DOI: https://doi.org/10.24297/jam.v23i.9600

REVIEW OF WπGR CLOSED SETS IN TOPOLOGICAL SPACES

[1]SREENA S. R[2]DR.SATHEESH E. N & [3]DR.VARUGHESE MATHEW

Department of Mathematics, Mar Thoma College, Thiruvalla

[1]sreenasr100@gmail.com[2] satheeshkizhakkemadom@gmail.com

[3]varughesemathewmtc@gmail.com

ABSTRACT:

In this paper we introduce a new class of sets called weakly π generalized regular closed (wπgr closed) sets. A subset A of X is called wπgr closed set if cl(int A) ⊆U whenever A⊆U and U is πgr open in X. The complement of wπgr-closed set is called wπgr-open set in X. We denote the family of all wπgr closed sets in X by wπGRC(X) and wπgr open sets in X by wπGRO(X)).

Key words : wπgr –closed sets, wπgr –open sets, πgr –closed sets

INTRODUCTION

Generalized closed sets play a very important role in general topology and they are now the research topics of many topologists worldwide. Generalized closed sets have been studied extensively in recent years by many topologists. The investigation of generalized closed sets has led to several new and interesting results. N.Levine [5] introduced the concept of generalized closed sets and studied their properties in 1970. Mashhour [7] [1982] introduced pre-open sets in topological spaces. Andrijevic [3] introduced one such new version called b-open sets in 1996. Ali [2] introduced the concept of gp closed sets in topological spaces. Usha Parameswari and Thangavelu [8] introduced b# opensets and studied its properties in 2014. Vidhya and Parimelazhagan [9] introduced g*b closed sets in 2012. In 2014,Elvina Mary[4] introduced (gs)*-closed sets in topological spaces and studied their properties . In 2015, Murugavalli & Pushpalatha [6] introduced and studied the properties of τ*- gλ–Closed Sets in Topological Spaces .R S Wali and Nirani Laxmi [11] [12] studied regular mildly generalized closed sets and open sets in 2016. Absanabanu and Pasunkilipandian [1] introduced b# generalized closed sets and its properties in 2017. Vithya and Thangavelu [10] introduced Generalization of b-closed Sets. In this paper, we introduce a new class of generalized closed sets called wπgr-closed sets in topological spaces.

Definition 2.1:A subset A of a topological space X is said to be

(i) pre-open if A⊆ int(cl(A)) and pre-closed if cl(int(A)) ⊆ A

(ii) semi open if A⊆ cl(int(A)) and semi-closed if int(cl(A) ⊆ A

(iii) regular open if A=int(cl(A)) and regular closed if

A=cl (int(A))

(iv) α- open if A⊆ int(cl(int(A))) and α-closed if cl(int((cl(A))) ⊆ A

(v) π-open if A is the finite union of regular open sets and the compliment of π-open is π-closed set in X .

Definition 2.2

A subset A of topological space X is said to be

(1) ω-closed set if cl(A) ⊆ U whenever A⊆ U and U ϵ SO(X).

(2) generalized closed set if cl(A) ⊆ U whenever A ⊆U and U ϵ O(X).

(3) regular generalized closed set if(briefly rg-closed set) if (cl(A) ⊆U whenever A ⊆ U and

U ϵ RO(X).

(4) weakly generalized closed set (briefly wg-closed) if cl(int(A)) ⊆ U whenever A ⊆U and

U ϵ O(X).

(5) π-generalized closed set (briefly πg –closed) if cl (A) ⊆U whenever A ⊆U and U ϵ π o(X).

(6) πgα-closed set if αcl (A) ⊆U whenever A ⊆U and U ϵ π O(X).

(7) regular α-open in X if there is a regular open set U such that U⊆A⊆αcl (U).

(8) regular generalized α-closed set (briefly rgα closed set) if αcl(A) U⊆,whenever A ⊆ U and U ϵ Rα O(X).

(9) regular weakly generalized closed set (briefly rwg closed) if cl(int(A))⊆U whenever A⊆ U and U ϵ RO(X).

(10) π*g-closed set if cl(int(A)) ⊆U whenever A⊆ U and U ϵ π O(X).

(11) πgp-closed set if pcl(A)⊆ U whenever A ⊆U and U ϵ π O(X).

(12) Pr-closed set in X if pcl(A) ⊆ U whenever A⊆U and U is regular semi open in X.

(13) rgw-closed set in X if cl(int(A))⊆U whenever A⊆U and U is regular semi open in X.

(14) generalized regular closed set (briefly g*r- closed set) if rcl(A) ⊆U whenever A⊆U and U is open.

(15) gp-closed set[2] if pcl(A) ⊆H whenever A ⊆ H and H is αg-open in (X,τ);

(16) generalized star b-closed set (briefly, g*b-closed) set[16] if bcl(A) ⊆H whenever A⊆ H where H is g-open in (X, τ)

(17) g#b-closed set [17] if bcl(A) ⊆ H whenever A⊆ H and H is αg open in X.

3. WπGR CLOSED SETS

Definition 3.1 :A subset A of X is called wπgr closed set if cl(int A)⊆U whenever A⊆U and U is πgr open in X. The complement of wπgr-closed set is called wπgr-open set in X.

We denote the family of all wπgr closed sets in X by wπGRC(X) and wπgr open sets in X by wπGRO(X)).

Result 3.2 : The union of two wπgr -closed sets need not be wπgr- closed

Example 3.3

Let X = { a , b ,c ,d } τ = { φ ,{a} , { c ,d } { a , c , d } , X } τ^c = { x , { b , c , d } , { a , b } , { b } , φ } ,wπgr – closed sets = { X , { b } ,{ c } ,{ d} , { a , b } ,{ b ,c } ,{ b ,d },{ a , b ,c } ,{ a ,b , d } ,{ b , c , d } , φ }.Let A={c} and B={d}. The set A and B are wπgr-closed and their union {c,d} is not wπgr-closed.

Result 3.4

The intersection of two wπgr- closed sets need not be wπgr- closed

Example 3.5

X = { a , b , c , d } τ = { φ , { a } ,{ d } , { a ,d } ,{ b ,d } ,{ a , b , d } , X} τ^c = { X , { b ,c , d } ,{ a , b , c } ,{ b , c } , { a , c } , { c } , φ } wπgr closed sets = { φ , { b } , { c } , { a , c } , { b , c } , { c , d } , { a , b , c } , { a , c , d } , { b , c ,d } ,X }

In this example {a , c } ∩ { a , d } = { a } is not wπgr- closed.

Theorem 3.6

If A is wπgr closed then cl(int(A))-A does not contain any non empty πgr closed set.

Proof Let F be a non empty πgr closed set such that F ⊆cl lint(A)-A ⇒ F⊆X-A ⇒ A⊆X –FSince A is wπgr closed X-F is πgr open.Since cl int (A)⊆ X-F ,F ⊆X- cl int(A). Thus F⊆clint (A) ∩ (X- cl int(A)) ⇒ F⊆φ, which is a contradiction. Thus F=φ and hence cl(int (A))-A does not contain non empty πgr closed set.

Theorem 3.7 : Let A⊆X is wπgr open iff F⊆ cl int (A) whenever A is πgr closed and F⊆A.

Proof Necessity: Let A be wπgr open. Let F be πgr closed set and F⊆A. Then X-A ⊆X-F where X-F is πgr open. Since A is wπgr open X-A is wπgr closed.Then cl(int(X-A))⊆X-F

We know that cl int(X-A)= X- int(cl (A)) ⇒ X-int(cl(A))⊆X-F ⇒ F⊆int(cl(A))

Sufficiency Suppose that F is πgr closed and F⊆A ⇒ F⊆ int cl(A)**,**Let X-A⊆U, where U is πgr open Then X-U⊆ A, X-U is πgr closed ,By hypothesis X-U⊆int cl (A) ⇒ X-int cl(A)⊆USince clint(X-A)=X-int cl(A)

The above implies cl int(X-A)⊆U whenever X-A is πgr open.Hence X-A is wπgr closed and A is wπgr open

Theorem 3.8: If A⊆X is wπgr closed then cl int(A)-A is wπgr open.

Proof : Let A be wπgr closed ,let F be a πgr closed set such that F⊆cl int(A) –A

Then F=φ (by theorem).So F⊆ int cl(clint(A) –A),for any A⊆X, int cl(clint(A) –A) =φ ⇒ cl int(A)-A is wπgr open

Definition 3.9

A space X is called a wπgr $T_{1/2}$ space if ever wπgr – closed set is closed.

Theorem 3.10: For a topological space the following conditions are equivalent.

(i) X is wπgr $T_{1/2}$ space.(ii) Every singleton of X either πgr closed or open

Proof :(i) ⇒ (ii): Let xϵ X and assume that { x } is not πgr closed. Then clearly X- { x }is not πgr open and X- { x } is trivially wπgr closed. Since X is wπgr $T_{1/2}$ space, every wπgr closed set is closed. ⇒ X- { x } is closed and hence { x } is open.

(ii) ⇒ (i) : Assume every singleton of X is either πgr closed or open . Let A⊆X be wπgr closed and obviously , A ⊆rcl (A) and let xϵ rcl(A).To prove rcl (A)⊆A.

Case (i) Let { x } be πgr closed. Suppose { x } does not belong to A . Then { x }⊆rcl(A)-A.

Since { x }ϵ A. Hence rcl (A)⊆A . The above implies rcl (A) =A . Hence A is closed.Thus every wπgr closed set is closed and hence X is wπgr $T_{1/2}$ space.

Case (ii) Let { x } be open. Since {x }ϵ rcl(A), we have {x } ∩ A ≠ φ .Hence {x }ϵ A . Therefore A is closed and hence every wπgr closed set is closed.

Theorem 3.11: (i) O(X))⊆wπGRO(X)

(ii) A space X is wπgr $T_{1/2}$ space iff O(X) = wπGRO(X)

Proof (i) Let A be open Then X- A is closed and so wπgr closed. ⇒ A is wπgr open .hence O (X) ⊆wπGRO(X)**.**(ii) Let X be wπgr $T_{1/2}$ - space . Let A ϵ wπGRO(X). Then X-A is wπgr closed. Since the space X is wπgr $T_{1/2}$ – space, X-A is closed. The above implies A is open in X.Hence O(X)= wπGRO(X).**Sufficiency**: Let O(X)= wπGRO(X).Let A be wπgr closed .Then X-A is wπgr open and X-A ϵ O(X). Hence A is closed and Hence a wπgr $T_{1/2}$ space.

CONCLUSION

During the last few years the study of generalized closed sets has found considerable interest among general topologists. One reason is these objects are natural generalizations of closed sets. More importantly, generalized closed sets suggest some new ideas which have been found to be very useful in the study of certain objects of digital topology.

The aim of this paper is to introduce the concepts of weakly π generalized regular closed sets.

CONFLICTS OF INTEREST

There is no conflicts of interest.

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