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**Some new results for αβ −statistical convergence through difference compact operator of fuzzy 2- normed spaces**

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**Abstract**:

The research aims to provide advanced compact for order αβ −converge of statistic in the normed spaces of fuzzy-2 and introduce universal the spaces's.

**Keywords:** αβ −statistical convergence, 2- 2-normed spaces, compact operator.

**1. Introduction**

converge of statistic was obtainable since in (1951) with support to the density of asymptotic can be: a sequence $b = b_x$ is named to be statistical convergence to τ if was for all $\varepsilon > 0$, the set $\{x \in N: |b_x - \tau| \geq \varepsilon\} = 0$ or $\frac{|\{x \leq \text{ɰ}: |b_x - \tau| \geq \varepsilon\}|}{\text{ɰ}} = 0$.[ 1]

We denoted $st - \lim b_x = \tau$ , and we symbolize statistical convergence by ʂƫʂ

On the other hand, it was presented idea related to ʂƫʂ is αβ − ʂƫʂ to was put it up by Aktuğlu [2]. Also, he gave a definition of the term of *αβ*- convergence of statistic eqution in between *αβ*-statistical pointwise of statistical and convergence of uniform, he worked on this this notion to get theorem of Korovkin approximation. Gokhan (2002) presented the pointwise definitions and he given uniform ʂƫʂ of sequences of real valued functions[3],[4] .

Fuzzy set theory is a very helpful applications in several purview. Katsaras definition found a fuzzy vector topology structure with fuzzy normal on linearspace construct on the space. [5]

After a period of time, some researchers were able to define fuzzy norms of the linear space there are many different opinions [6],[7].

Let $\text{ƕ} = \text{ƕ}_{\text{ɰ}}$ *and* $\text{ꝑ} = \text{ꝑ}_{\text{ɰ}}$ the following condition satisfy with to sequences of positive numbers :

- ƕ *and* ꝑ are both nondecreasing
- $\text{ꝑ} \geq \text{ƕ}$
- $\text{ꝑ}_{\text{ɰ}} - \text{ƕ}_{\text{ɰ}} \to \infty \; as \; \text{ɰ} \to \infty$

We impose a set that satisfies the previous conditions for ƕ *and* ꝑ, We denote this set byɱ

**2. Definition and Preliminaries**

**Definition 2.1. [9]:** we can assume ɏ as the linear space that belongs to dimension greater than one, Ꝁ a triangle function, and let $N$ be a mapping from ɏ×ɏ into $D^+$.

The following conditions are satisfied for all $y_1, y_2, y_3$ ∈ ɏ and:

(1) $N(y_1, y_2; t) = H_o$ if $y_1$ *and* $y_2$ are linearly dependent,

(2) $N\left(y_1, y_2; t\right) \neq H_o$ if $y_1$ *and* $y_2$ are linearly independent,

(3) $N\left(y_1, y_2; t\right) = N\left(y_2, y_1; t\right)$ for every $y_1, y_2 \in$ ɏ

(4) $N\left(\sigma y_1, y_2; t\right) = N\left(y_1, y_2; \frac{t}{|\sigma|}\right)$ for every $t > 0, \sigma \neq 0$ *and* $y_1, y_2 \in$ ɏ.

(5) $N\left(y_1 + y_2, y_3; t_1 + t_2\right) \geq N\left(y_1, y_3; t_1\right) * N\left(y_2, y_3; t_2\right)$ for all $y_1, y_2, y_3 \in$ ɏ and $t_1 + t_2 \in R^+$.

Triple (ɏ, $N$, *) denoted to the fuzzy 2-normed space (abbreviated as, FTNS). Add to that,

$t > 0$, $\left(y_1, y_2\right) \rightarrow N\left(y_1, y_2; t\right)$ is a continuous function on ɏ×ɏ, then to (ɏ, $N$, *) named tough fuzzy 2-normtive space (denoted , strong FTNS).

A $f: R \rightarrow R_o^+$ considered function of distribution if it is not decreasing and continuing left with $inf_{t \in R} f(t) = 0$ *and* $sup_{t \in R} f(t) = 1$ . we can point the set of all function of distribution by $D^+$, such that $f(0) = 0$ .

If ɨ$\in R_o^+$, then $H_ɨ \in D^+$, in which

$$H_ɨ(t) = \{1 \quad if\ t > ɨ\ 0 \quad if\ t \leq ɨ$$

It is clear that $H_o \geq f$ for every $f \in D^+$

**Definition 2-2** [8] Let ɏ *considered* linear space in field ℝ with dimension ≥ 2 . A function ||.,. ||: ɏ×ɏ→$R \geq 0$

is called a 2-norm over ɏ if:

(1)$\|y_1, y_2\| \geq 0$ to every $y_1, y_2 \in$ ɏ , $\|y_1, y_2\| = 0$ if and on condition $y_1$ *and* $y_2$ are depend linearly.

(2) $\|y_1, y_2\| = \|y_2, y_1\|$ *to* every $y_1, y_2 \in$ ɏ.

(3) $\|\sigma y_1, y_2\| = |\sigma| \|y_1, y_2\|$ to every $y_1, y_2 \in$ ɏ and$\sigma \in R$ .

(4) $\|y_1, y_2 + y_3\| \leq \|y_1, y_2\| + \|y_1, y_3\|$ *to* every $y_1, y_2, y_3 \in$ ɏ.

The pair (ɏ, ||.,. ||) named a 2 −normed space

**Definition2-3**: b is sequence to be *αβ*-₷ƫ₷ for *γ* to τ, expressed by $st_{\alpha\beta}^{\gamma} - b_x$ = τ.

if for every $\epsilon > 0$,

$$st^{\alpha,\beta}\left(\left\{x: \left|b_x - \tau\right| \geq \epsilon\right\}, \gamma\right) = \frac{\left|\left\{x \in Ɒ_{ɯ}^{\alpha,\beta}: \left|b_x - \tau\right| \geq \epsilon\right\}\right|}{\left(\beta_{ɯ} - \alpha_{ɯ} + 1\right)^{\gamma}} = 0$$

For $\gamma = 1$, we can considered that $b$ is *αβ*-₷ƫ₷ to τ, this is refer to

$st_{\alpha\beta}^{\gamma}$ - $\lim_{ɯ \rightarrow \infty} b_x = \tau$ .

where $Đ_{ɯ}^{α,β}$ is the closed interval $[α_{ɯ}, β_{ɯ}]$

All $αβ$-ȿŧȿ are set for a sequence of fuzzy 2- normed of numbers of order γ will be refer $ȝ_{γ}^{αβ}(b)$

**Definition .2.4.[9]** $B$ so be it a non-empty subset of a FTNS($ɏ, N, *$), For every $y_1 \in ɏ$ , $t > 0$ and nonzero $y_3 \in ɏ$ , let

$$N(y_1 - B, y_3; t) = \sup sup \{(y_1 - y_2, y_3; t): y_2 \in B\}$$

An element $y_o \in B$ is named to be a ϼ - best approximating to $y_2 from\ B$ if

$$N(y_1 - y_o, y_3; t) = N(y_1 - B, y_3; t)$$

By $P_B^t(y_1, y_3)$, we refer the set of elements of ϼ - good approximation of $y_1$ by the elements of the $B$,i.e.

$$P_B^t(y_1, y_3) = \{y_2 \in B: N(y_1 - B, y_3; t) = N(y_1 - y_2, y_3; t)\}$$

Also we define

$$e_B^t(y_1, y_3) = 1 - N(y_1 - B, y_3; t)$$

**Definition 2.5** :A sequence in $b_x$ difference compact operator of fuzzy 2- normed spaces is said ϼ - best approximation to be αβ − ȿŧȿ on ɏ of order γ to τ if for every $\epsilon > 0$

$$st^{α,β}\left(\left\{x: N\|b_x - τ\|_B - t \ge \epsilon\right\}, γ\right) = \frac{\left|\left\{x \in Đ_{ɯ}^{α,β}: N\|b_x - τ\|_B - t \ge \epsilon\right\}\right|}{(β_{ɯ} - α_{ɯ} + 1)^{γ}} = 0$$

**Proposition 2-1** : let $(α, β) \in ϑ, 0 < γ \le 1$, by the two sequence $b_x\ and\ ɱ_x$ of fuzzy 2- normed numbers, where $b_x$, $ɱ_x \in B$, we can give:

**1-** If $st_{γ}^{α,β}(ɏ, N, *) - lim_{ɯ\to\infty} b_x = b_0$ for every constant in positive real number then

$st_{γ}^{α,β}(ɏ, N, *) - lim_{ɯ\to\infty} Ψ b_x = Ψ b_0 , for\ all\ Ψ \in R^{+}$

**Proof**: if this case holds where $Ψ = 0$, suppose the $Ψ \ne 0$

$$\frac{\left|\left\{x \in Đ_{ɯ}^{α,β}: N\|Ψ b_x - Ψτ\|_B - Ψ t \ge \epsilon\right\}\right|}{(β_{ɯ} - α_{ɯ} + 1)^{γ}} \le \frac{\left|\left\{x \in Đ_{ɯ}^{α,β}: N\|b_x - τ\|_B - t \ge \epsilon/Ψ\right\}\right|}{(β_{ɯ} - α_{ɯ} + 1)^{γ}}$$

By ɯ→∞ ,we get

$$\left(\frac{\left|\left\{x \in Đ_{ɯ}^{α,β}: N\|Ψ b_x - Ψτ\|_B - Ψ t \ge \epsilon\right\}\right|}{(β_{ɯ} - α_{ɯ} + 1)^{γ}}\right) \to 0$$

Hence $st_{γ}^{α,β}(ɏ, N, *) - lim_{ɯ\to\infty} Ψ b_x = Ψ b_0$

**2-** If $st_{\gamma}^{\alpha,\beta}(ɏ, N, *) - lim b_x = b_0$ and $st_{\gamma}^{\alpha,\beta}(ɏ, N, *) - lim ɱ_x = ɱ_0$ then

$st_{\gamma}^{\alpha,\beta}(ɏ, N, *) - lim(b_x + ɱ_x) = b_0 + ɱ_0$

**Proof:** If suppose that $st_{\gamma}^{\alpha,\beta}(ɏ, N, *) - lim b_x = b_0$ and $st_{\gamma}^{\alpha,\beta}(ɏ, N, *) - lim ɱ_x = ɱ_0$ we get

$$\frac{\left|\left\{x \in Đ_{ɯ}^{\alpha,\beta} : N\|\left((b_x + ɱ_x),(b_0 + ɱ_0)\right) - \tau\|_B - t \geq \epsilon\right\}\right|}{(\beta_{ɯ} - \alpha_{ɯ} + 1)^{\gamma}}$$

$$\leq \frac{\left|\left\{x \in Đ_{ɯ}^{\alpha,\beta} : N\|(b_x + b_0) - \tau\|_B - t \geq \epsilon/2\right\}\right|}{(\beta_{ɯ} - \alpha_{ɯ} + 1)^{\gamma}} + \frac{\left|\left\{x \in Đ_{ɯ}^{\alpha,\beta} : N\|(ɱ_x + ɱ_0) - \tau\|_B - t \geq \epsilon/2\right\}\right|}{(\beta_{ɯ} - \alpha_{ɯ} + 1)^{\gamma}}$$

By $ɯ \to \infty$ ,we get $st_{\gamma}^{\alpha,\beta}(ɏ, N, *) - lim(b_x + ɱ_x) = b_0 + ɱ_0$

**Definition 2-6** : A sequence $b = b_x$ of fuzzy 2- normed numbers is called to be strongly ρ - best approximation for be $\alpha\beta - ȿƀȿ$ on ɏ of order γ to τ if there is a fuzzy 2- normed numbers ` such that

$$\frac{1}{(\beta_{ɯ} - \alpha_{ɯ} + 1)^{\gamma}} \sum_{x \in Đ_{ɯ}^{\alpha,\beta}, y_1, y_3 \in ɏ} \left\{\left(\left(N\|b_x(y_1) - \tau\|_{y_0 \in B}\right)^{\rho} \cup \left(N\|b_x(y_3) - \tau\|_{y_0 \in B}\right)^{\rho}\right) - t \geq \epsilon\right\} = 0, \ for\ all\ nonzero, y_3$$

where ρ is positive real number and $0 < \gamma \leq 1$. By $M_{\gamma,\rho}^{\alpha,\beta}(b)$ we symbolize the set

for all strongly ρ- best approximation to be $\alpha\beta - ȿƀȿ$ on ɏ of order γ for fuzzy sequence ,we write $M_{\gamma,\rho}^{\alpha,\beta} - lim\, b_x = \tau$.

**Theorem2-1** :on the suppose $\gamma, ɦ$ be real numbers such that $\gamma, ɦ \in (0, 1]$, $\gamma \leq ɦ$ , and $0 < \rho < \infty$. Then $M_{\gamma,\rho}^{\alpha,\beta}(b) \subseteq З_{ɦ}^{\alpha,\beta}(b)$.

**Proof:** Let the $= b_x \in M_{\gamma,\rho}^{\alpha,\beta}(b)$. $\forall \epsilon > 0$ ,We get

$$\sum_{x \in Đ_{ɯ}^{\alpha,\beta}} \left(\left(N\|b_x - \tau\|_B - t\right)^{\rho}\right) = \sum_{x \in Đ_{ɯ}^{\alpha,\beta}\ x : \left(N\|b_x - \tau\|_B - t\right) \geq \epsilon} \left(\left(N\|b_x - \tau\|_B - t\right)^{\rho}\right) + \sum_{x \in Đ_{ɯ}^{\alpha,\beta}\ x : \left(N\|b_x - \tau\|_B - t\right) < \epsilon} \left(\left(N\|b_x - \tau\|_B - t\right)^{\rho}\right)$$

$$\geq \sum_{x \in Đ_{ɯ}^{\alpha,\beta}\ x : \left(N\|b_x - \tau\|_B - t\right) \geq \varepsilon} \left(\left(N\|b_x - \tau\|_B - t\right)^{\rho}\right)$$

$$\geq \left|\left\{x \in Đ_{ɯ}^{\alpha,\beta} : N\|(b_x) - \tau\|_B - t \geq \epsilon\right\}\right| \epsilon^{\rho} \qquad (1)$$

By (1),

$$\sum_{x \in Đ_{ɯ}^{\alpha,\beta}} \frac{\left(N\|b_x - \tau\|_B - t\right)^{\rho} \geq \epsilon}{(\beta_{ɯ} - \alpha_{ɯ} + 1)^{\gamma}}$$

$$\geq \frac{\left|\left\{x\in Ð_{ɰ}^{\alpha,\beta}:\left(N\|b_x-\tau\|_B-t\right)^{\rho}\geq\epsilon\right\}\right|}{(\beta_{ɰ}-\alpha_{ɰ}+1)^{\gamma}}\epsilon^{\rho}$$

$$\geq \frac{\left|\left\{x\in Ð_{ɰ}^{\alpha,\beta}:\left(N\|b_x-\tau\|_B-t\right)^{\rho}\geq\epsilon\right\}\right|}{(\beta_{ɰ}-\alpha_{ɰ}+1)^{ɓ}}\epsilon^{\rho}$$

Now , we obtain $b = b_x \in ʒ_{ɓ}^{\alpha,\beta}(b)$.

**Theorem2-2** : Let $(\alpha,\beta), (ɑ,ƥ) \in ɱ$ such that $ɑ(ɰ)\leq\alpha(ɰ)$ *and* $\beta(ɰ)\leq ƥ(ɰ)$, for all $ɰ\in N$ and

$\gamma, ɓ \in(0,1]$, $\gamma\leq ɓ$

**1)** If $\liminf_{ɰ\to\infty}\left(\frac{(\beta_{ɰ}-\alpha_{ɰ}+1)^{\gamma}}{(\beta_{ɰ}-\alpha_{ɰ}+1)^{ɓ}}\right) > 0$, then $ʒ_{ɓ}^{ɑ,ƥ}(b) \subseteq ʒ_{\gamma}^{\alpha,\beta}(b)$

**Proof**: since $ɑ(ɰ)\leq\alpha(ɰ)$ *and* $\beta(ɰ)\leq ƥ(ɰ)$, for all $ɰ\in N$, we get

$$\left\{x\in Ð_{ɰ}^{\alpha,\beta}:\left(N\|b_x-\tau\|_B-t\right)^{\rho}\geq\epsilon\right\}\subset\left\{x\in Ð_{ɰ}^{ɑ,ƥ}:\left(N\|b_x-\tau\|_B-t\right)^{\rho}\geq\epsilon\right\}$$

We obtain from our assertion $\beta_{ɰ}-\alpha_{ɰ}+1\leq ƥ_{ɰ}-ɑ_{ɰ}+1$ for all $ɰ\in N$

$$\frac{(\beta_{ɰ}-\alpha_{ɰ}+1)^{\gamma}}{(\beta_{ɰ}-\alpha_{ɰ}+1)^{ɓ}}\frac{\left|\left\{x\in Ð_{ɰ}^{\alpha,\beta}:N\|b_x-\tau\|_B-t\geq\epsilon\right\}\right|}{(\beta_{ɰ}-\alpha_{ɰ}+1)^{\gamma}}\leq\frac{\left|\left\{x\in Ð_{ɰ}^{ɑ,ƥ}:N\|b_x-\tau\|_B-t\geq\epsilon\right\}\right|}{(\beta_{ɰ}-\alpha_{ɰ}+1)^{ɓ}}$$

By the limit for two sides mentioned contrast as $ɰ\to\infty$and by $\liminf_{ɰ\to\infty}\left(\frac{(\beta_{ɰ}-\alpha_{ɰ}+1)^{\gamma}}{(\beta_{ɰ}-\alpha_{ɰ}+1)^{ɓ}}\right) > 0$, we prove this $ʒ_{ɓ}^{ɑ,ƥ}(b) \subseteq ʒ_{\gamma}^{\alpha,\beta}(b)$.

**2)** If $\lim_{ɰ\to\infty}\left(\frac{\beta_{ɰ}-\alpha_{ɰ}+1}{(\beta_{ɰ}-\alpha_{ɰ}+1)^{ɓ}}\right) = 1$ then $ʒ_{\gamma}^{\alpha,\beta}(b) \subseteq ʒ_{ɓ}^{ɑ,ƥ}(b)$

**Proof:** Suppose $b = b_x \in ʒ_{\gamma}^{\alpha,\beta}(b)$ *and* $\lim_{ɰ\to\infty}\left(\frac{\beta_{ɰ}-\alpha_{ɰ}+1}{(\beta_{ɰ}-\alpha_{ɰ}+1)^{ɓ}}\right) = 1$be satisfied where

$ɑ(ɰ)\leq\alpha(ɰ)$ *and* $\beta(ɰ)\leq ƥ(ɰ)$, for all $ɰ\in N$

$$\frac{\left|\left\{x\in Ð_{ɰ}^{ɑ,ƥ}:N\|b_x-\tau\|_B-t\geq\epsilon\right\}\right|}{(\beta_{ɰ}-\alpha_{ɰ}+1)^{ɓ}}=\frac{\left|\left\{ɑ(ɰ)\leq x\leq\alpha(ɰ)-1:N\|b_x-\tau\|_B-t\geq\epsilon\right\}\right|}{(\beta_{ɰ}-\alpha_{ɰ}+1)^{ɓ}}$$

$$+\frac{\left|\left\{\alpha(ɰ)\leq x\leq\beta(ɰ)-1:N\|b_x-\tau\|_B-t\geq\epsilon\right\}\right|}{(\beta_{ɰ}-\alpha_{ɰ}+1)^{ɓ}}+\frac{\left|\left\{\beta(ɰ)+1\leq x\leq ƥ(ɰ)-1:N\|b_x-\tau\|_B-t\geq\epsilon\right\}\right|}{(\beta_{ɰ}-\alpha_{ɰ}+1)^{ɓ}}\leq$$

$$\frac{\alpha_{ɰ}-ɑ_{ɰ}}{(\beta_{ɰ}-\alpha_{ɰ}+1)^{ɓ}}+\frac{ƥ_{ɰ}-\beta_{ɰ}}{(\beta_{ɰ}-\alpha_{ɰ}+1)^{ɓ}}+\frac{\left|\left\{x\in Ð_{ɰ}^{\alpha,\beta}:N\|b_x-\tau\|_B-t\geq\epsilon\right\}\right|}{(\beta_{ɰ}-\alpha_{ɰ}+1)^{ɓ}}$$

$$=\frac{(\beta_{ɯ}-\eth_{ɯ}+1)-(\beta_{ɯ}-\alpha_{ɯ}+1)}{(\beta_{ɯ}-\alpha_{ɯ}+1)^{\delta}}+\frac{\left|\left\{x\in Đ_{ɯ}^{\alpha,\beta}:N\|b_{x}-\tau\|_{B}-t\geq\epsilon\right\}\right|}{(\beta_{ɯ}-\alpha_{ɯ}+1)^{\delta}}\leq\left(\frac{(\beta_{ɯ}-\eth_{ɯ}+1)}{(\beta_{ɯ}-\alpha_{ɯ}+1)^{\delta}}-1\right)+\frac{\left|\left\{x\in Đ_{ɯ}^{\alpha,\beta}:N\|b_{x}-\tau\|_{B}-t\geq\epsilon\right\}\right|}{(\beta_{ɯ}-\alpha_{ɯ}+1)^{\delta}}$$

By the limit for two sides mentioned contrast as $ɯ\to\infty$ and by $lim_{ɯ\to\infty}\left(\frac{\beta_{ɯ}-\alpha_{ɯ}+1}{(\beta_{ɯ}-\alpha_{ɯ}+1)^{\delta}}\right)=1$ ,

$st_{\gamma}^{\alpha\beta}(b)-\lim_{ɯ\to\infty} b_{x}=\tau$ then get $st_{\delta}^{\eth,\beta}(b)-\lim_{ɯ\to\infty} b_{x}=\tau$ , $b=b_{x}\in ȝ_{\delta}^{\eth,\beta}(b)$ .

**Theorem 2-3 :** suppose $(\alpha,\beta),(\eth,\beta)\in ጦ$ such that $\eth(ɯ)\leq\alpha(ɯ)$ and $\beta(ɯ)\leq\beta(ɯ)$, for all $ɯ\in N$ and $\gamma,\delta\in(0,1]$, $\gamma\leq\delta$

**1)** On the impose $lim\,inf_{ɯ\to\infty}\left(\frac{(\beta_{ɯ}-\alpha_{ɯ}+1)^{\gamma}}{(\beta_{ɯ}-\alpha_{ɯ}+1)^{\delta}}\right)>0$ ,satisfied then $M_{\delta,\rho}^{\eth,\beta}(b)\subset M_{\gamma,\rho}^{\alpha,\beta}(b)$

**Poof:** since $lim\,inf_{ɯ\to\infty}\left(\frac{(\beta_{ɯ}-\alpha_{ɯ}+1)^{\gamma}}{(\beta_{ɯ}-\alpha_{ɯ}+1)^{\delta}}\right)>0$ , give us $ȝ_{\delta}^{\eth,\beta}(b)\subseteq ȝ_{\gamma}^{\alpha,\beta}(b)$ **(**Theorem 2-2**)**

And using **(**on the suppose $\gamma,\delta$ be real numbers such that $\gamma,\delta\in(0,1]$, $\gamma\leq\delta$ , and $0<\rho<\infty$**.** Then $M_{\gamma,\rho}^{\alpha,\beta}(b)\subseteq ȝ_{\delta}^{\alpha,\beta}(b)$ **). (**Theorem 2-1**)**

We get $M_{\delta,\rho}^{\eth,\beta}(b)\subseteq ȝ_{\delta}^{\eth,\beta}(b)$ , So this ends the proof .

**2)** On the impose $lim_{ɯ\to\infty}\left(\frac{\beta_{ɯ}-\alpha_{ɯ}+1}{(\beta_{ɯ}-\alpha_{ɯ}+1)^{\delta}}\right)=1$ and $b=b_{x}$ be bounded sequence of fuzzy mappings then $M_{\gamma,\rho}^{\alpha,\beta}(b)\subset M_{\delta,\rho}^{\eth,\beta}(b)$ .

**Proof**: let $b=b_{x}\in M_{\gamma,\rho}^{\alpha,\beta}(b)$ be bounded sequence of fuzzy mappings then there exists some $\Gamma>0$ such that

$(N\|b_{x}-\tau\|_{B}-t)\leq\Gamma$ *for all* $x\in N$ and $lim_{ɯ\to\infty}\left(\frac{\beta_{ɯ}-\alpha_{ɯ}+1}{(\beta_{ɯ}-\alpha_{ɯ}+1)^{\delta}}\right)=1$ is holds we obtain

$$\sum_{x\in Đ_{ɯ}^{\eth,\beta}}\frac{\left(N\|b_{x}-\tau\|_{B}-t\right)^{\rho}}{(\eth_{ɯ}-\beta_{ɯ}+1)^{\delta}}=\sum_{x\in Đ_{ɯ}^{\frac{\eth,\beta}{\alpha,\beta}}}\frac{\left(N\|b_{x}-\tau\|_{B}-t\right)^{\rho}}{(\eth_{ɯ}-\beta_{ɯ}+1)^{\delta}}+\sum_{x\in Đ_{ɯ}^{\alpha,\beta}}\frac{\left(N\|b_{x}-\tau\|_{B}-t\right)^{\rho}}{(\beta_{ɯ}-\alpha_{ɯ}+1)^{\gamma}}$$

$$\leq\left(\frac{(\beta_{ɯ}-\eth_{ɯ}+1)-(\beta_{ɯ}-\alpha_{ɯ}+1)}{(\beta_{ɯ}-\alpha_{ɯ}+1)^{\delta}}\right)\Gamma^{\rho}+\frac{\left(N\|b_{x}-\tau\|_{B}-t\right)^{\rho}}{(\beta_{ɯ}-\alpha_{ɯ}+1)^{\delta}}$$

$$\leq\left(\frac{(\beta_{ɯ}-\eth_{ɯ}+1)-(\beta_{ɯ}-\alpha_{ɯ}+1)^{\delta}}{(\beta_{ɯ}-\alpha_{ɯ}+1)^{\delta}}\right)\Gamma^{\rho}+\frac{\left(N\|b_{x}-\tau\|_{B}-t\right)^{\rho}}{(\beta_{ɯ}-\alpha_{ɯ}+1)^{\delta}}\leq\left(\frac{(\beta_{ɯ}-\eth_{ɯ}+1)}{(\beta_{ɯ}-\alpha_{ɯ}+1)^{\delta}}-1\right)\Gamma^{\rho}+\frac{\left(N\|b_{x}-\tau\|_{B}-t\right)^{\rho}}{(\beta_{ɯ}-\alpha_{ɯ}+1)^{\delta}}\qquad(2)$$

Since $b=b_{x}\in M_{\gamma,p}^{\alpha,\beta}(b)$ by passing to the limit as $ɯ\to\infty$ in the(2) and by $lim_{ɯ\to\infty}\left(\frac{\beta_{ɯ}-\alpha_{ɯ}+1}{(\beta_{ɯ}-\alpha_{ɯ}+1)^{\delta}}\right)=1$

We get $b_{x}\in M_{\delta,\rho}^{\eth,\beta}(b)$ , so $M_{\gamma,\rho}^{\alpha,\beta}(b)\subset M_{\delta,\rho}^{\eth,\beta}(b)$ .

**References**

[1] Fast, H.: Sur la convergence statistique. Colloq. Math. 2, 241–244 (1951) .

[2] Aktuğlu, H.: Korovkin type approximation theorems proved via $\alpha\beta$-statistical convergence. J. Comput. Appl. Math.259, 174–181 (2014).

[3] Gokhan, A, Gungor, M: On pointwise statistical convergence. Indian J. Pure Appl. Math. 33(9), 1379-1384 (2002)

[4] Gungor, M, Gokhan, A: On uniform statistical convergence. Int. J. Pure Appl. Math. 19(1), 17-24 (2005)

[5] A.K. Katsaras, Fuzzy topological vector spaces, Fuzzy Sets and Systems 12 (1984) 143–154.

[6] C. Feblin, Finite dimensional fuzzy normed linear spaces, Fuzzy Sets and Systems 48 (1992) 239–248.

[7] J.Z. Xiao, X.H. Zhu, Fuzzy normed spaces of operators and its completeness, Fuzzy Sets and Systems 133 (2003) 135–146.

[8] S. Gähler, 2-metrische Räume und ihre topologische Struktur, Math. Nachr. 26 (1963) 115–148.

[9] S.A.Mohiuddine, Some new results on approximation in fuzzy 2-normed spaces, Mathematical and computer Modelling 53 (2011) 574-580.