codirectional angular speed ωOA = ωAB ,$\overline{OA} = \overline{AB}$

$$\begin{cases} \omega OA = \omega AB = \frac{\pi}{18}\ \mathrm{Rad/Sec} \\ t = 1\ \mathrm{sec} \\ \overline{OA} = \overline{AB} \\ \omega OB = ? \\ \alpha = \frac{\pi}{18}\ \mathrm{Rad} \end{cases}$$

$$\hat{OAB} \Rightarrow (\pi - \alpha) + \gamma + \gamma = \pi \Rightarrow -\alpha + 2\gamma = 0 \Rightarrow \gamma = \frac{\alpha}{2}$$

ωOB = α + γ = α +$\frac{\alpha}{2} = \frac{3}{2}\alpha = \frac{3}{2} \cdot \frac{\pi}{18} = \frac{\pi}{12}$

t=1 sec.

…..

ωOB= 3α + $\frac{3}{2}$ α = $\frac{9\alpha}{2} = \frac{9}{2} \cdot \frac{\pi}{18} = \frac{\pi}{4}$

t=3 sec.

…..

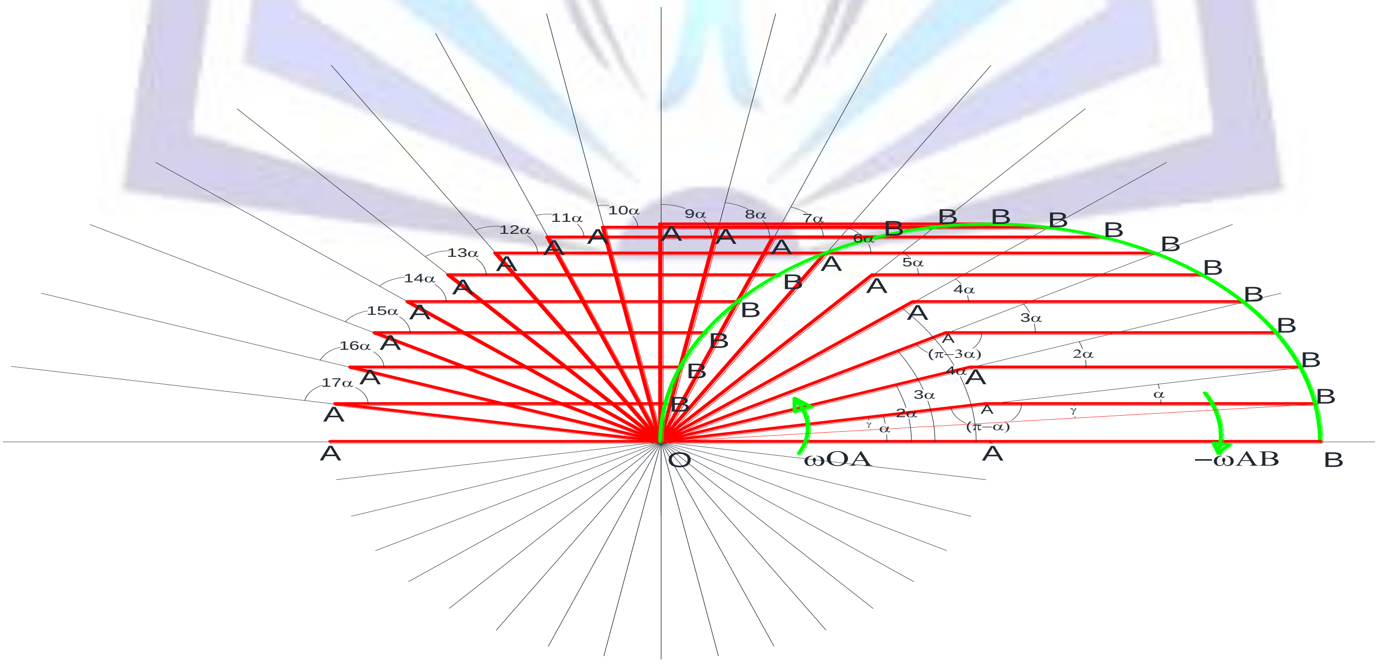
.....

ωOB = n α +$\frac{n}{2}$ α = $\frac{2n\alpha + n\alpha}{2}$

t=n sec

ωOB = nα + $\frac{n}{2}$ α = nα ($\frac{3}{2}$) $= n \cdot \frac{\pi}{18} \cdot \frac{3}{2}$

t= n sec.



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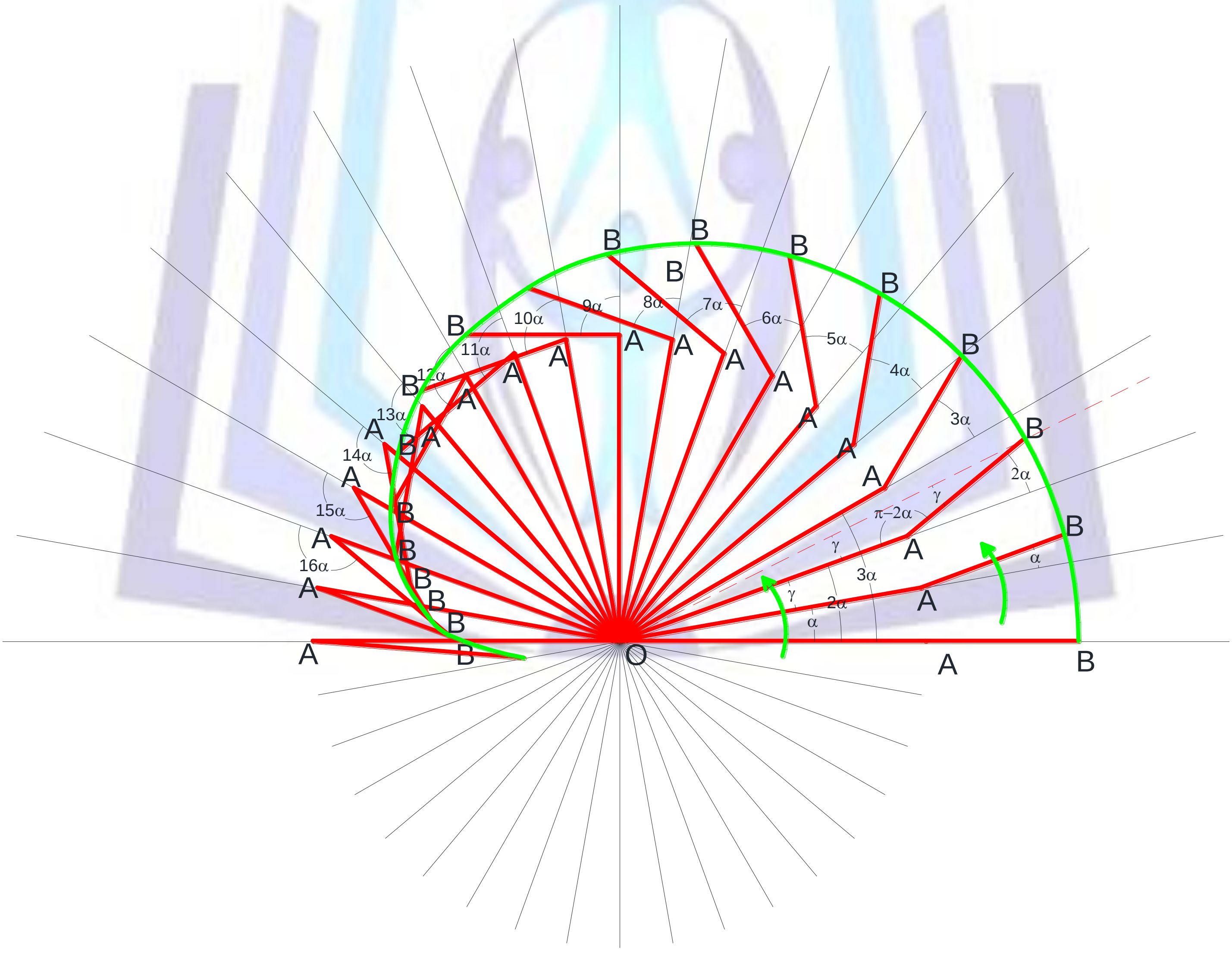
**Ex 2**

**Codirectional angular speed**

$$\begin{cases} \omega OA = \omega AB = \dfrac{\pi}{18} \text{Rad/Sec} \\ t = 1 \text{ sec} \\ \overline{OA} = 2.\overline{AB} \\ \omega OB = ? \\ \alpha = \dfrac{\pi}{18} \text{ Rad} \end{cases}$$

$$\overline{OB}^2 = \overline{OA}^2 + \overline{AB}^2 - 2.\overline{OA}.\overline{AB}.\mathrm{Cos}(\pi - 2\alpha)$$

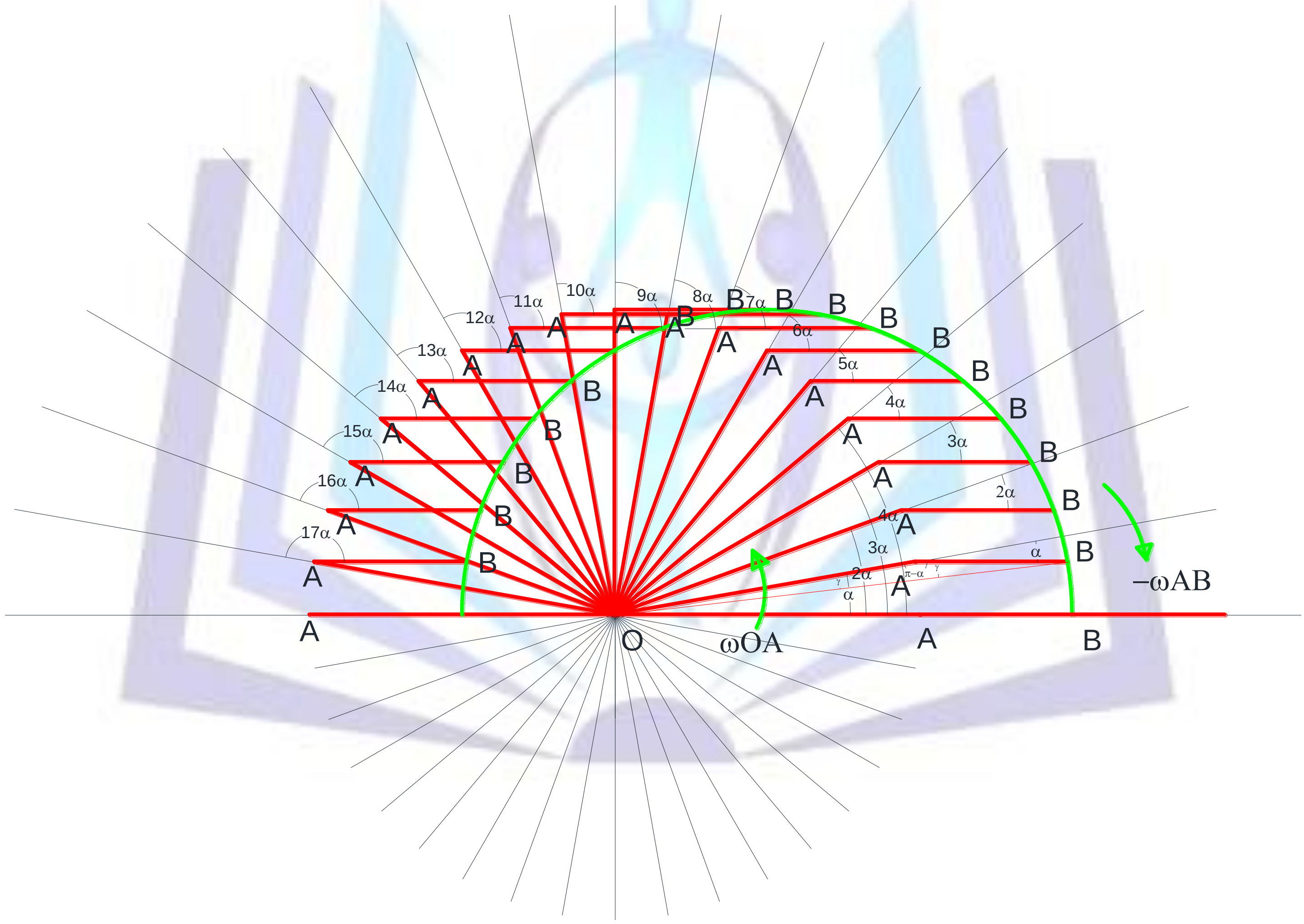


**Ex 3**

**Counterdirectional angular speed**

$$\begin{cases} \omega OA = -\omega AB = \dfrac{\frac{\pi}{18}\,\mathrm{Rad}}{\mathrm{Sec}} \\ t = 1\,\mathrm{sec} \\ \overline{OA} = 2.\overline{AB} \\ \omega OB = ? \\ \alpha = \dfrac{\pi}{18}\ \mathrm{Rad} \end{cases}$$

$$\overline{OB}^2 = \overline{OA}^2 + \overline{AB}^2 - 2.\overline{OA}.\overline{AB}.\mathrm{Cos}(\pi - \alpha)$$



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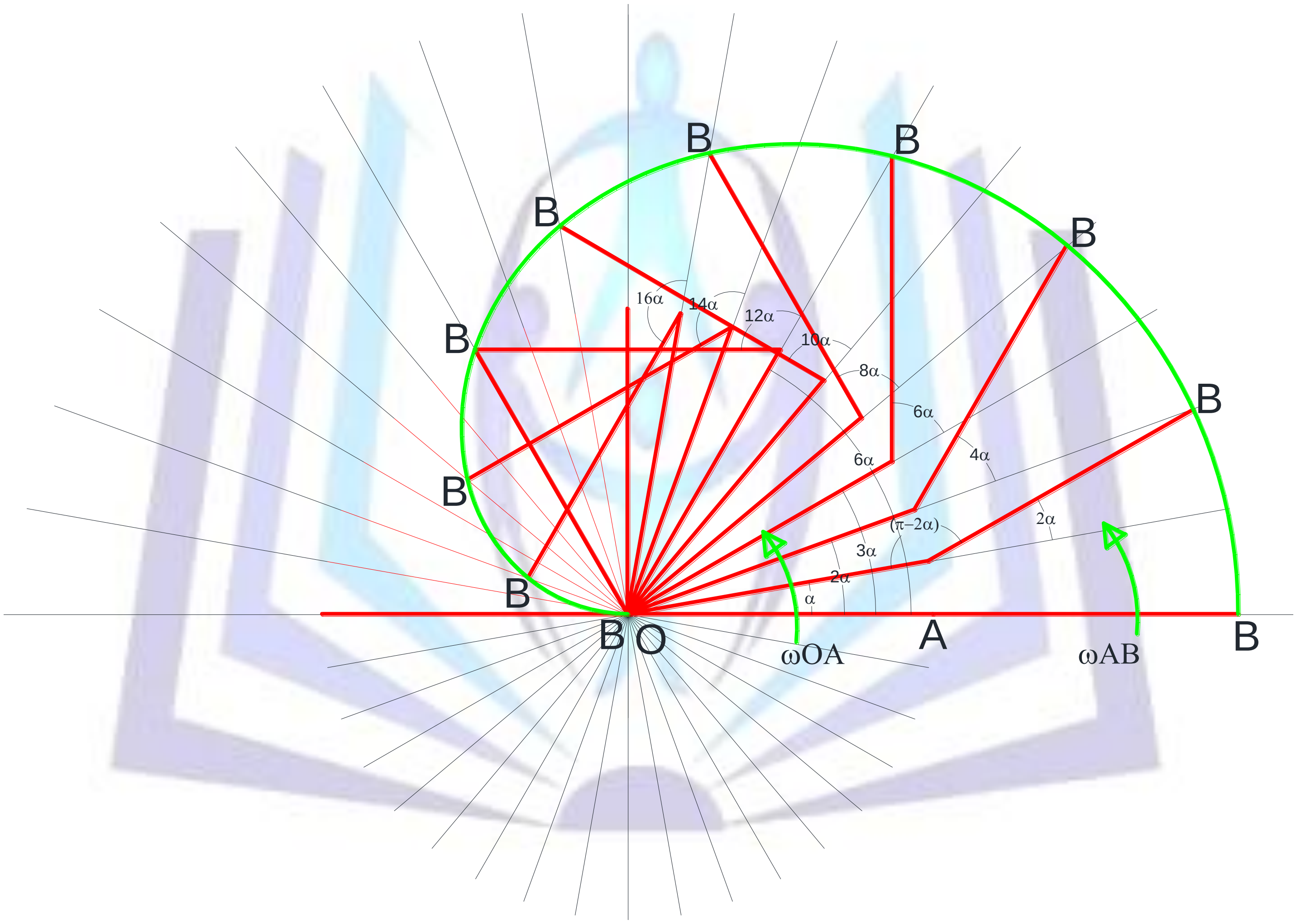
**Ex 4**

**Codirectional angular speed**

$$\begin{cases} \omega AB = 2.\omega OA \Rightarrow \omega OA = \dfrac{\pi}{18} \text{Rad/Sec} \\ t = 1\,\text{sec} \\ \overline{OA} = \overline{AB} \\ \omega OB = ? \\ \alpha = \dfrac{\pi}{18}\ \text{Rad} \end{cases}$$



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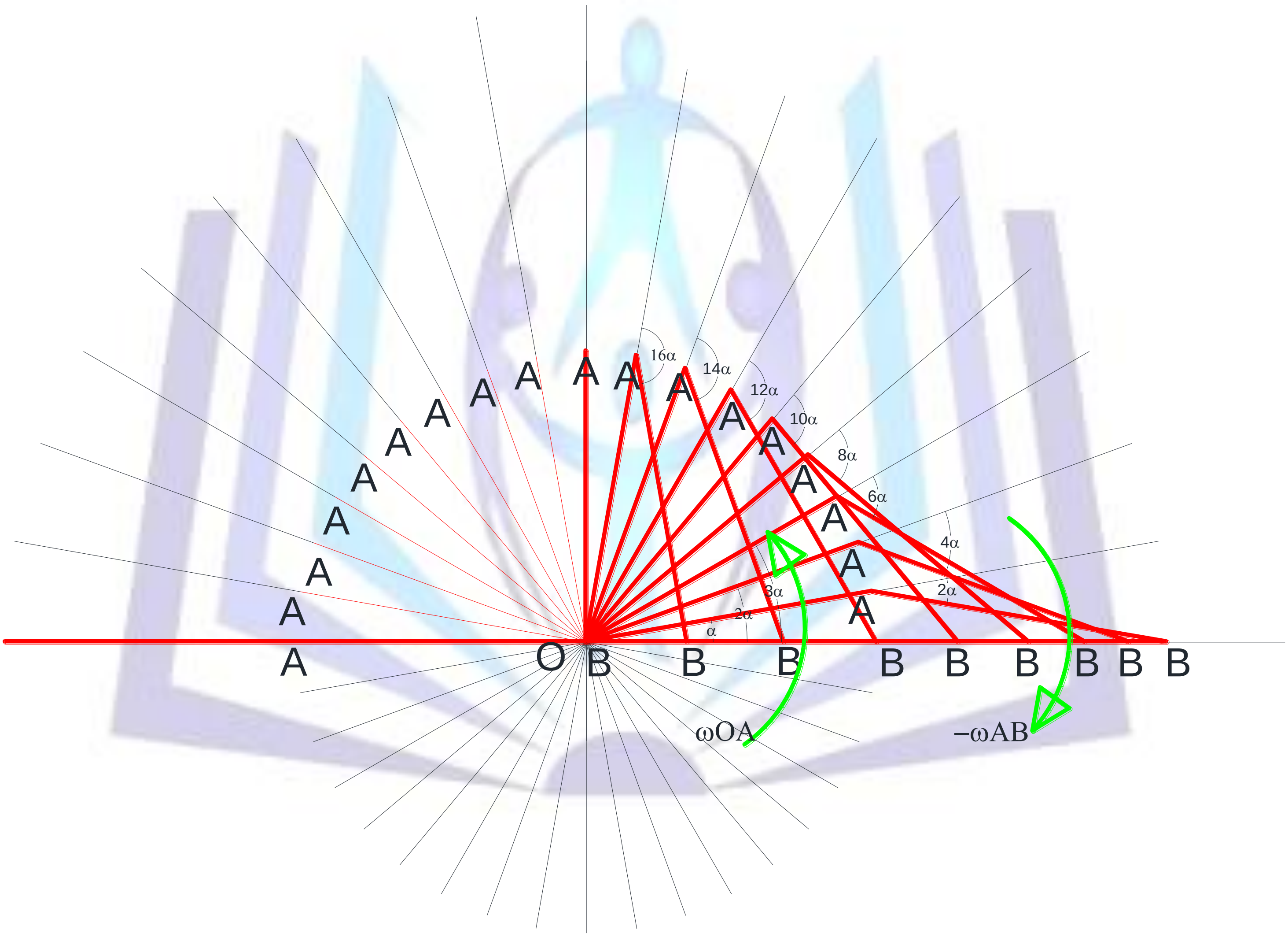
**Ex 5**

**Counterdirectional angular speed**

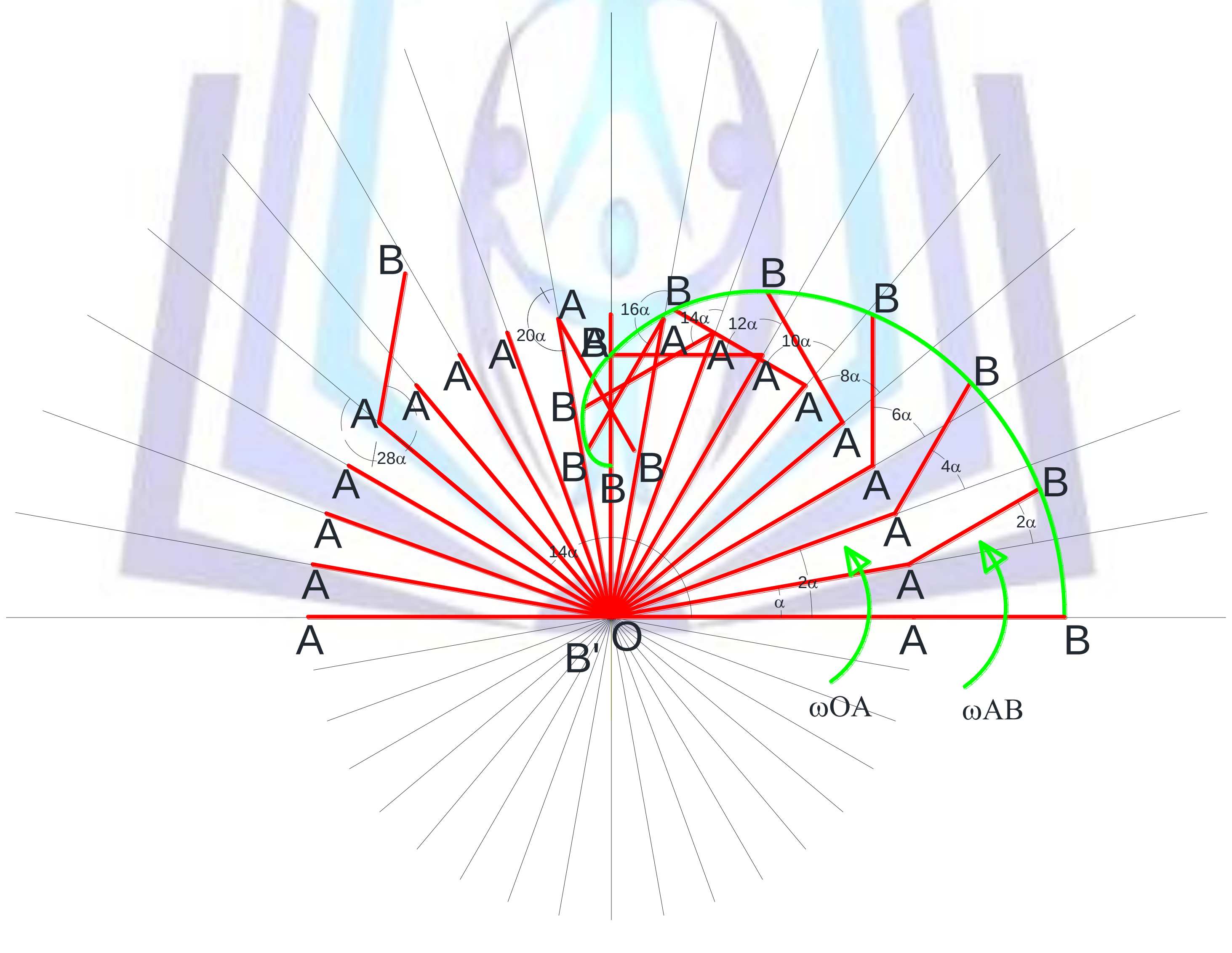
$$\begin{cases} -\omega AB = 2.\omega OA = \omega AO = \dfrac{\pi}{18} \text{Rad/Sec} \\ t = 1\,\text{sec} \\ \overline{OA} = \overline{AB} \\ \omega OB = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



**Ex 6**

**Codirectional angular speed**

$$\begin{cases} \omega AB = 2.\omega OA \Rightarrow \omega AB = \dfrac{\pi}{18} Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{2.AB} \\ \omega OB = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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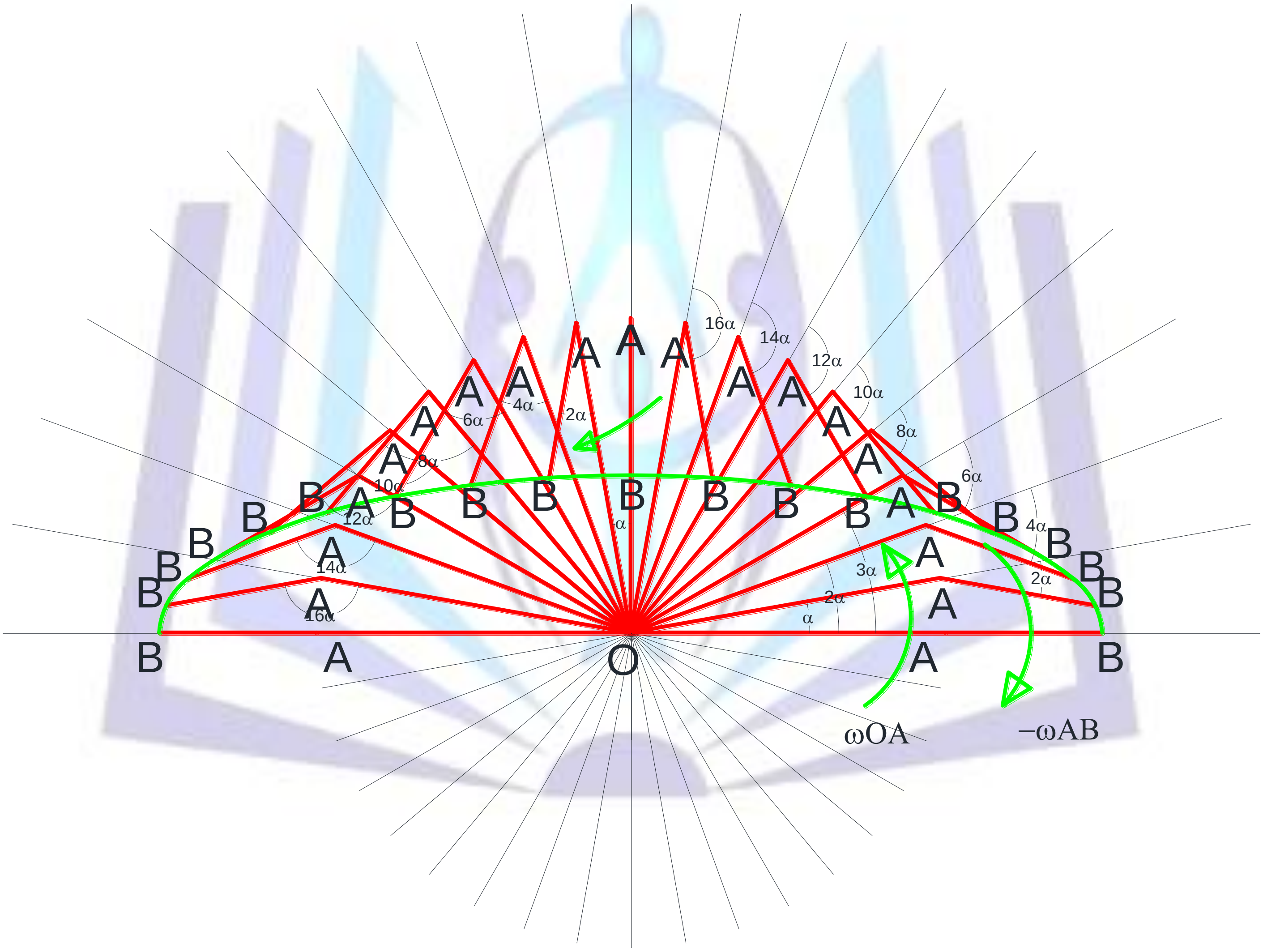
Ex 7

$$\begin{cases} \omega AB = 2.\omega OA \Rightarrow \omega OA = \dfrac{\pi}{18} Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{2.AB} \\ \omega OB = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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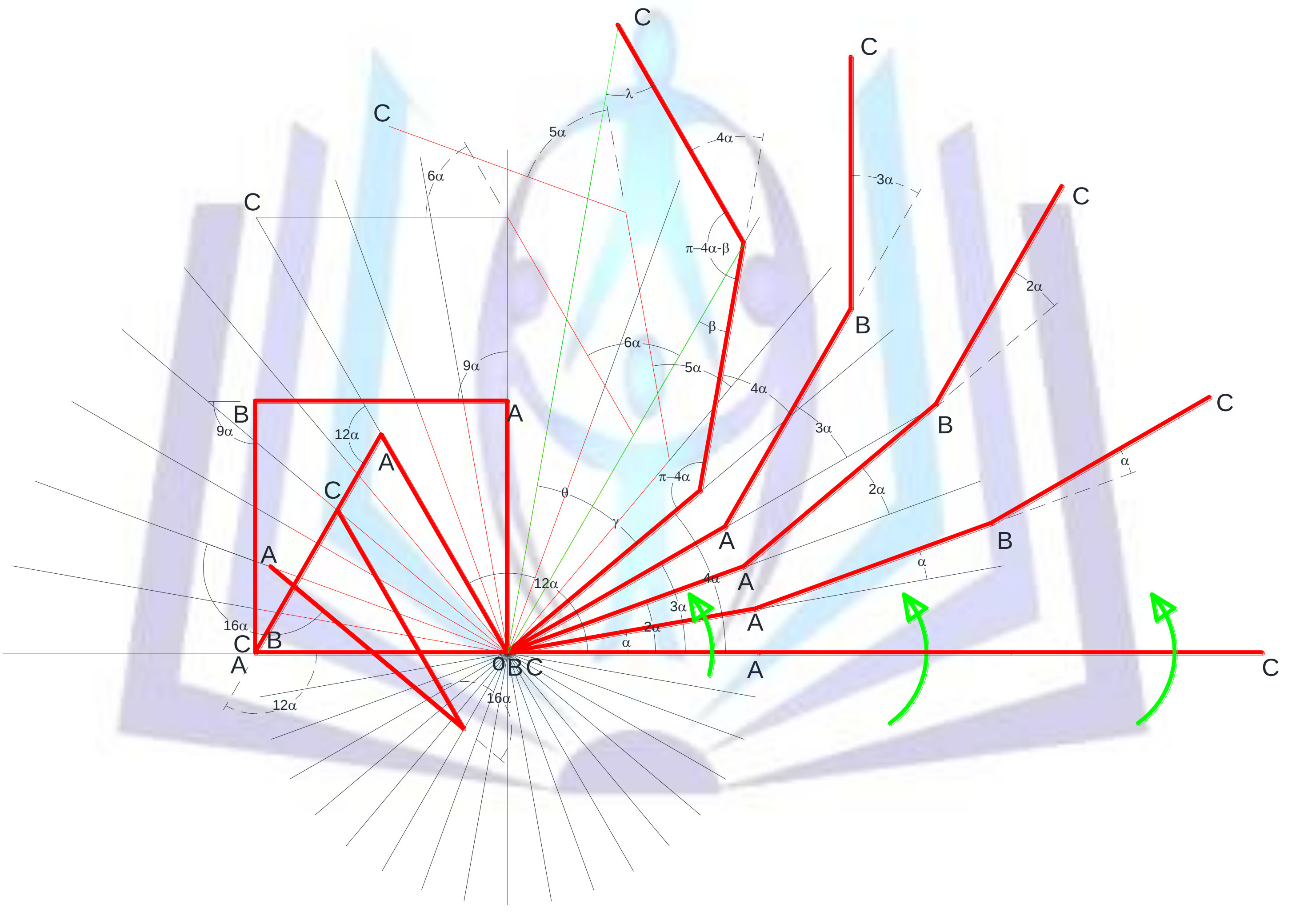
**Ex 8**

$$\begin{cases} \omega BC = \omega AB = \omega OA \Rightarrow \omega OA \dfrac{\pi}{18} Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$

$WOC = (4\alpha + \gamma + \theta)$

T=4 Sec



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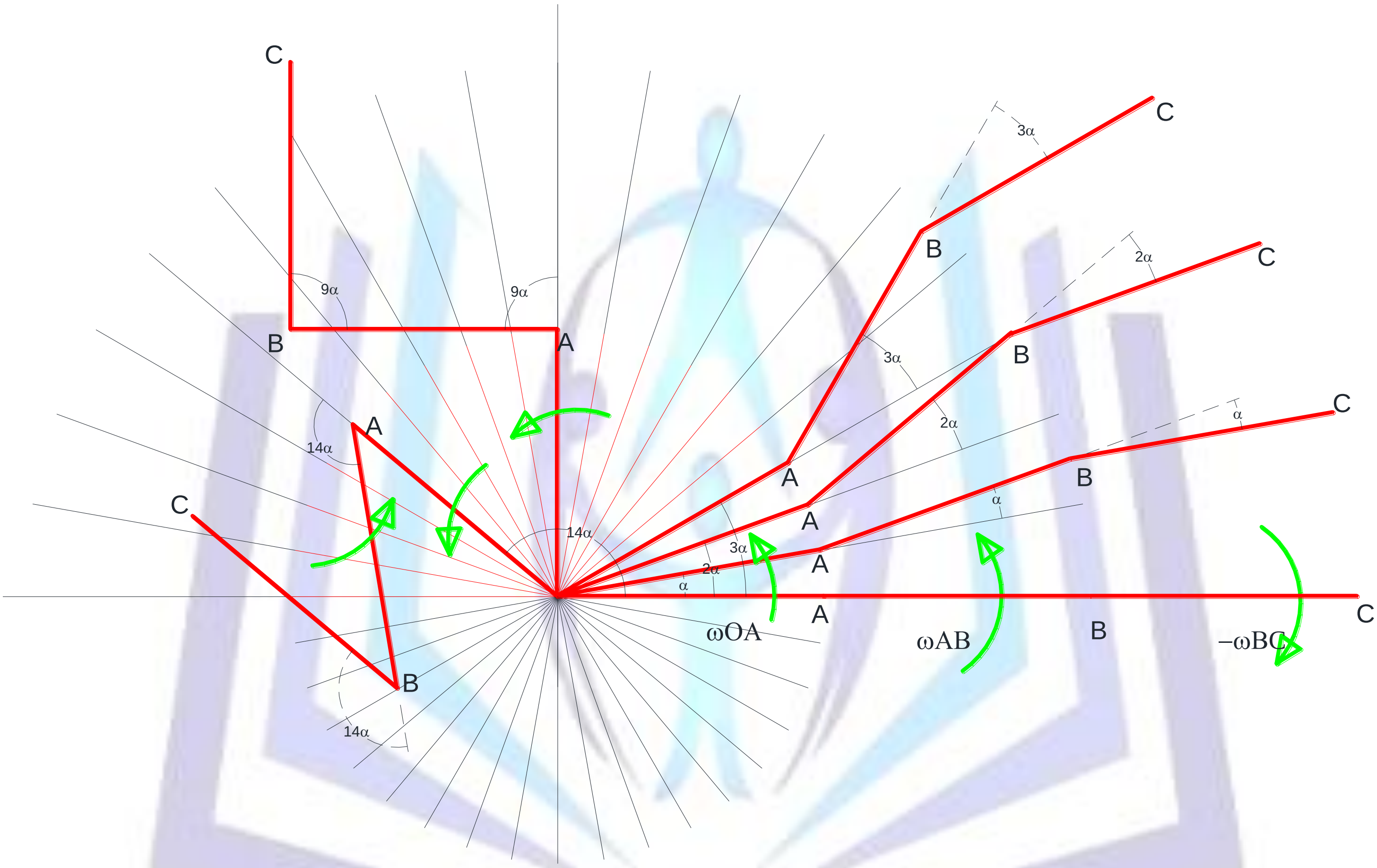
**Ex 9**

$$\begin{cases} -\omega BC = \omega AB = \omega OA \Rightarrow \omega OA \dfrac{\pi}{18} Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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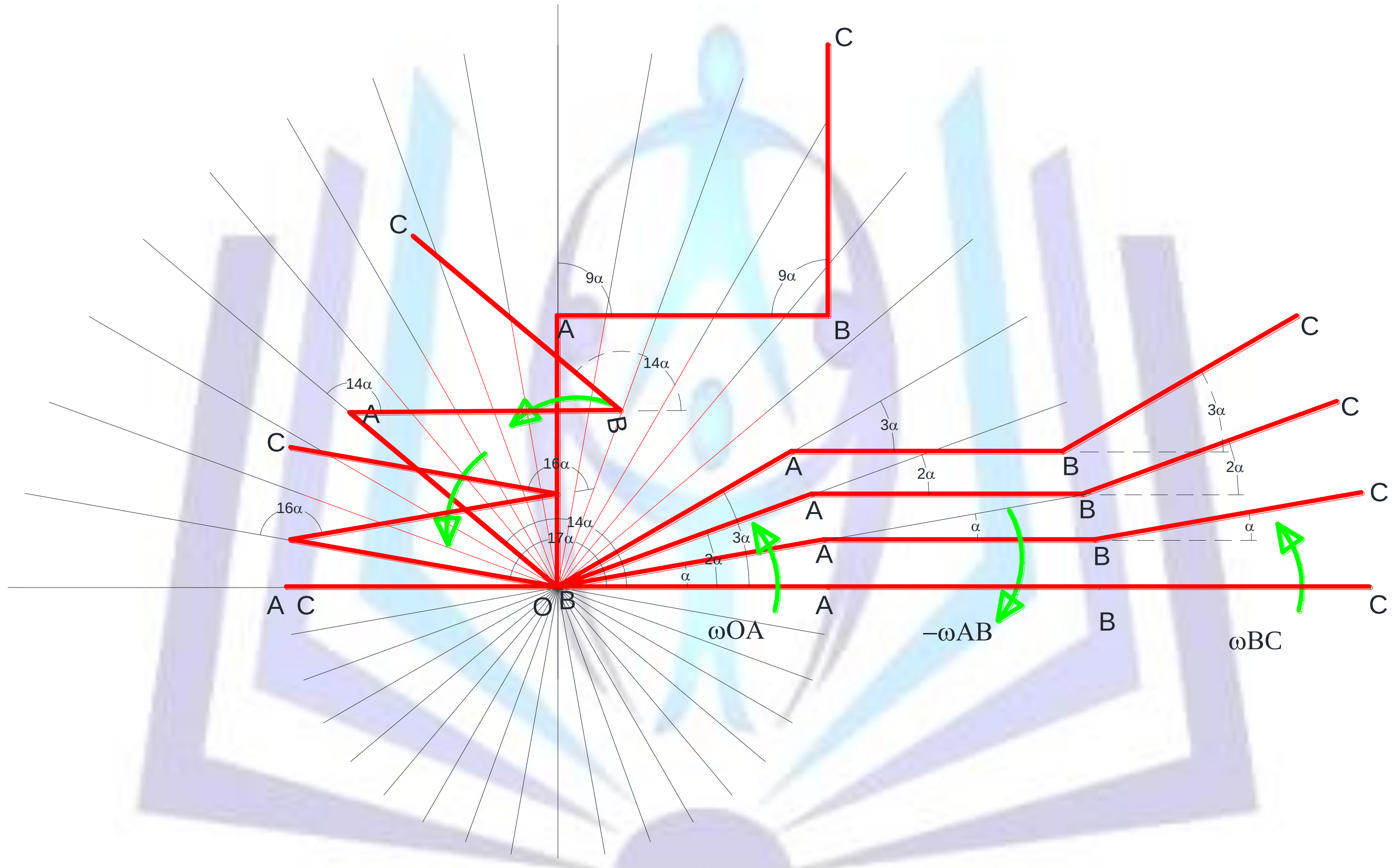
**Ex 10**

$$\begin{cases} \omega BC = -\omega AB = \omega OA \Rightarrow \omega OA \dfrac{\pi}{18} Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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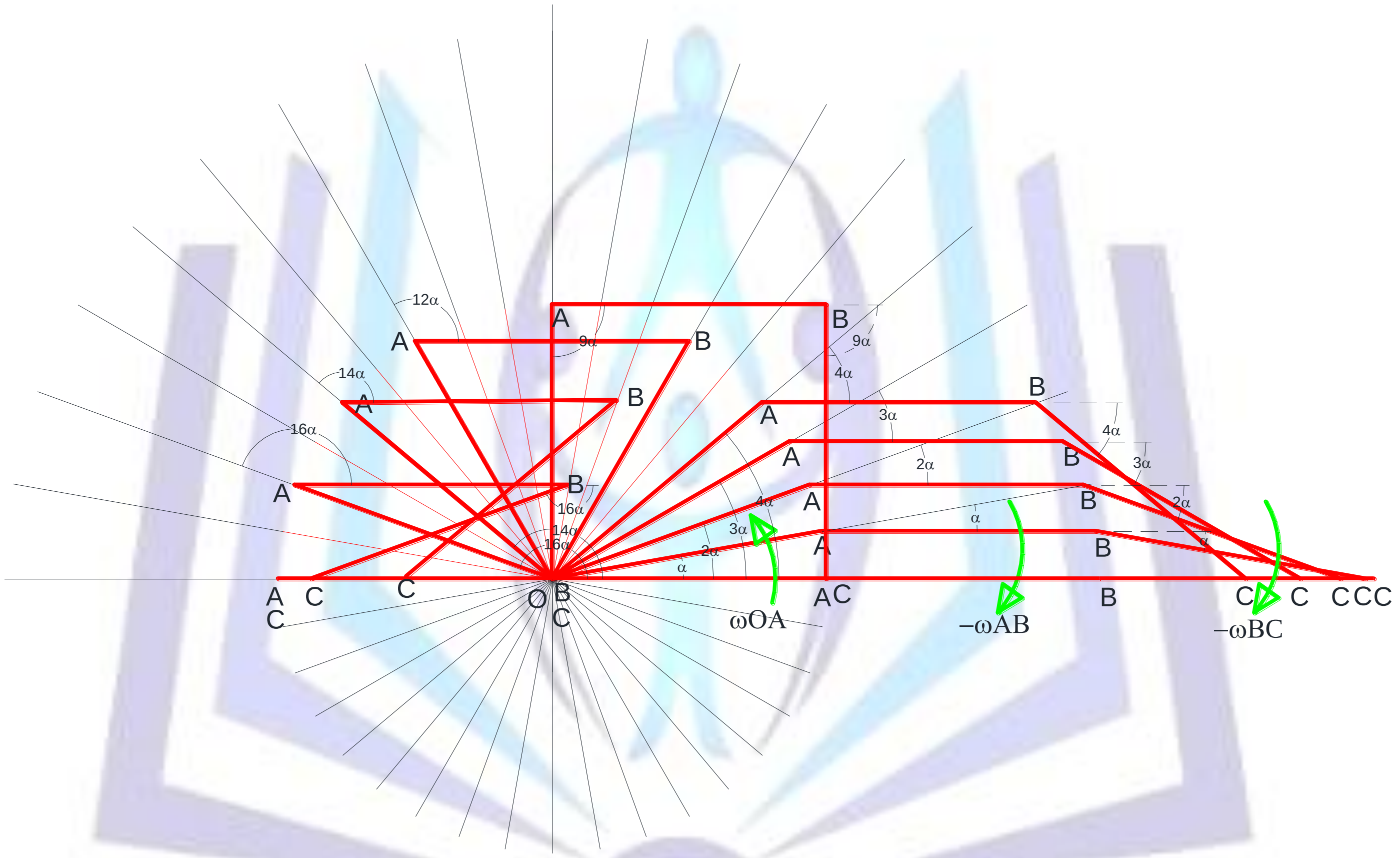
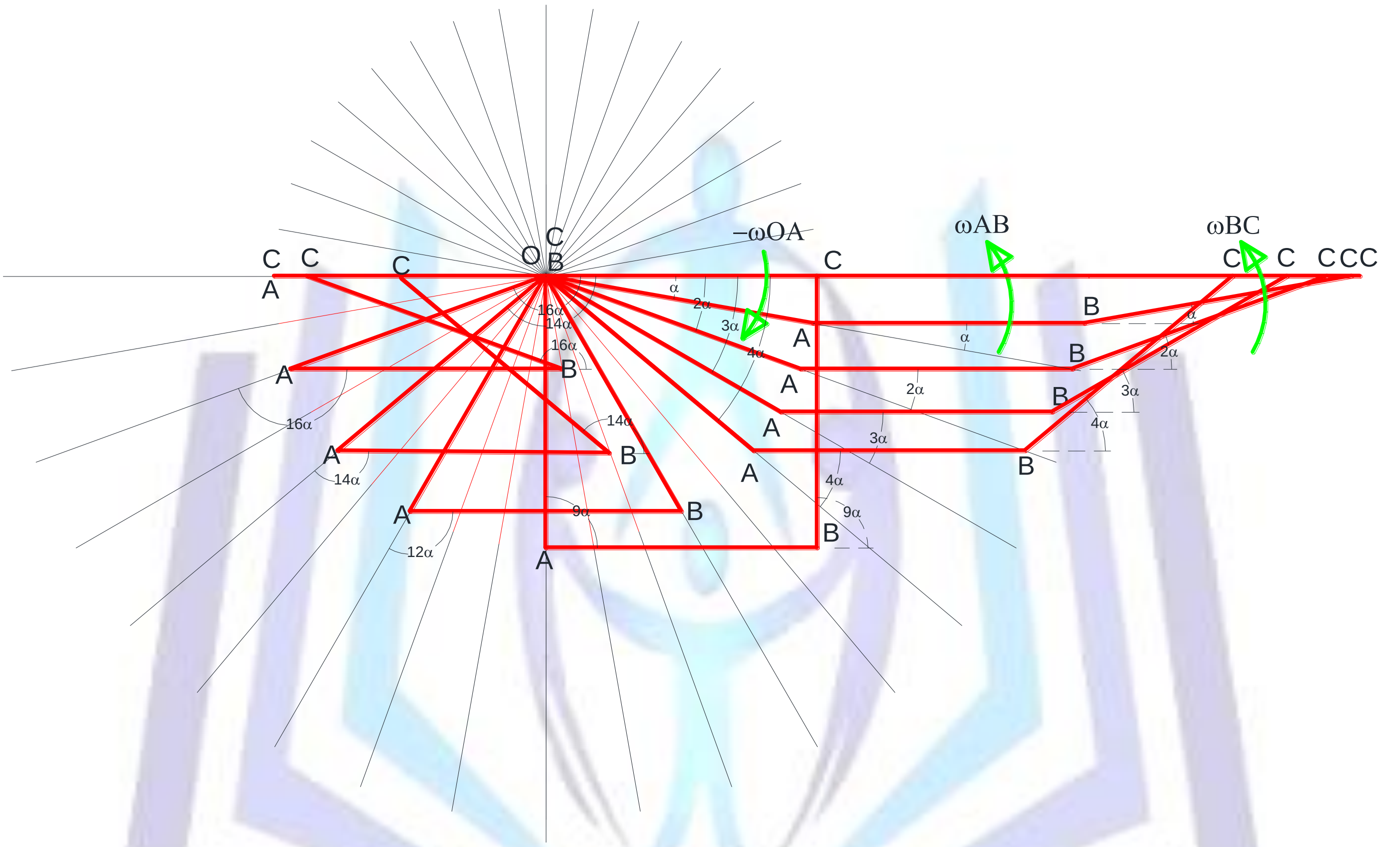
**Ex 11**

$$\begin{cases} -\omega BC = -\omega AB = \omega OA \Rightarrow \omega OA \dfrac{\pi}{18} Rad/Sec \\ t = 1\, sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



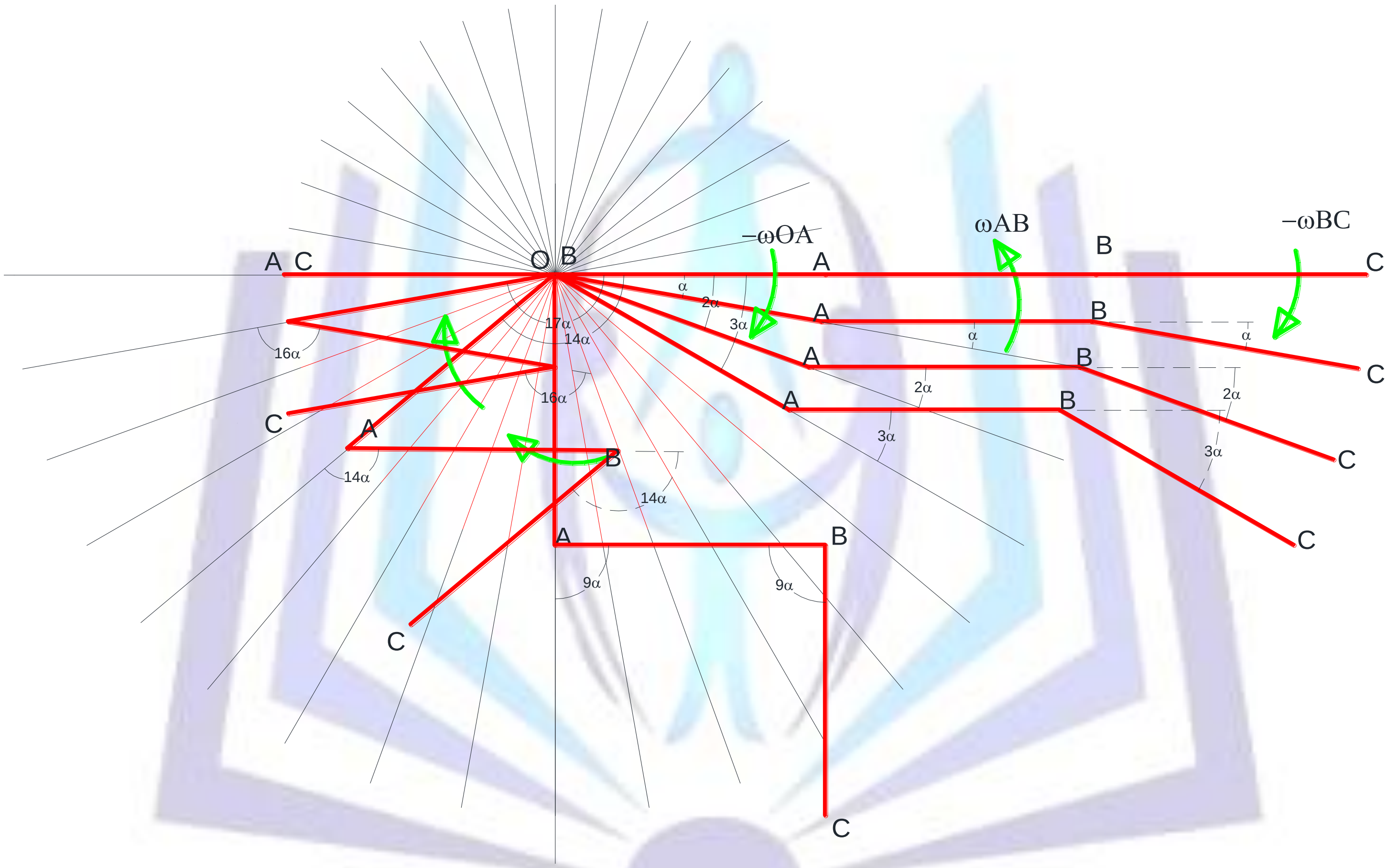
**Ex 12**

$$\begin{cases} \omega BC = \omega AB = -\omega OA \Rightarrow \omega OA = \dfrac{-\pi}{18} Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



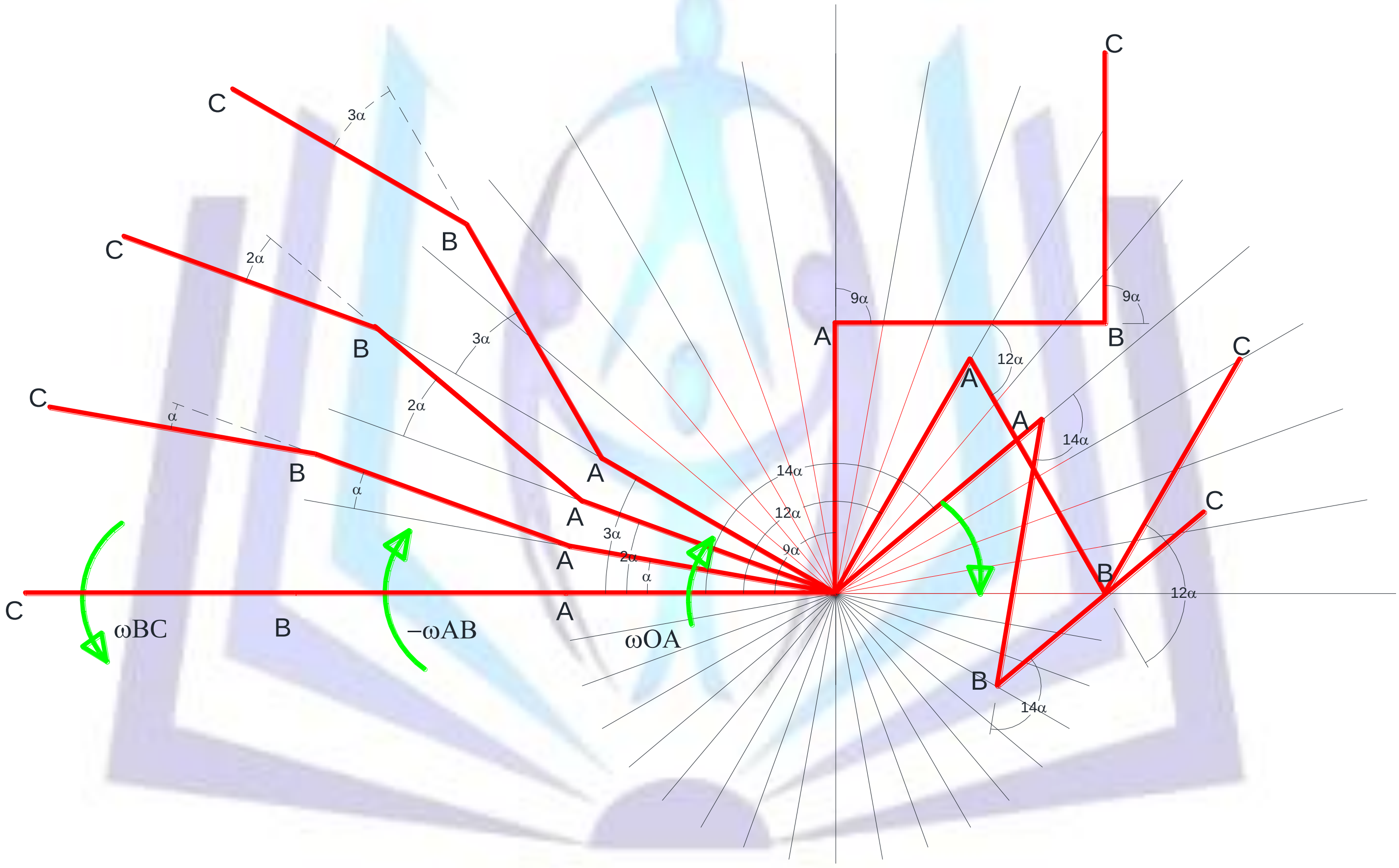
**Ex 13**

$$\begin{cases} -\omega BC = \omega AB = -\omega OA \Rightarrow \omega OA = \dfrac{-\pi}{18} Rad/Sec \\ t = 1\ sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



**Ex 14**

$$\begin{cases} \omega BC = -\omega AB = -\omega OA \Rightarrow \omega OA = \dfrac{\pi}{18} Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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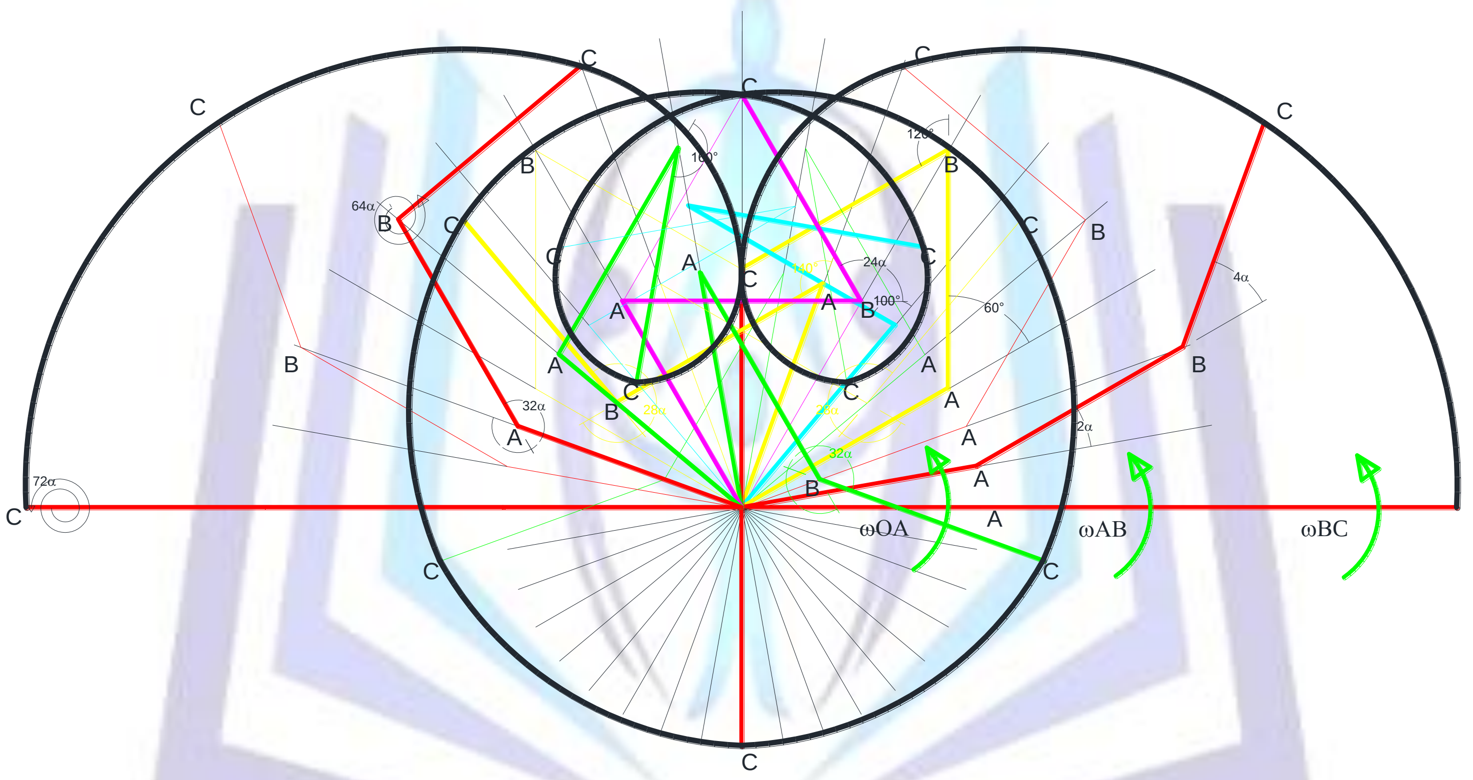
**Ex 15**

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18} Rad/Sec \\ t = 1\, sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \boldsymbol{\omega} BC = 2.\omega AB \ \Rightarrow \omega AB = 2.\omega OA \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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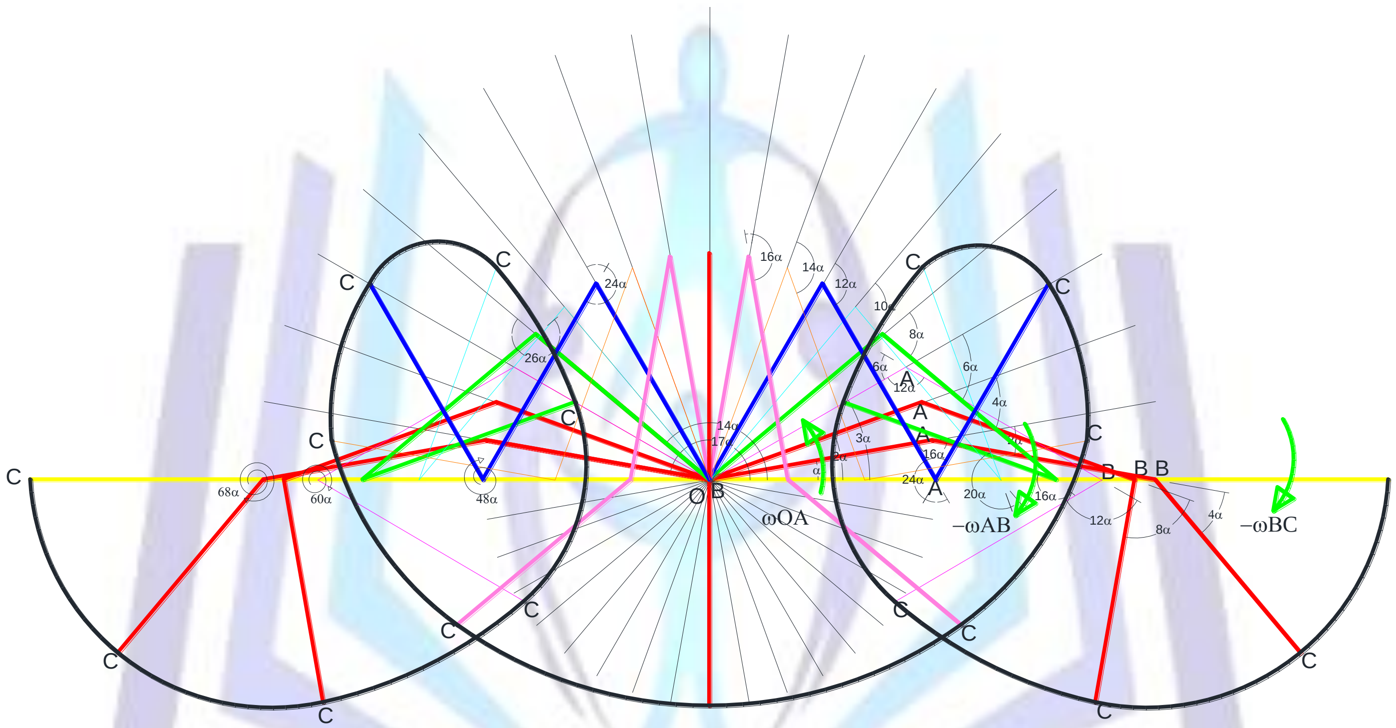
**Ex 16**

$$\begin{cases} \omega OA = \dfrac{\pi}{18} Rad/Sec \\ t = 1\, sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ -\omega BC = 2.\omega AB \Rightarrow 2.\omega AB = -\omega OA \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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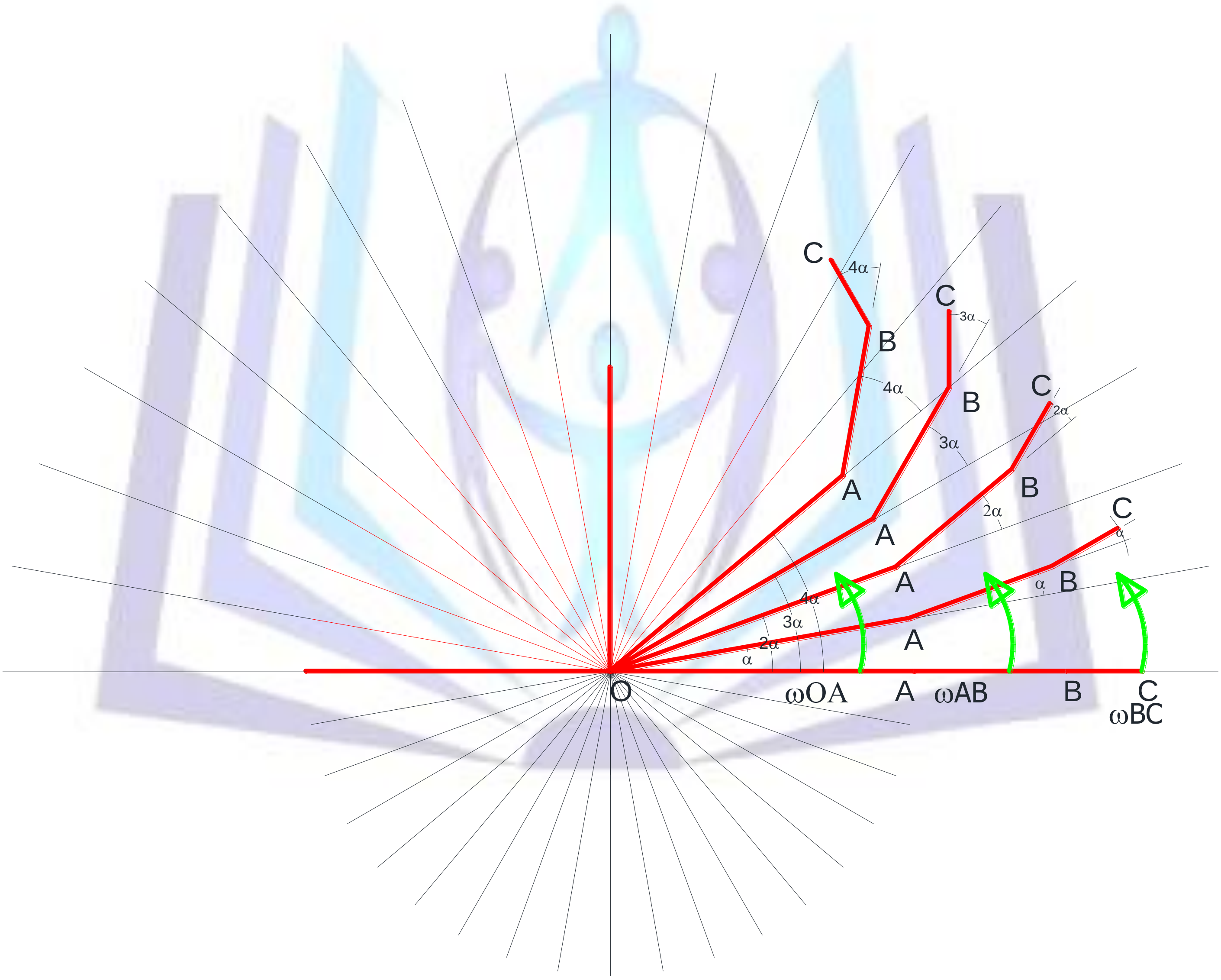
**Ex 17**

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18} Rad/Sec \\ t = 1\, sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ \omega BC = \omega AB = \omega OA \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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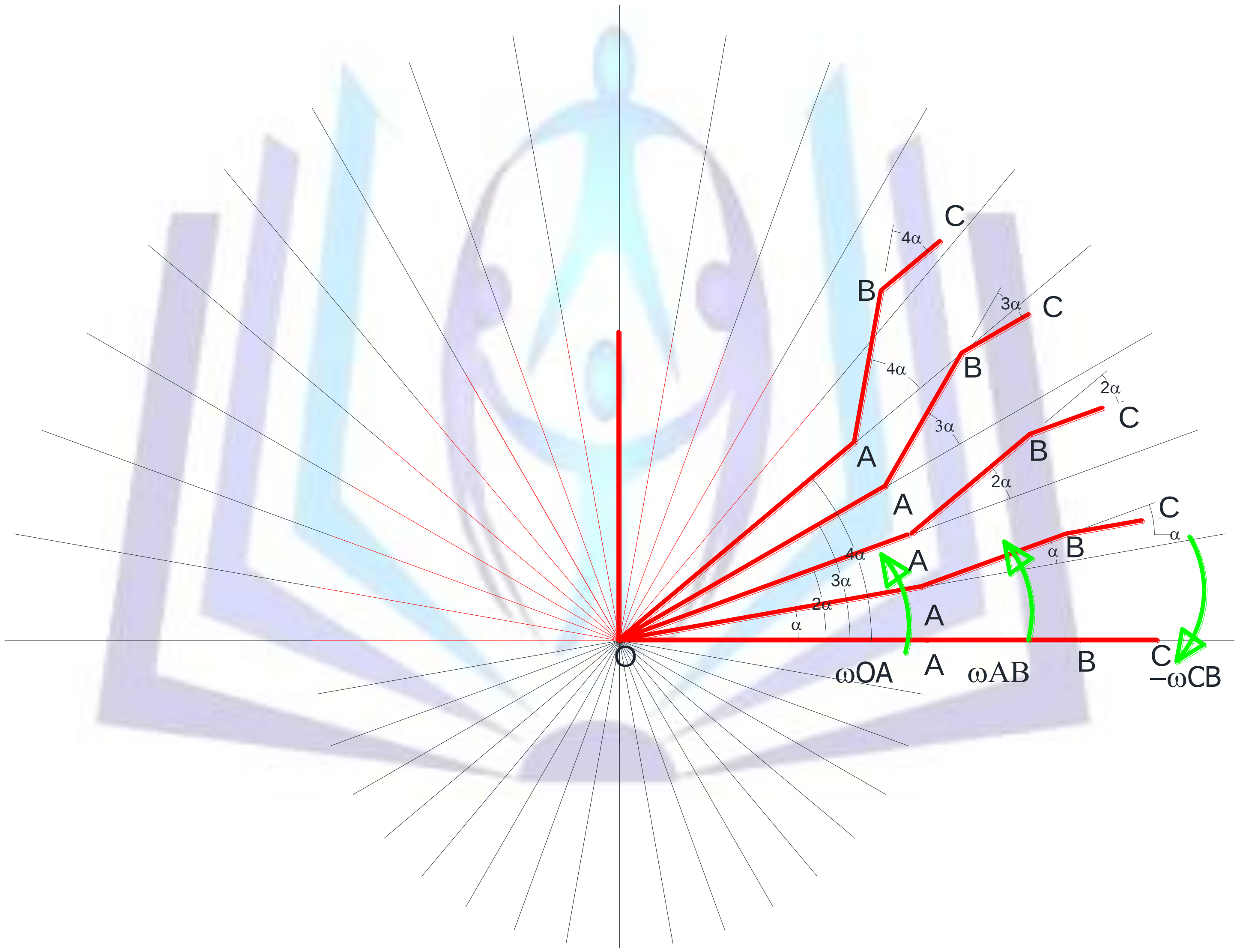
**Ex 18**

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18} Rad/Sec \\ t = 1\, sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ \omega BC = \omega AB = \omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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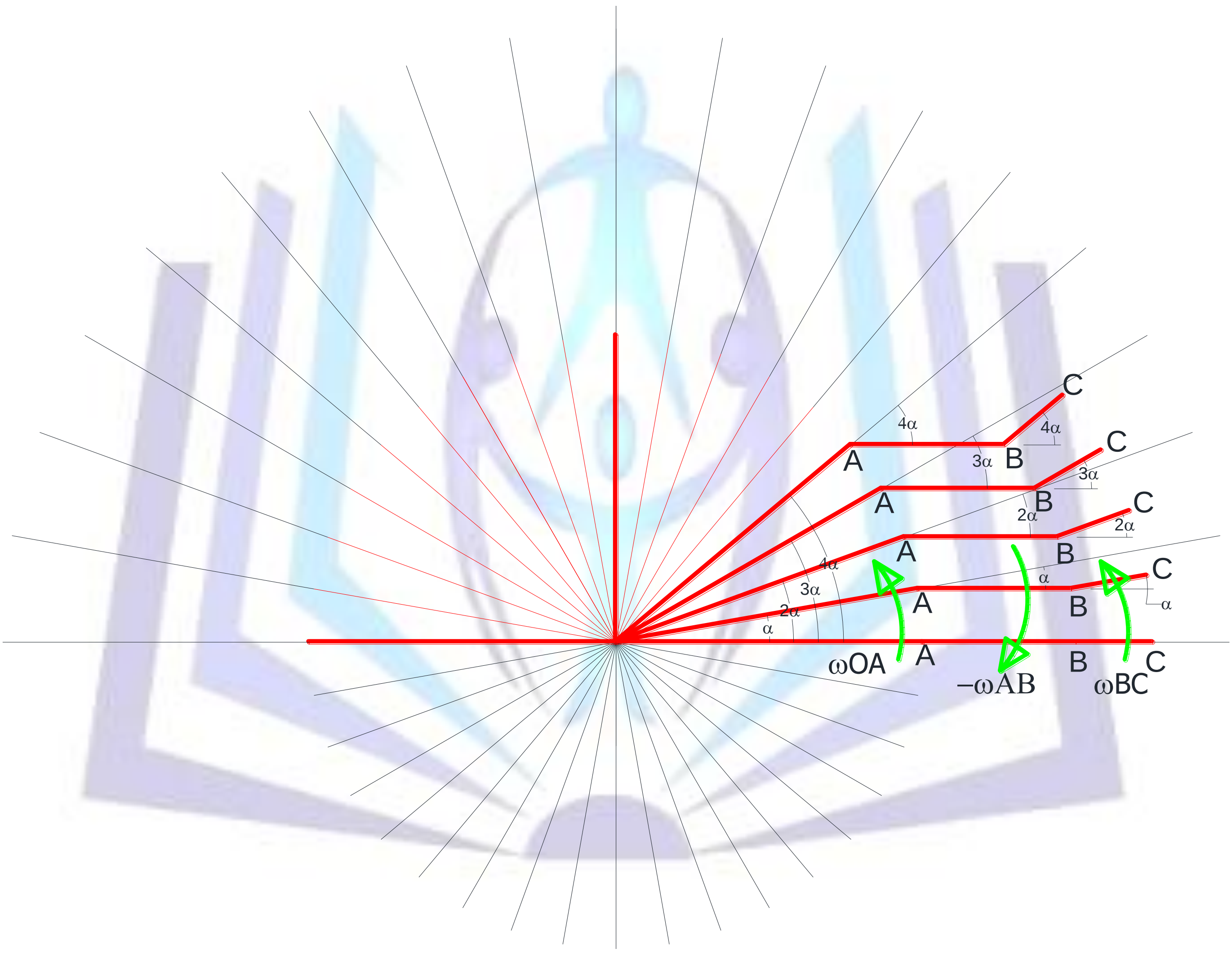
**Ex 19**

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18} Rad/Sec \\ t = 1\, sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ \omega OA = -\omega AB = \omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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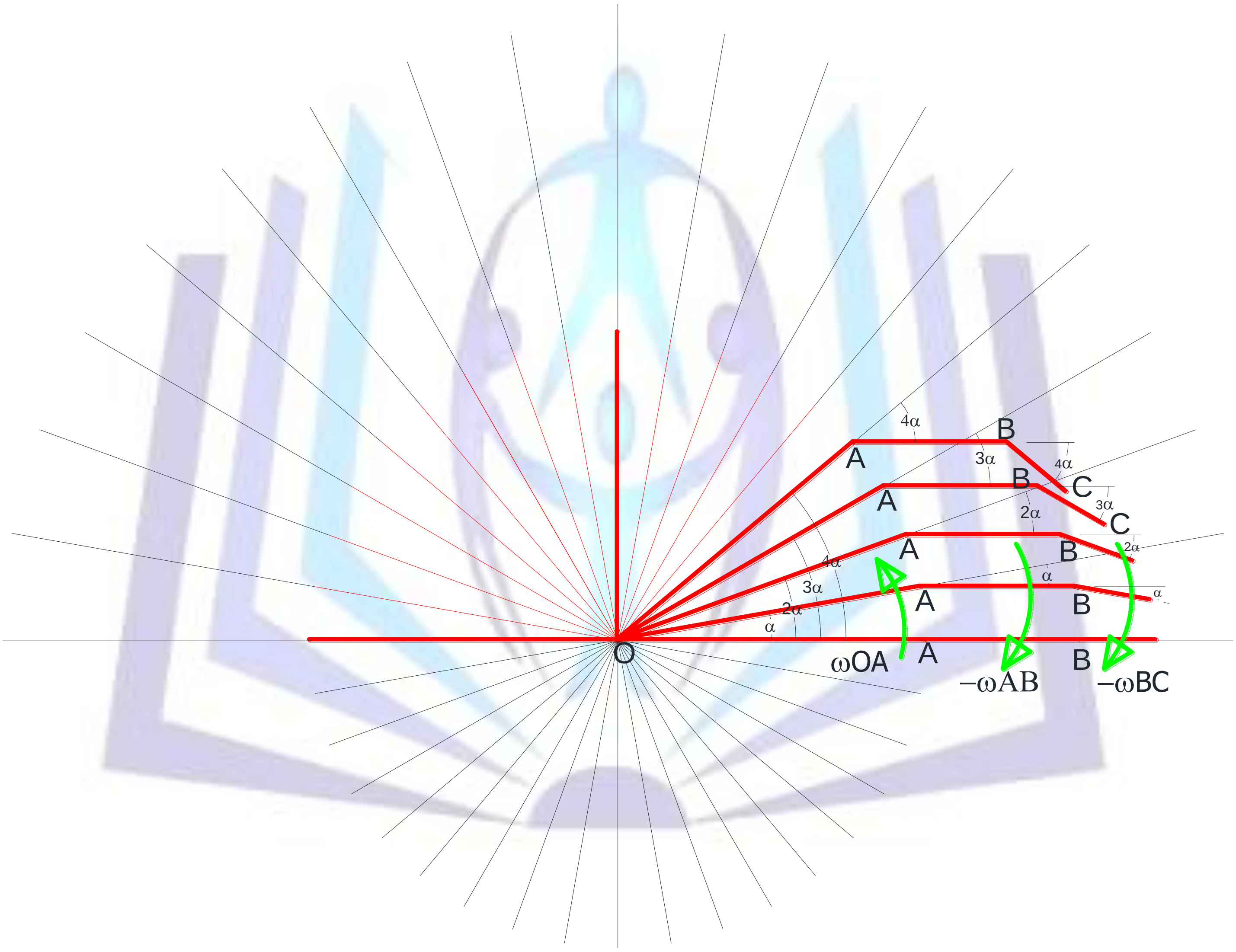
**Ex 20**

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18} Rad/Sec \\ t = 1\, sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ \omega OA = -\omega AB = -\omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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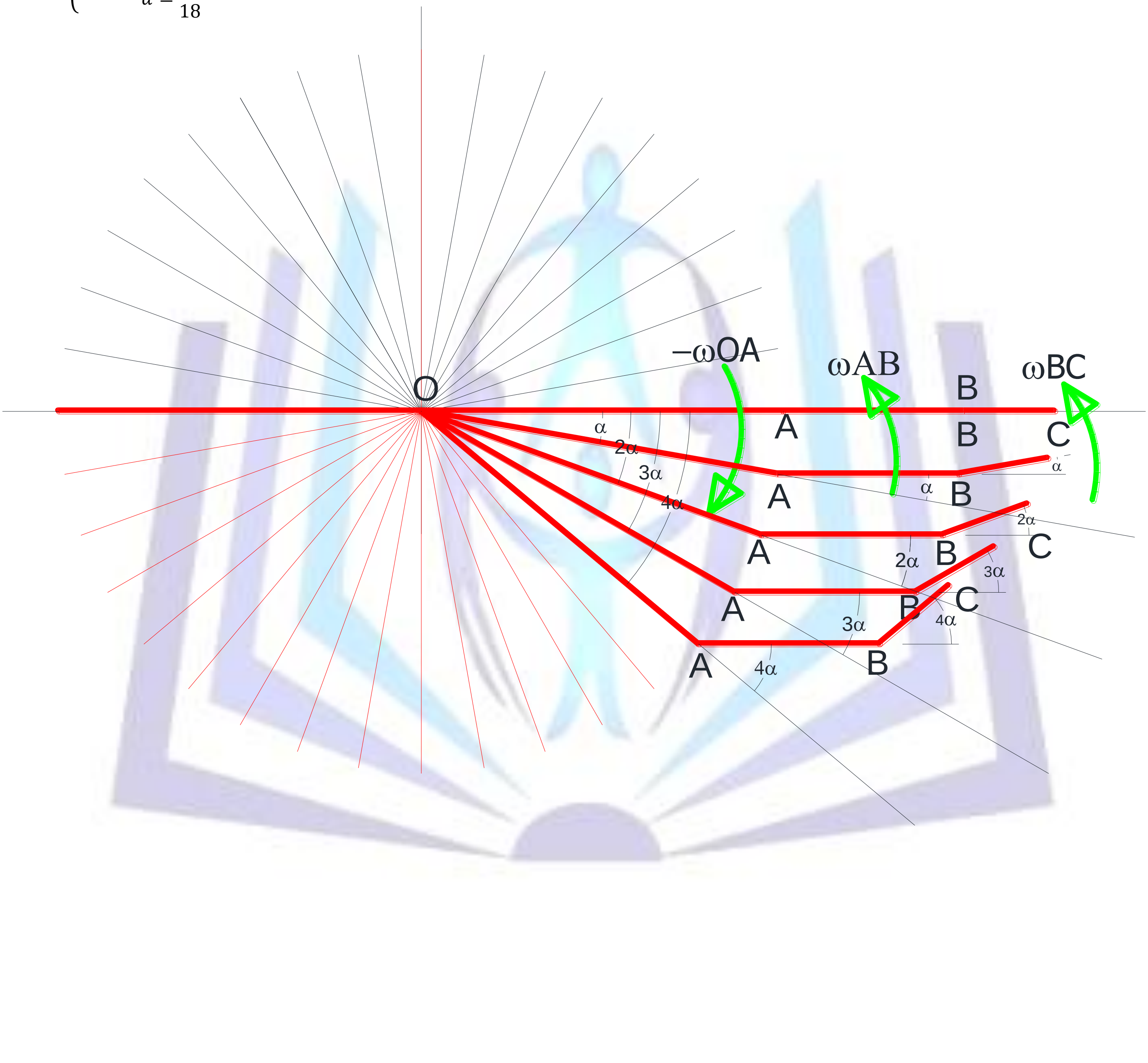
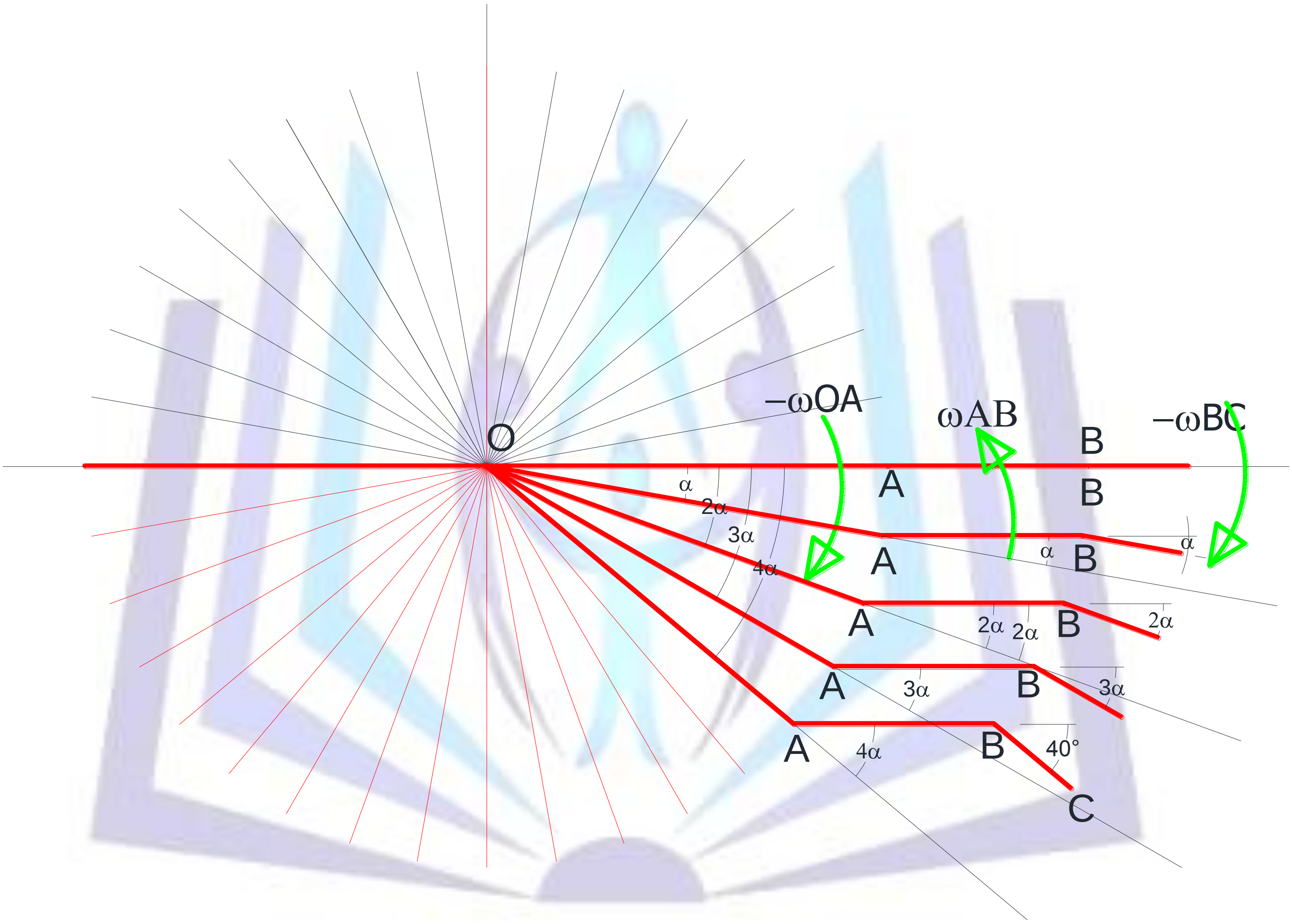
**Ex 21**

$$\begin{cases} \boldsymbol{\omega}OA = \dfrac{-\pi}{18} Rad\ /Sec \\ \quad t = 1\,sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ -\omega OA = \omega AB = \omega BC \\ \omega OC = ? \\ \quad \alpha = \dfrac{\pi}{18} \end{cases}$$



**Ex 22**

$$\begin{cases} \boldsymbol{\omega} OA = \frac{\pi}{18} Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ -\omega OA = \omega AB = -\omega BC \\ \omega OC = ? \\ \alpha = \frac{\pi}{18} \end{cases}$$



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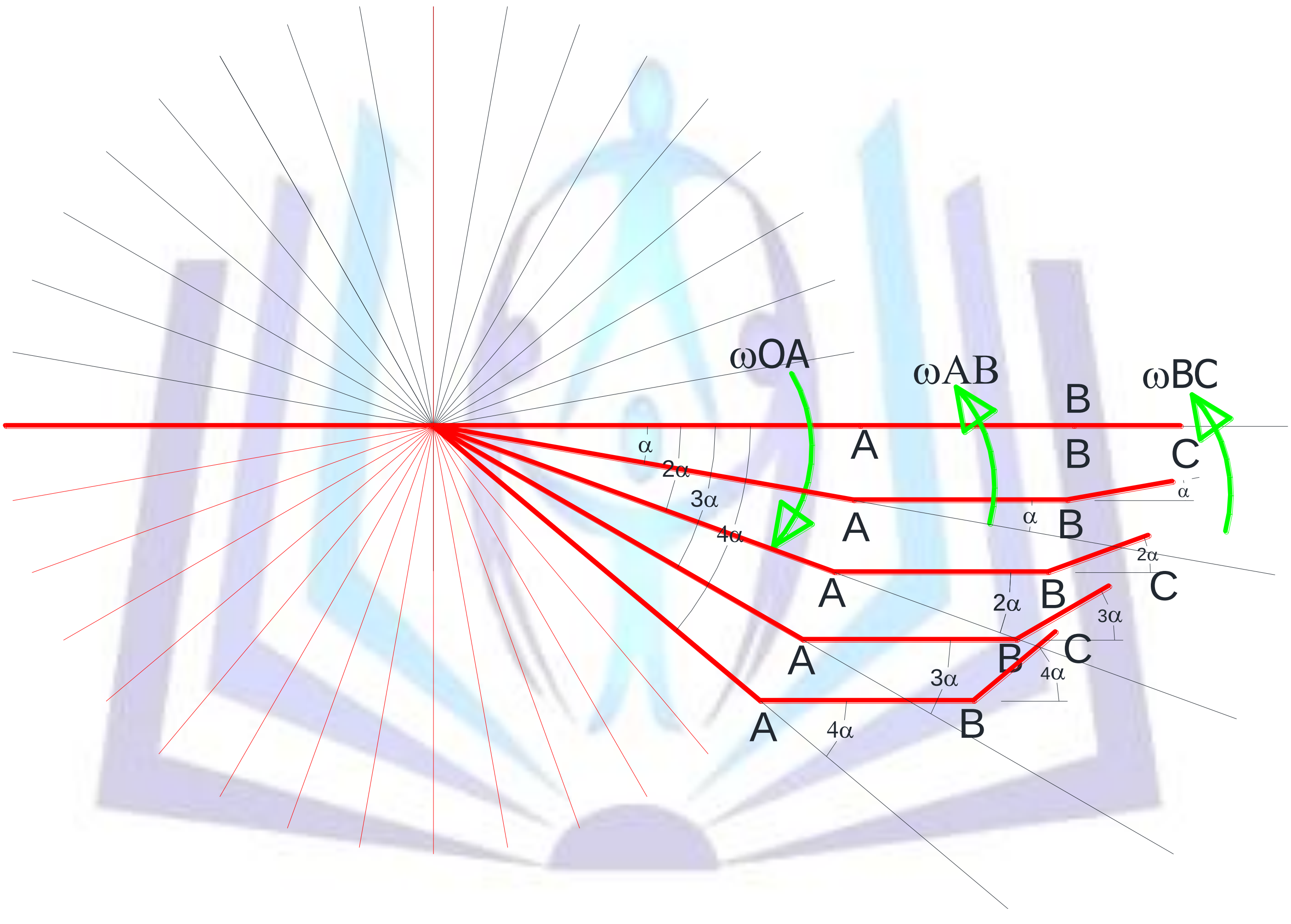
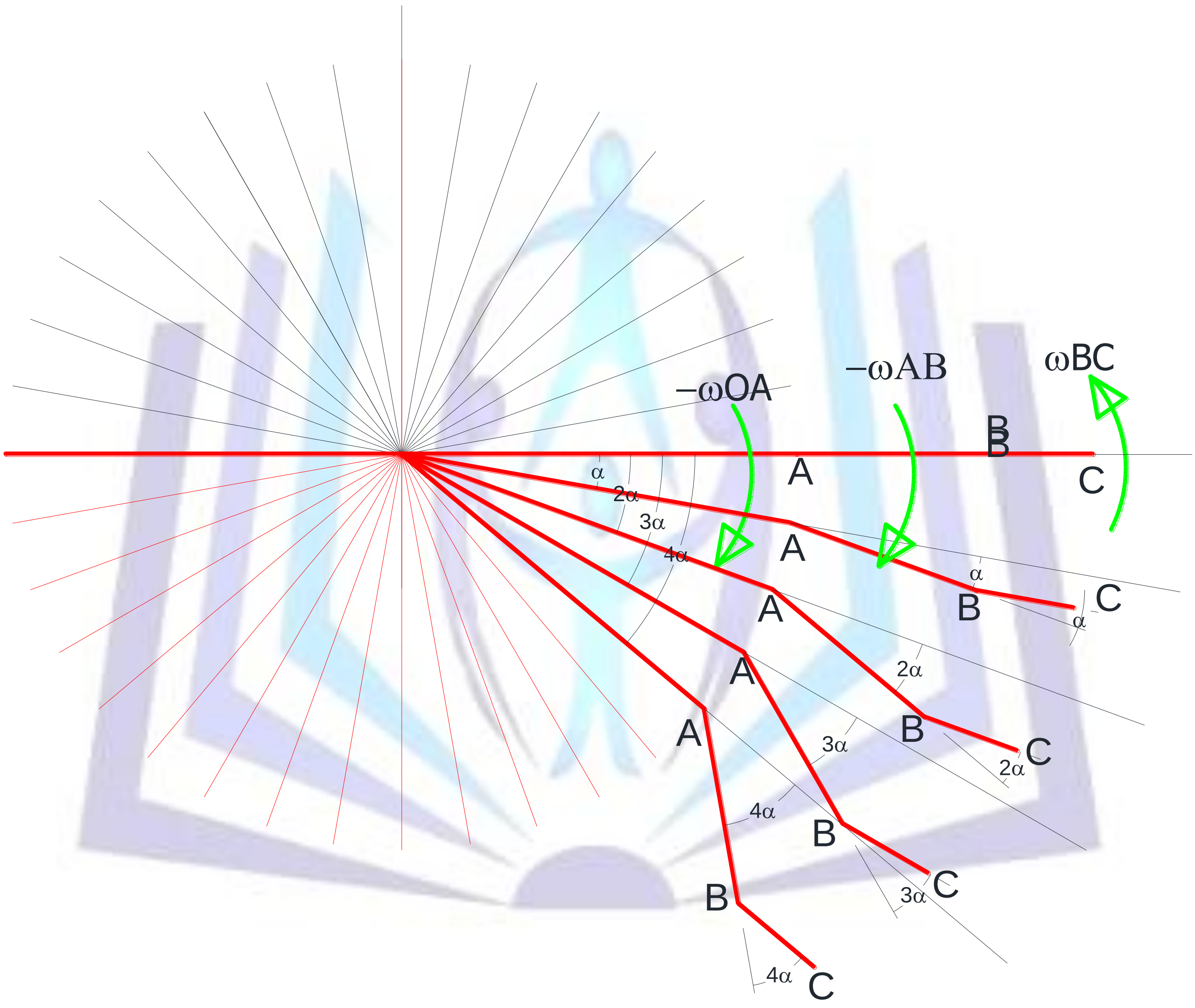
Ex 23

$$\begin{cases} \boldsymbol{\omega} OA = \frac{\pi}{18} Rad/Sec \\ t = 1\, sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ -\omega OA = \omega AB = \omega BC \\ \omega OC = ? \\ \alpha = \frac{\pi}{18} \end{cases}$$



**Ex 24**

$$\begin{cases} \boldsymbol{\omega} OA = \frac{-\pi}{18} Rad/Sec \\ t = 1\,sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ -\omega OA = -\omega AB = \omega BC \\ \omega OC = ? \\ \alpha = \frac{\pi}{18} \end{cases}$$



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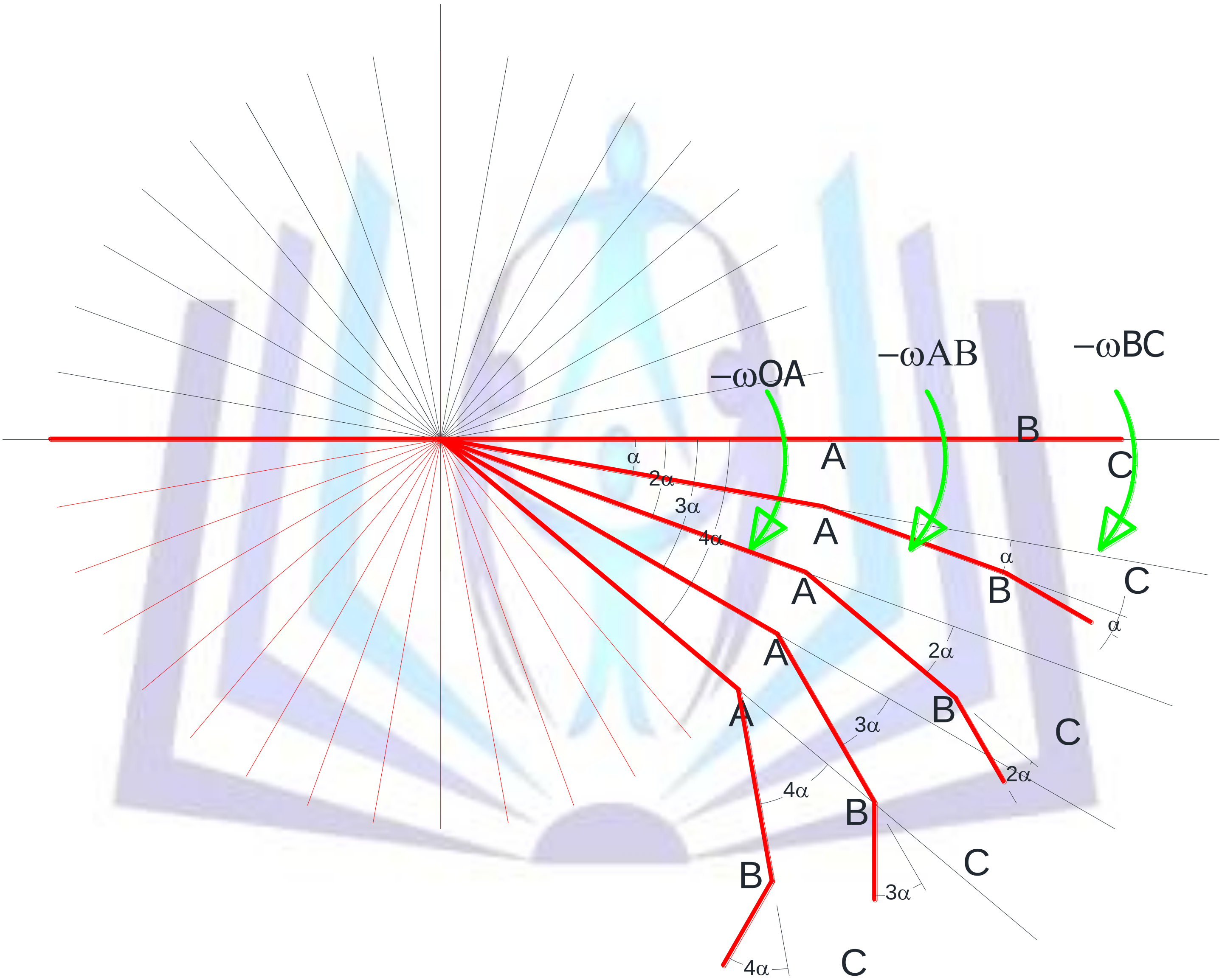
**Ex 25**

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{-\pi}{18} Rad/Sec \\ t = 1\, sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ -\omega OA = -\omega AB = -\omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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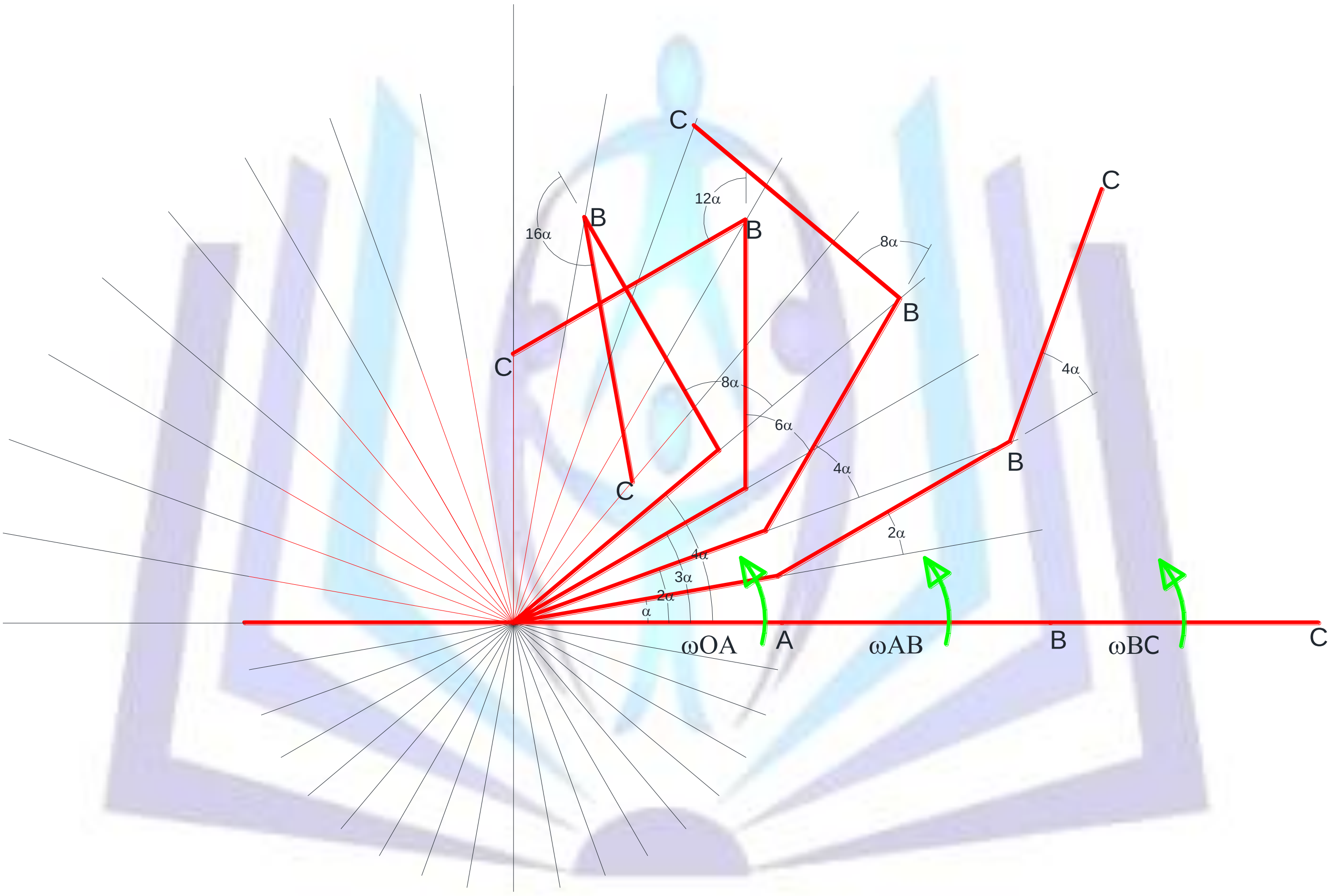
Ex 26

$$\begin{cases} \boldsymbol{\omega} OA = \frac{\pi}{18} Rad/Sec \\ t = 1\, sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OA = 2.\omega AB = 4.\omega BC \\ \omega OC = ? \\ \alpha = \frac{\pi}{18} \end{cases}$$



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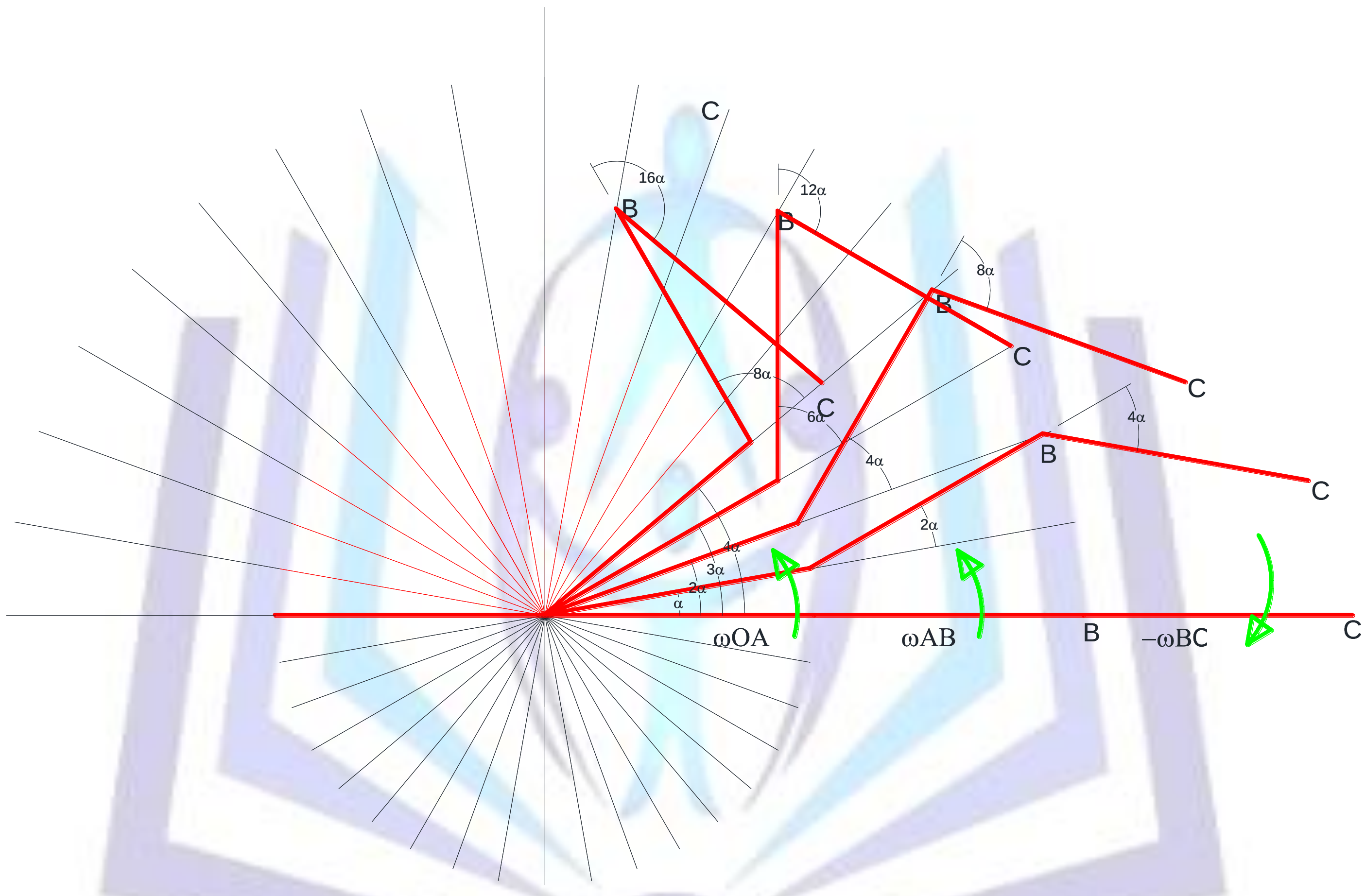
**Ex 27**

$$\begin{cases} \boldsymbol{\omega} OA = \frac{\pi}{18} Rad/Sec \\ t = 1\ sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OA = 2.\omega AB = 4.\omega BC \\ \omega OC = ? \\ \alpha = \frac{\pi}{18} \end{cases}$$



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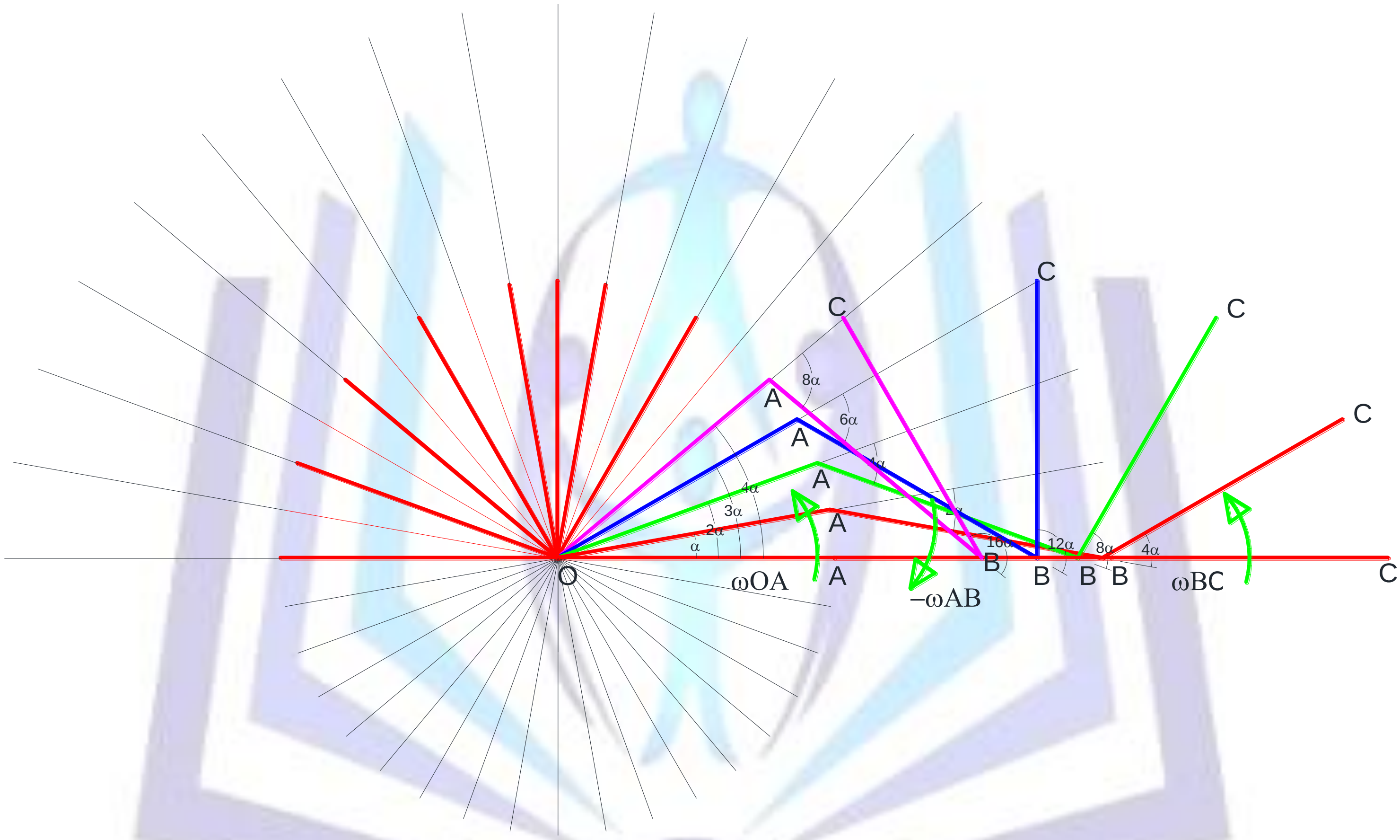
**Ex 28**

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18} Rad/Sec \\ t = 1 \ sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OA = -2.\omega AB = 4.\omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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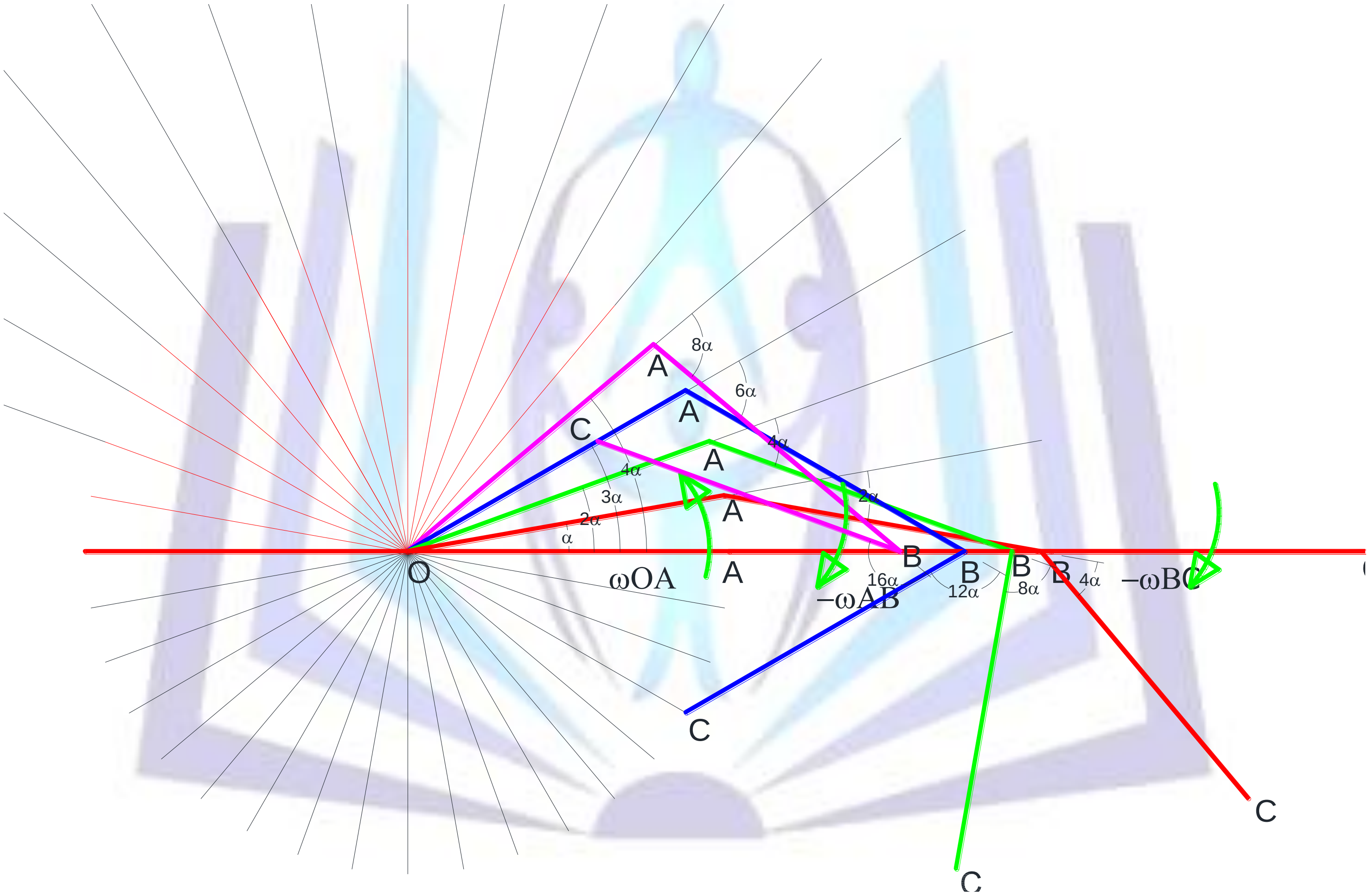
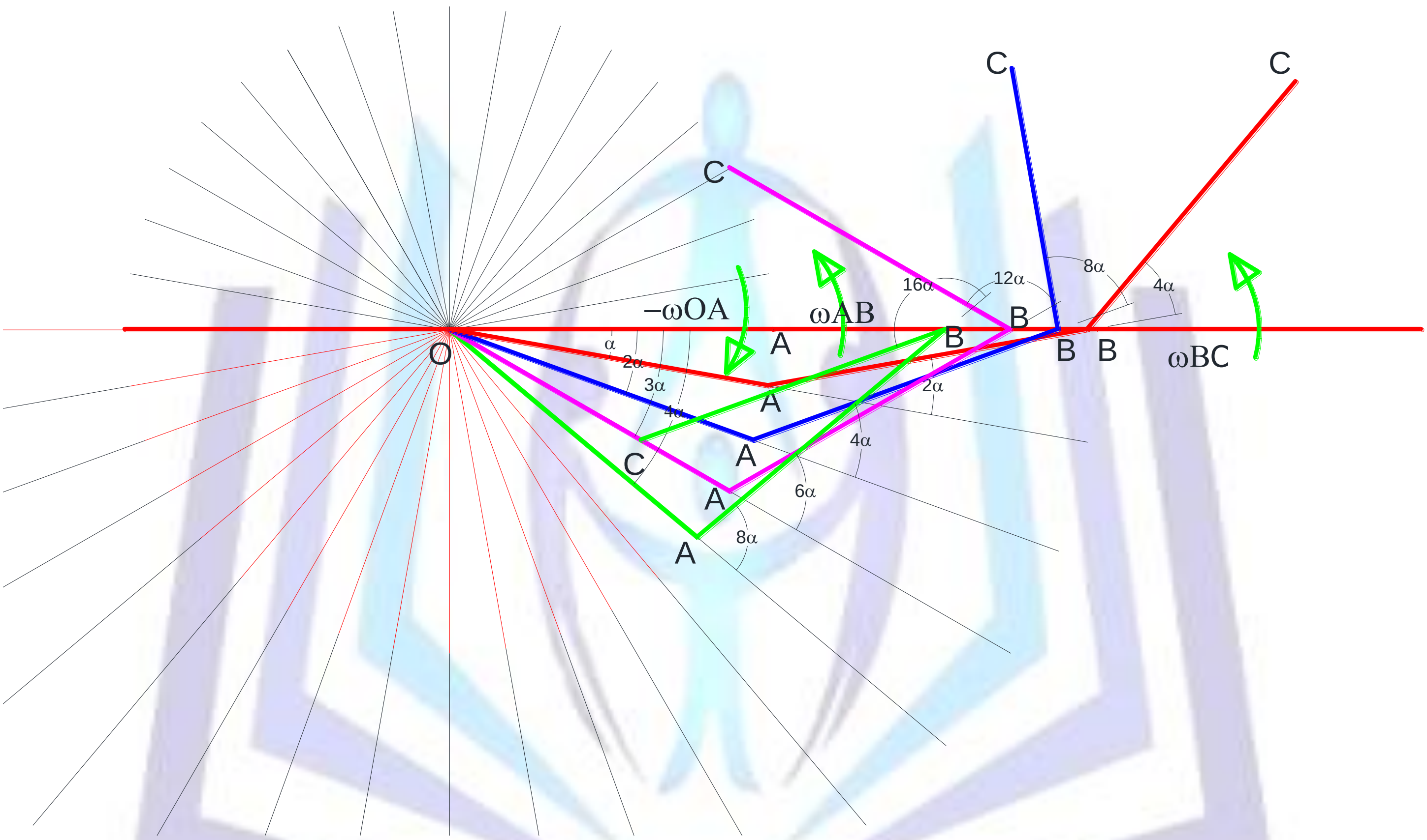
Ex 29

$$\begin{cases} \boldsymbol{\omega} OA = \frac{\pi}{18} Rad/Sec \\ t = 1\ sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ \omega OA = -2.\omega AB = -4.\omega BC \\ \omega OC = ? \\ \alpha = \frac{\pi}{18} \end{cases}$$



Ex 30

$$\begin{cases} -\boldsymbol{\omega} OA = \dfrac{\pi}{18} Rad\ /Sec \\ t = 1\ sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ -\omega OA = 2.\omega AB = 4.\omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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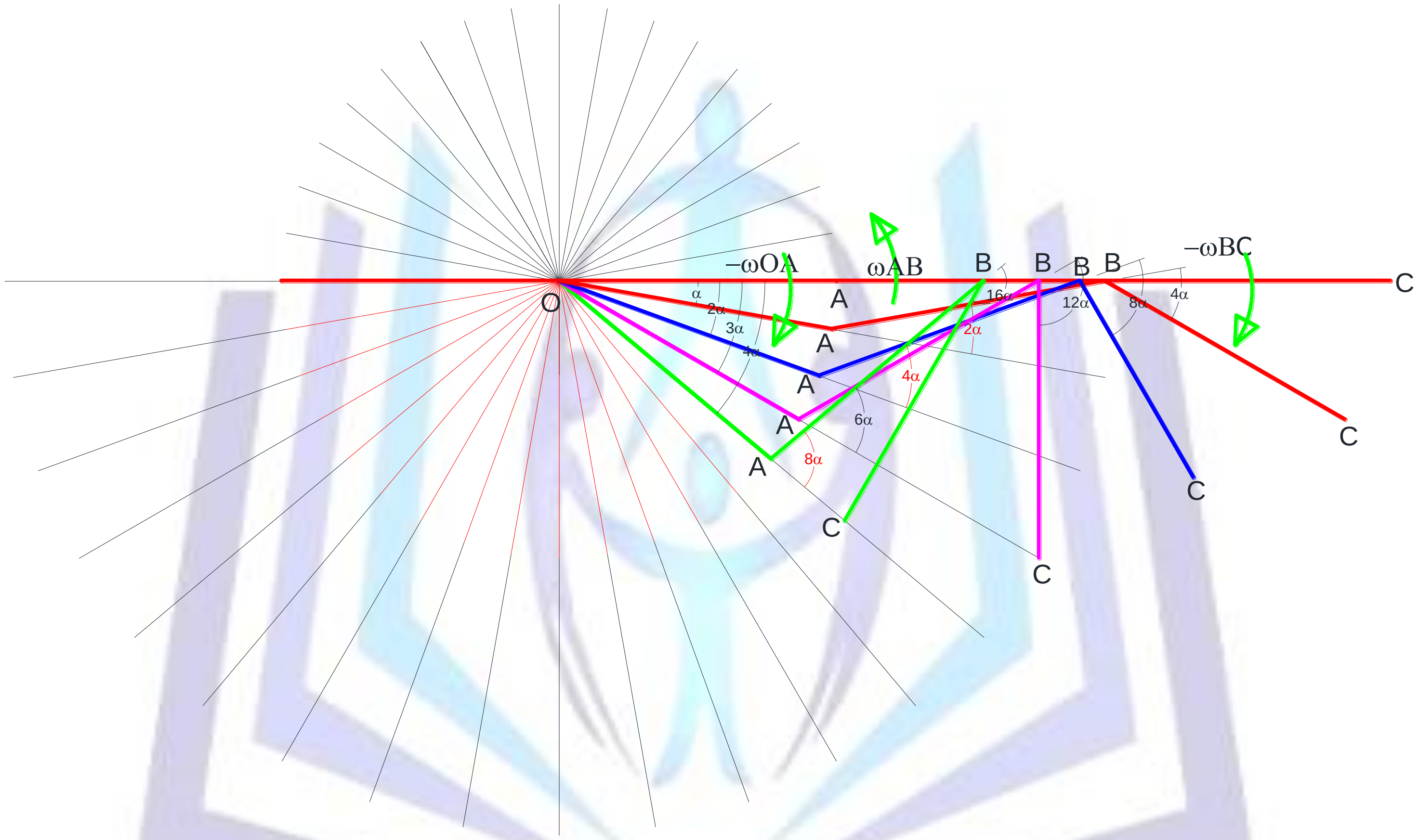
Ex 31

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{-\pi}{18} Rad/Sec \\ t = 1 \ sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ -\omega OA = 2.\omega AB = -4.\omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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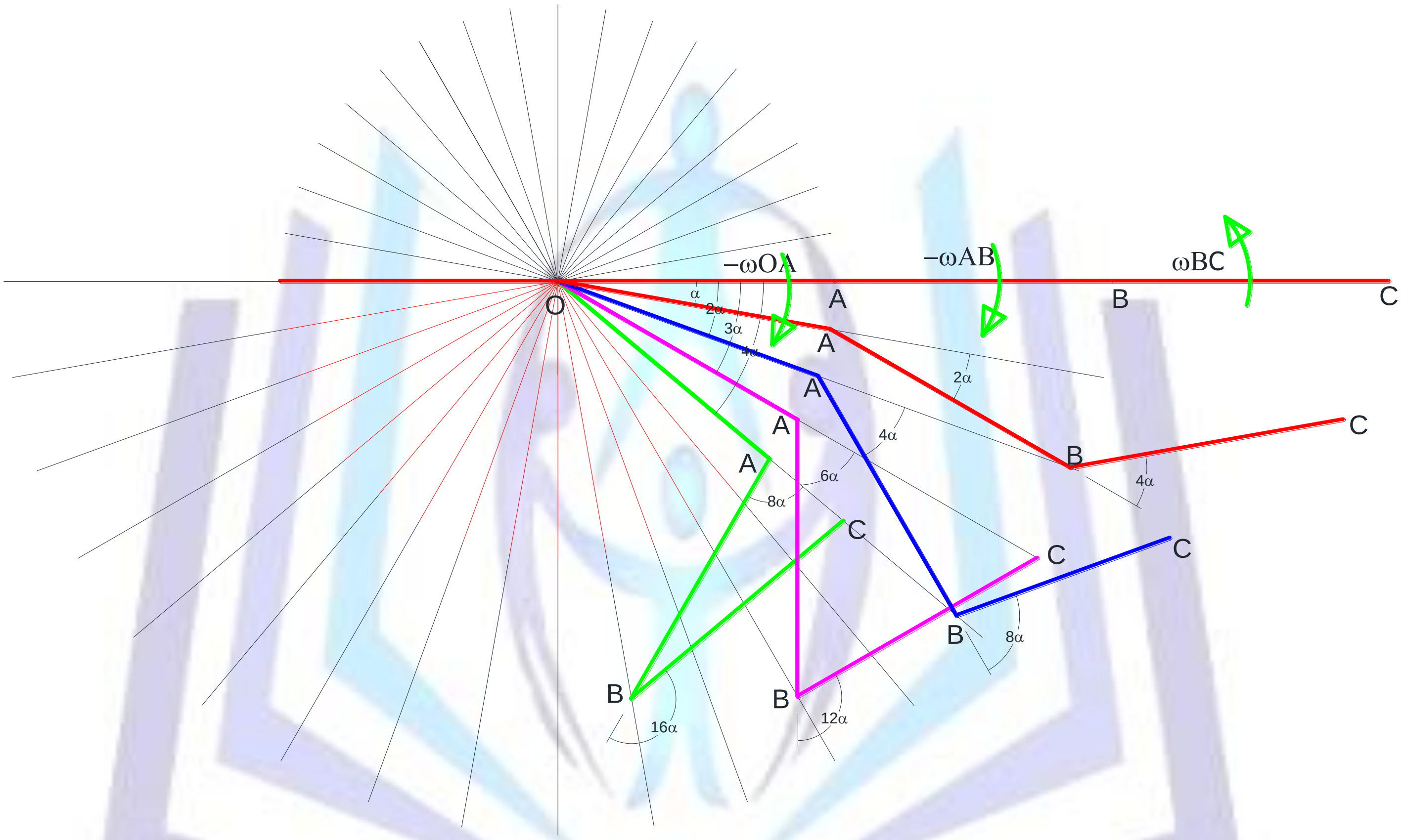
Ex 32

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{-\pi}{18} Rad/Sec \\ t = 1 \ sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ -\omega OA = -2.\omega AB = -4.\omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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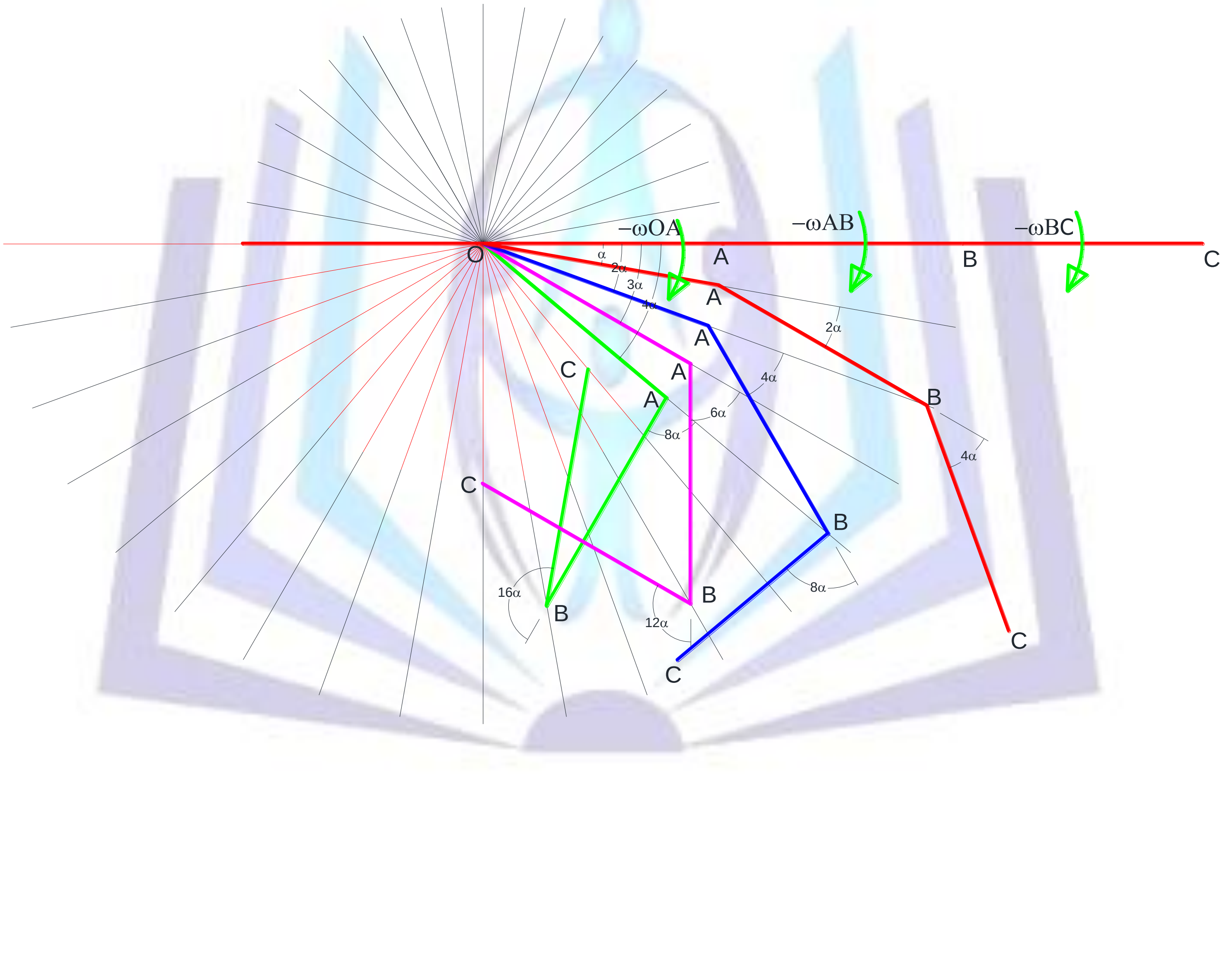
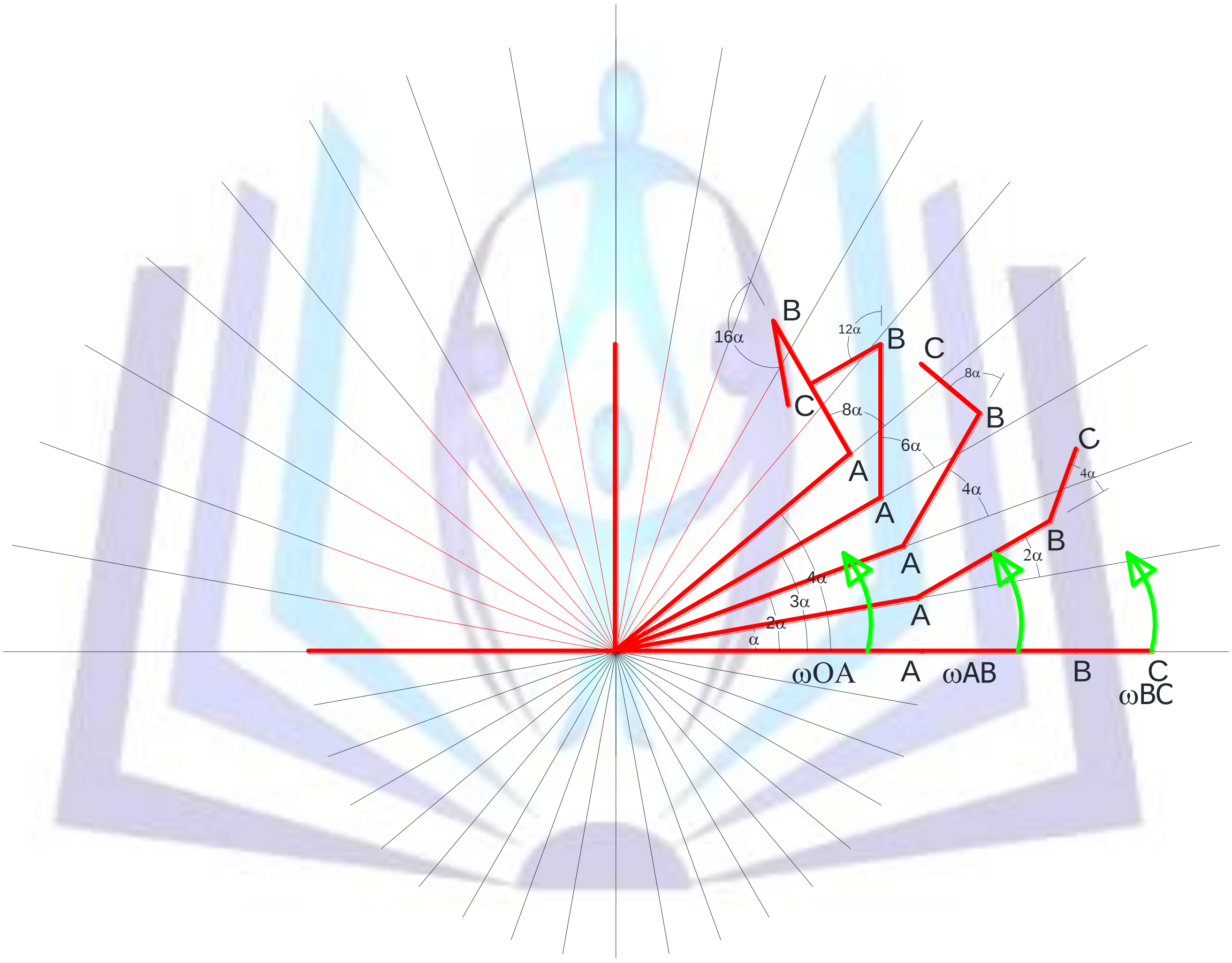
Ex 33

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{-\pi}{18} Rad/Sec \\ t = 1\ sec \\ \overline{OA} = \overline{AB} = \overline{BC} \\ -\omega OA = 2.\omega AB = -4.\omega BC \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



**Ex 34**

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18} Rad\ /Sec \\ \quad t = 1\ sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ \omega OA = \dfrac{\omega AB}{2} = \dfrac{\omega BC}{4} \\ \omega OC = ? \\ \quad \alpha = \dfrac{\pi}{18} \end{cases}$$



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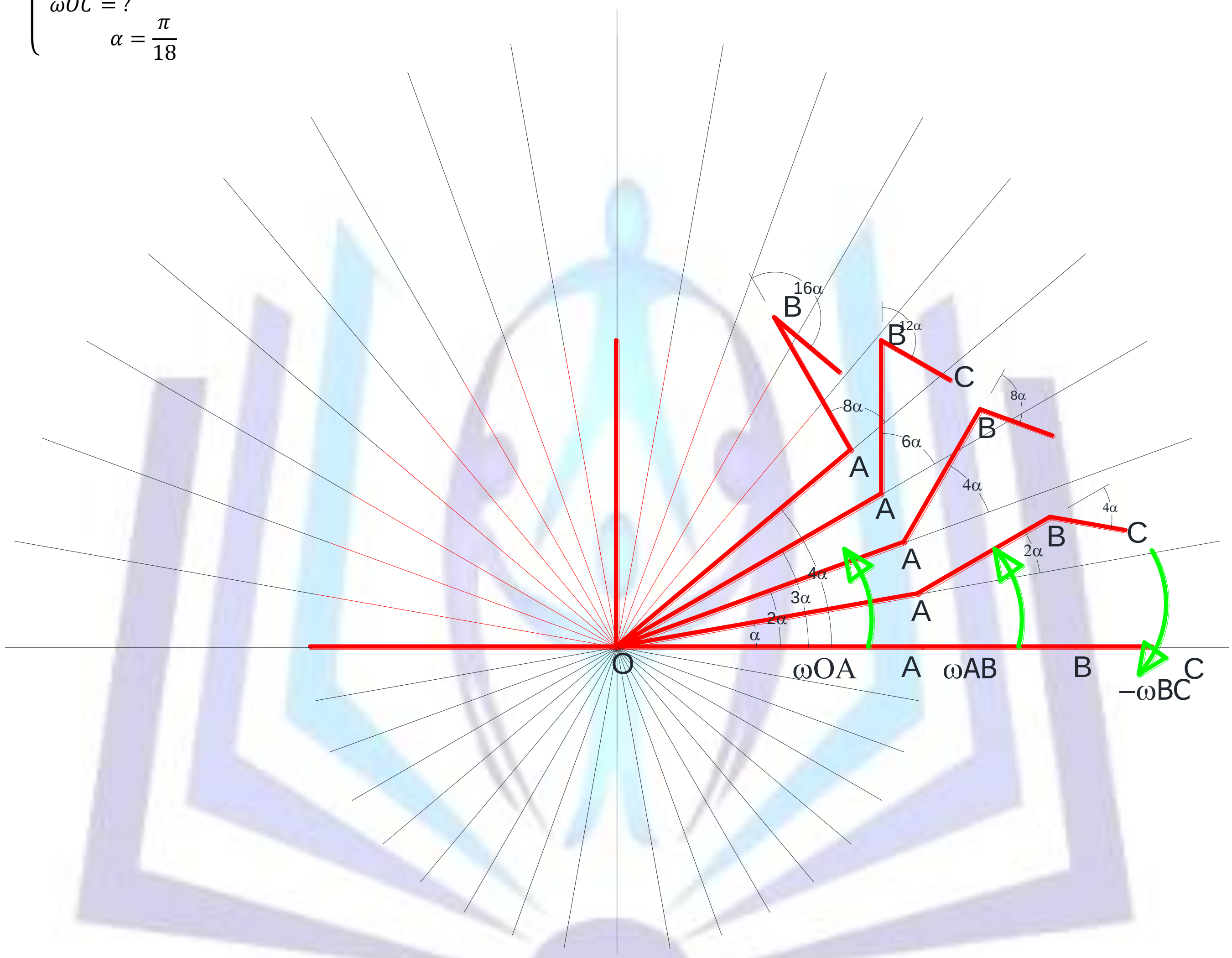
Ex 35

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18} Rad/Sec \\ t = 1 \ sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ \omega OA = \dfrac{\omega AB}{2} = \dfrac{-\omega BC}{4} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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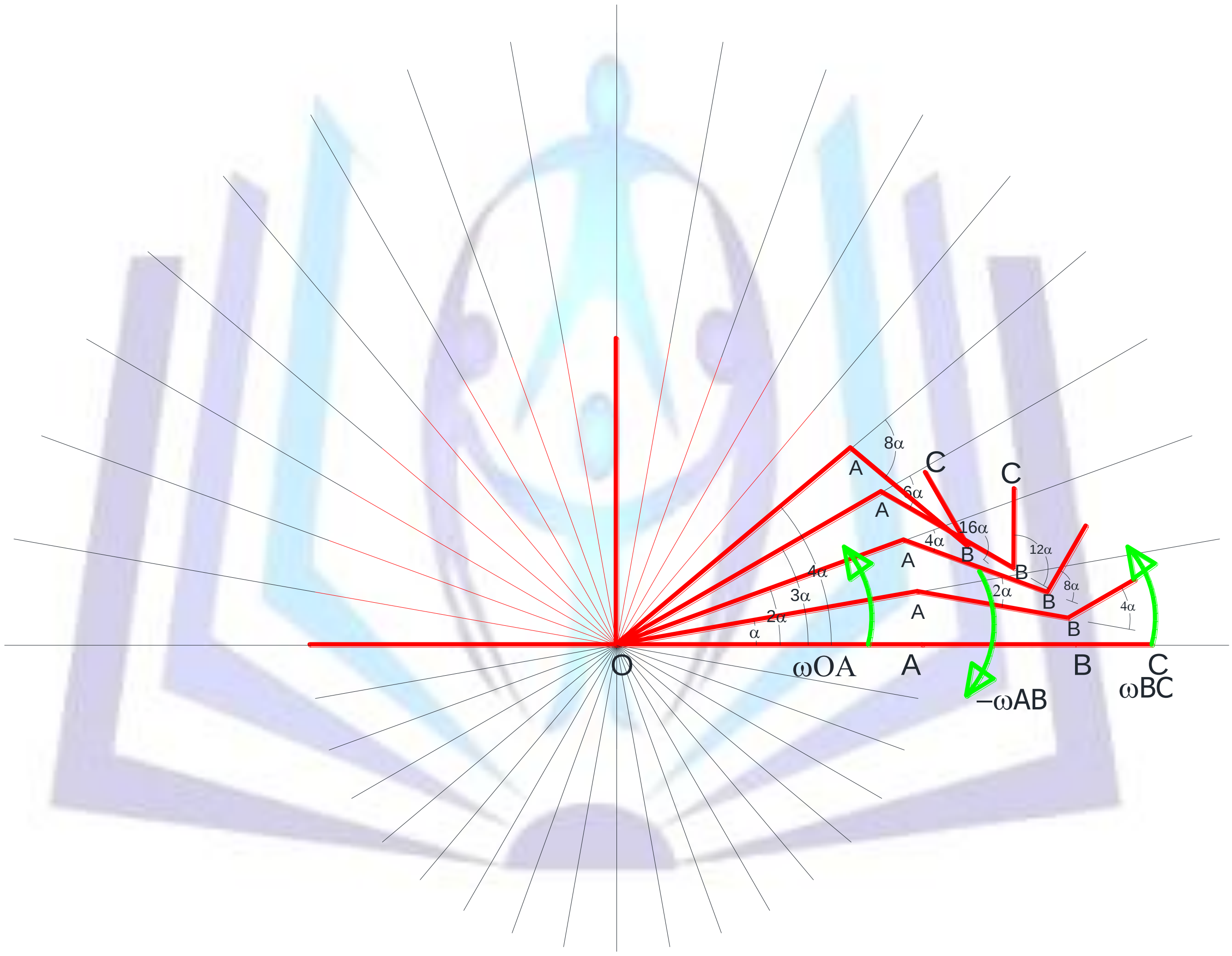
**Ex 36**

$$\begin{cases} \boldsymbol{\omega} OA = \frac{\pi}{18} Rad\ /Sec \\ t = 1\ sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ \omega OA = \frac{-\omega AB}{2} = \frac{\omega BC}{4} \\ \omega OC = ? \\ \alpha = \frac{\pi}{18} \end{cases}$$



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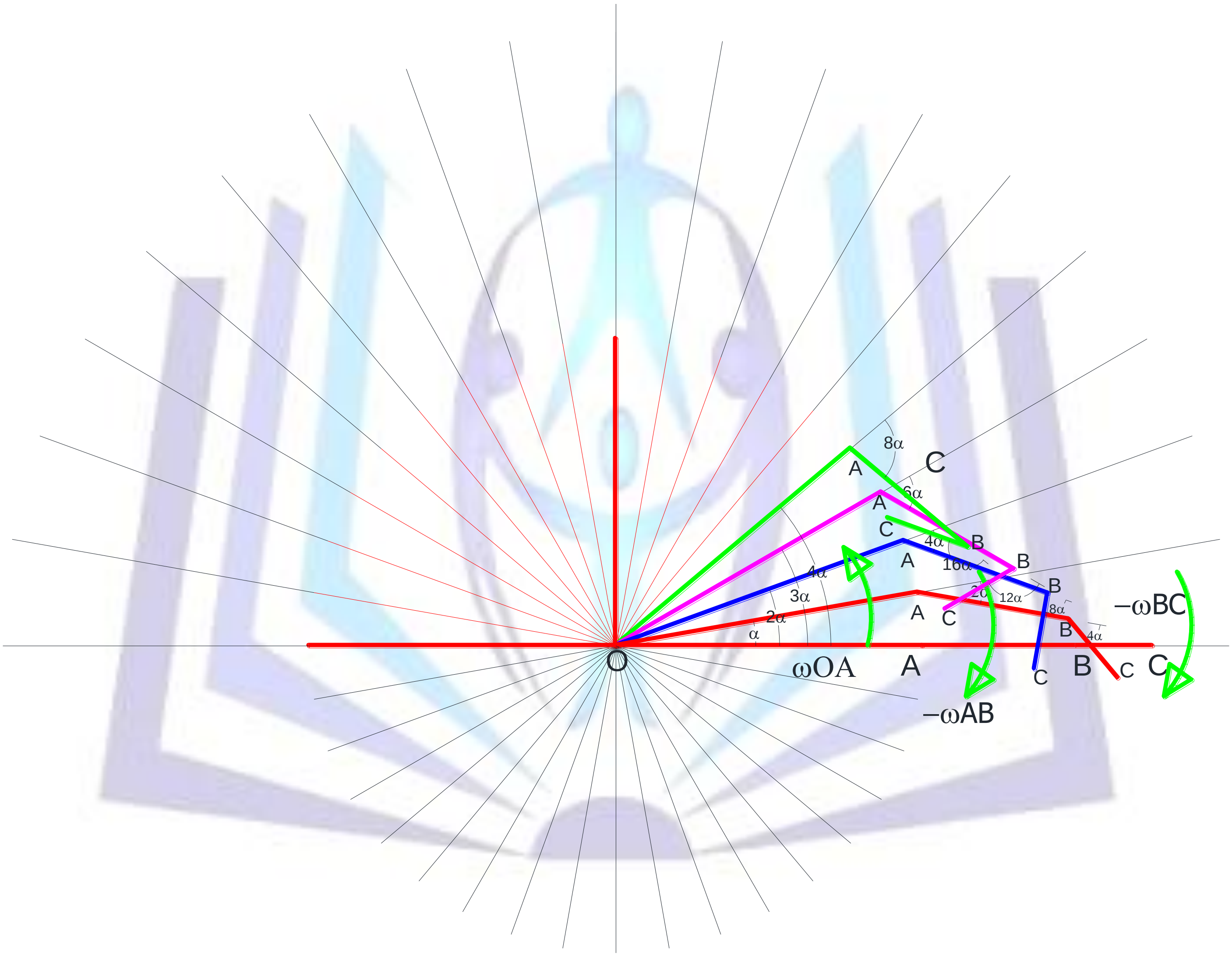
Ex 37

$$\begin{cases} \boldsymbol{\omega} OA = \dfrac{\pi}{18} Rad\ /Sec \\ t = 1\ sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ \omega OA = \dfrac{-\omega AB}{2} = \dfrac{-\omega BC}{4} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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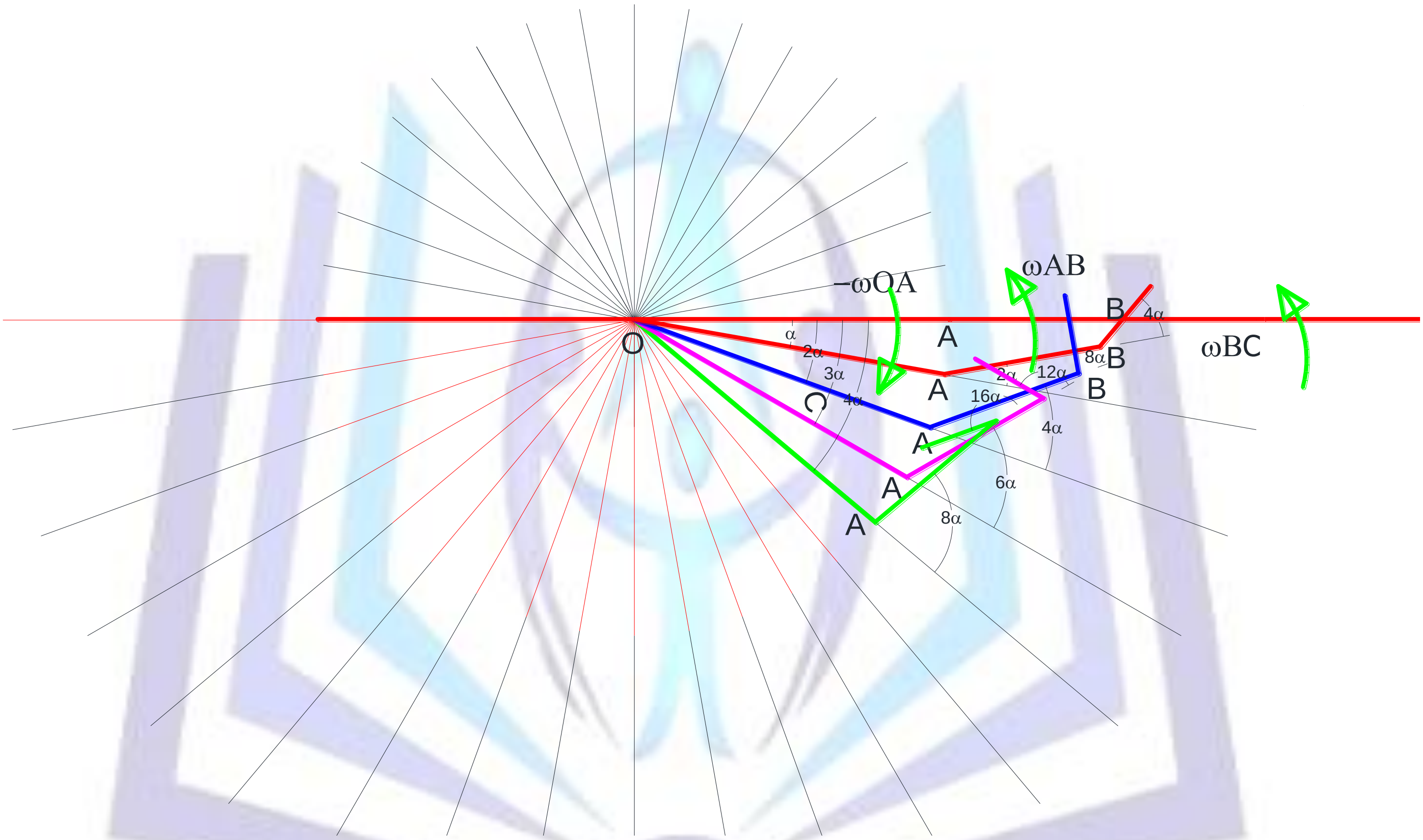
Ex 38

$$\begin{cases} -\boldsymbol{\omega} OA = \dfrac{\pi}{18} Rad/Sec \\ \quad t = 1\ sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ -\omega OA = \dfrac{\omega AB}{2} = \dfrac{\omega BC}{4} \\ \omega OC = ? \\ \quad \alpha = \dfrac{\pi}{18} \end{cases}$$



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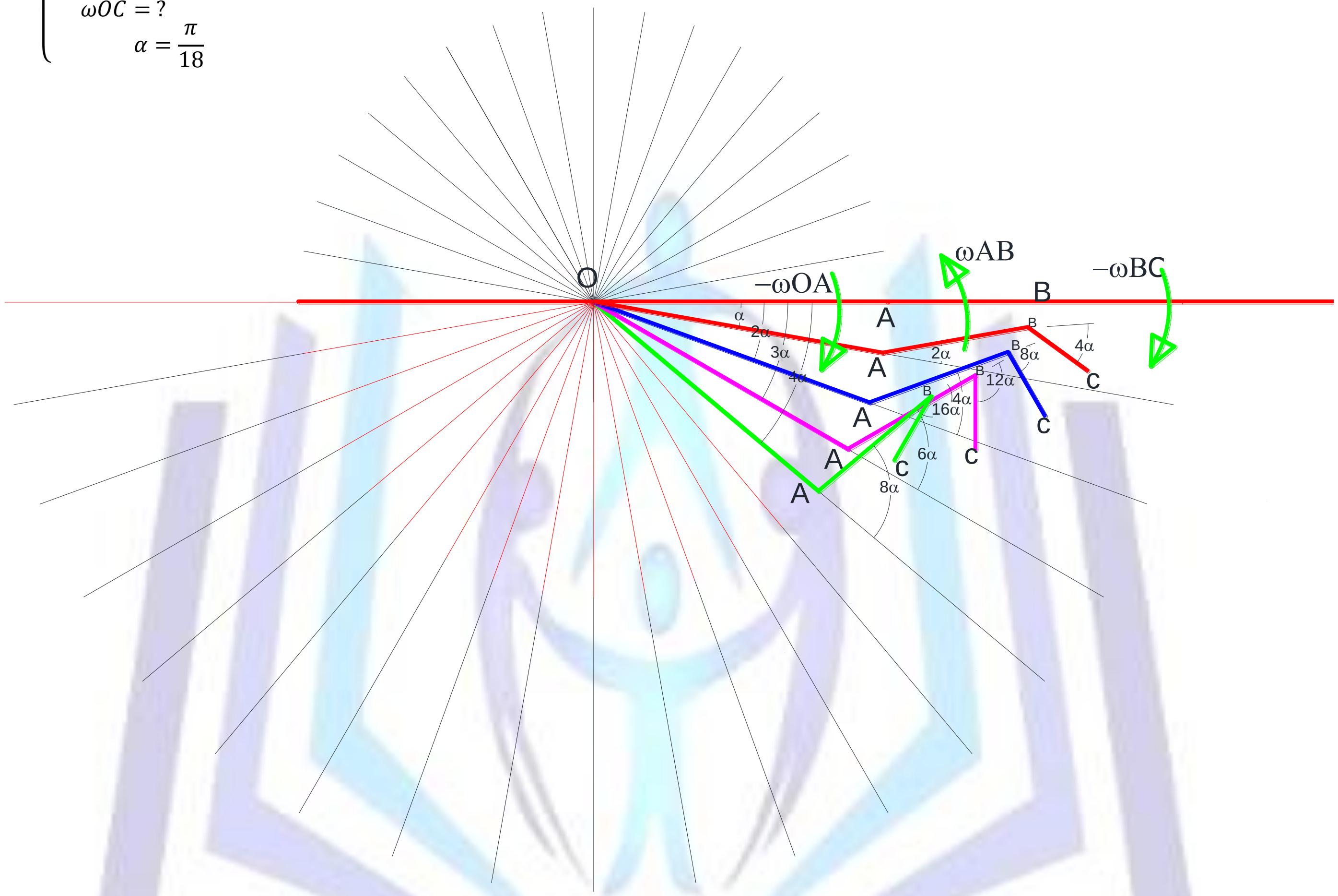
Ex 39

$$\begin{cases} -\boldsymbol{\omega} OA = \dfrac{\pi}{18} Rad/Sec \\ t = 1 \ sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ -\omega OA = \dfrac{\omega AB}{2} = \dfrac{-\omega BC}{4} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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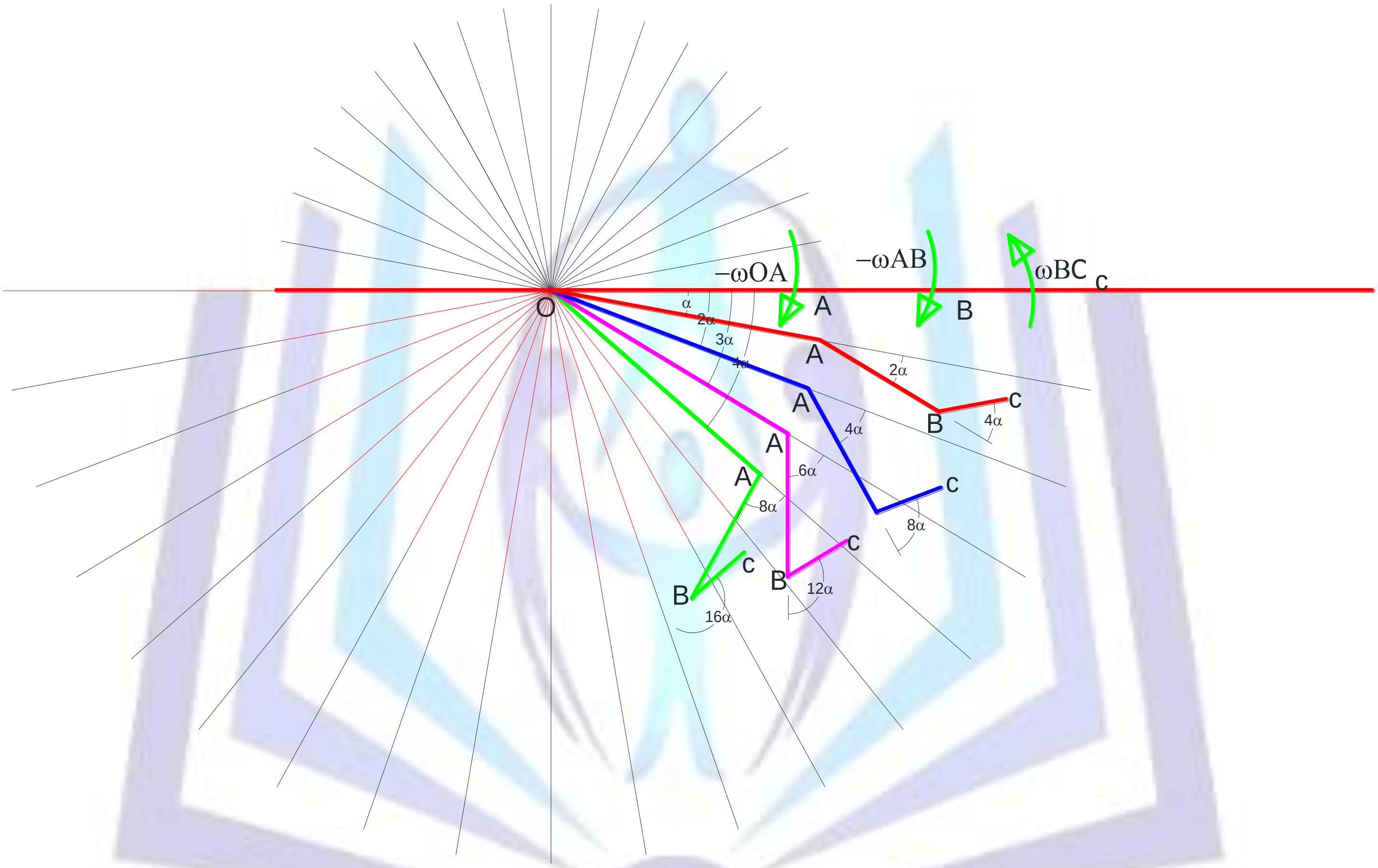
**Ex 40**

$$\begin{cases} -\boldsymbol{\omega} OA = \dfrac{\pi}{18} Rad/Sec \\ t = 1\ sec \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ -\omega OA = \dfrac{-\omega AB}{2} = \dfrac{\omega BC}{4} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



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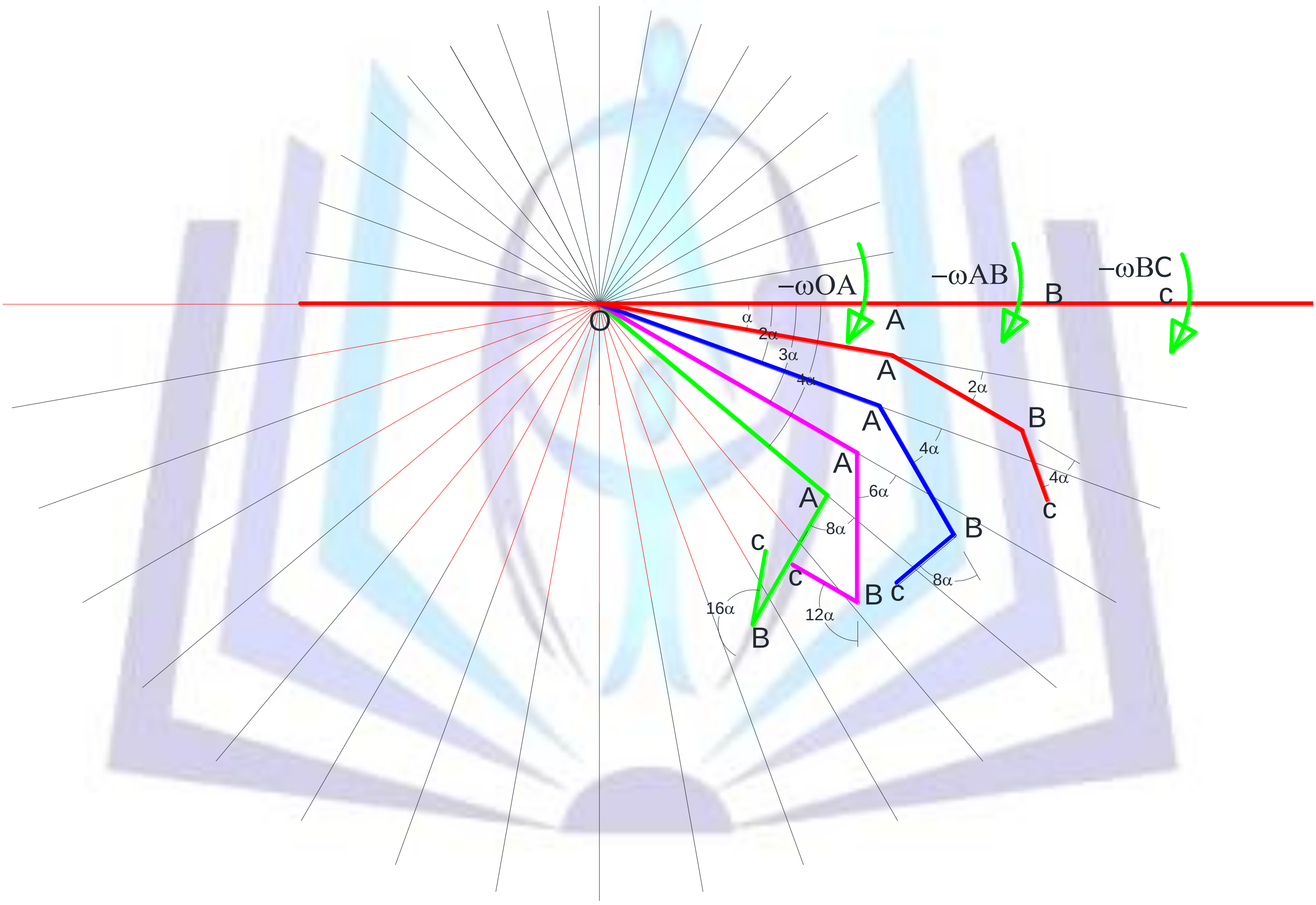
**Ex 41**

$$\begin{cases} -\boldsymbol{\omega} OA = \dfrac{\pi}{18} Rad/Sec \\ t = 1 \text{ sec} \\ \overline{OA} = \overline{2.AB} = \overline{4.BC} \\ -\omega OA = \dfrac{-\omega AB}{2} = \dfrac{-\omega BC}{4} \\ \omega OC = ? \\ \alpha = \dfrac{\pi}{18} \end{cases}$$



ISSN 2347-1921

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## Author' biography with Photo

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Member ofthe scientific boardof Isfahan Universityand head of thetechnicalofficeMedicalSciencesIsfahan University and Iran'sofficialexpertin the field ofheavy machinery. Hefieldsof Physics, Mechanics,Mathematicshaspublishedseveral articlesin Iranian universitiesandthis article and 3the followingarticlein