DOI: https://doi.org/10.24297/jam.v22i.9543

Suggested Approach to Solve Nonlinear Ordinary Differential Equations

Noor A. Hussein[1] , Firas Jawad Obaid[2]

[1] Department of Mathematics, College of Education, University of Al-Qadisiyah, Diwaniya-Iraq

[2] Department of Mathematics, College of Education, University of Al-Qadisiyah, Diwaniya-Iraq

[1]noor.ali@qu.edu.iq [2]fieras.joad@qu.edu.iq

Abstract:

In this research a suggested method has been proposed to solve the non-linear ordinary differential equations, This method is somewhat similar to the Adomain decomposition method of solving equations but it is quite different in the using steps to find the non-linear boundaries , in the suggested method does not use an unknown parameter to calculate non-linear boundaries as well as the steps used to calculate non-linear boundaries are very simple and do not need many and complex steps. we can be called this approach by Fatima style to solve nonlinear ODE.

Keywords: suggested approach, nonlinear terms, nonlinear ordinary differential equations.

Introduction

In many scientific disciplines, including hydrodynamics, solid-state physics, and plasma physics, complex processes are often described using nonlinear differential equations (NLDEs), as they are known. It is of great importance to search for the exact solutions of these nonlinear equations. As a result, several effective techniques have been introduced. For example, homogeneous equilibrium method [1], Jacobi elliptic function method [2], algebraic method [3], inverse scattering method [4], contrast method [5, 10, 11], decomposition method [6], Backlund transform [7], sine-Gordon expansion method [8], Hirota's bilinear method [9] and so on to solve these differential equations. In contrast to earlier methods, particularly the Adomain decomposition method, we describe a fresh approach to the resolution of equations of nonlinear differential type that makes it simple to calculate nonlinear terms and produce accurate solutions.

Description of the suggested approach (Fatima approach)

The steps of the suggested approach are described in this section. First, it's described in terms of the ordinary differential equation; take the following into consideration for the nonlinear ordinary differential equation;

$$L(y(t)) + R(y(t)) + N(y(t)) = g(t) \quad (1)$$

R denotes the rest of the differential operator, nonlinear terms are expressed by N(y), and an inhomogeneous term is denoted by g(t). The differential operator L can be assumed to be the highest-order derivative in this equation.. Consider the operator L is

$$L = \frac{d^n}{dt^n}, \quad n = 1, 2, 3, ... \quad (2)$$

Next, let's assume that L may be inverse, and the inverse operator for L-1 will be written out as

$$L^{-1}(.) = \underbrace{\int_0^t \int_0^t ... \int_0^t (.)dtdt...dt}_{n-times} \quad (3)$$

When L^{-1} is applied to both sides of eq. (1), the result is

$$y(t) = \omega_0 + L^{-1}g(t) - L^{-1}R(y) - L^{-1}N(y) , \quad (4)$$

where

$$\omega_0 = \{y(0) \quad for\ L = \frac{d}{dt},\ y(0) + ty'(0) \quad for\ L = \frac{d^2}{dt^2},\ y(0) + ty'(0) + \frac{1}{2!}t^2 y''(0)\ for\ L = \frac{d^3}{dt^3}, \quad (5)$$

And so on . The equation which most accurately captures the solution can be summed up as an infinite series

$$y(t) = \sum_{k=0}^{\infty} y_k \quad (6)$$

$$y_0 = \omega_0 + L^{-1}g(t),$$

$$y_{k+1} = -L^{-1}R\left(y_k\right) - L^{-1}N_k$$

as well as the nonlinear factor N(y), which can be calculated as

$$N(y) = \sum_{k=0}^{\infty} N_k \quad , \tag{5}$$

where

$$N_k = \frac{1}{k!}\left[\frac{d^k N(y)}{dt^k}\right]_{t=0} \quad , \; where \;\; \frac{d^k}{dt^k}y = k!\, y_k, \;\; k = 0, 1, 2, \ldots$$

Application

In this section we introduce two examples by using this method, where these examples showed simplicity and convergence this method if compare with Adomain decomposition method.

Example 1: Take into account the differential equation, which is

$$y' - y^2 = 1, \;\; y(0) = 0 \tag{6}$$

Write the eq.(6) by operator L, where $L = \frac{dy}{dt}$

$L(y) = 1 + y^2$, taking L^{-1} we get

$y = t + L^{-1}\left(y^2\right), \;\; y^2 = N(y)$

$y_0(t) = t$

$y_{j+1} = L^{-1}\left(N_j(y)\right)$

$N_0 = {y_0}^2$

$N_1 = 2yy' = 2y_0y_1$

$N_2 = \frac{1}{2!}\left(2yy'' + 2(y')^2\right) = 2y_0y_2 + {y_1}^2, \ldots$

Thus, $y_1 = \frac{1}{3}t^3, y_2 = \frac{2}{15}t^5, \ldots$

So, $y(t) = t + \frac{1}{3}t^3 + \frac{2}{15}t^5 + \ldots$

The close solution is $y(t) = tan(t)$

Example 2: To solve the equation

$$y' - e^y = 0, \; y(0) = 1 \tag{7}$$

$y(x) = 1 + L^{-1}\left(e^y\right)$

$y_0(t) = 1$

$y_{j+1} = L^{-1}\left(N_j(y)\right)$

$N_0 = e^{y_0}$

$N_1 = y'e^y = y_1e^{y_0}$

$N_2 = \frac{1}{2!}\left(y'' + (y')^2\right)e^y = \left(y_2 + \frac{1}{2}{y_1}^2\right)e^{y_0}, \ldots$

Thus,

$y_0(t) = 1$

$y_1(t) = et$

$y_2(t) = \frac{e^2}{2}t^2$

$y_3(t) = \frac{e^3}{3}t^3, \ldots$

$So, y(t) = 1 + et + \frac{e^2}{2}t^2 + \frac{e^3}{3}t^3 + \ldots$

The exact solution is $y(t) = 1 - ln(1 - et), \quad -1 \le et \le 1$

Conclusion

For the purpose of this article, the typical non-linear differential equations were solved using suggested approach. The straightforward procedures allowed for the effortless acquisition of accurate solutions to the non-linear equations. This is a unique way of calculating the non-linear boundaries in the differential equation when compared with other methods, particularly the Adomian decomposition approach because it produced the same findings without utilizing any unknown parameters or complicated rules to calculate those boundaries. This makes it a standout method for calculating the non-linear boundaries in the differential equation.

References

1. Sheama Amer, Homogeneous Balance Method for Solving Nonlinear Dispersive Dissipative Equation, Al-Mustansiriyah J. Sci Vol. 22, No. 6, 2011
2. Mohammed K. Elboree, The Jacobi elliptic function method and its application for two component BKP hierarchy equations, Computers and Mathematics with Applications 62 (2011) 4402–4414, journal homepage: www.elsevier.com/locate/camwa .
3. Golomb, M. (1965). An Algebraic Method in Differential Equations. The American Mathematical Monthly, 72(10), 1107. doi:10.2307/2315962.
4. M.J. Ablowitz and P.A. Clarkson, Solitons, Nonlinear Evolution Equations and Inverse Scattering, Cambridge University Press, Cambridge, (1991).
5. J.H.He, Variational iteration method for autonomous ordinary differential systems, Appl. math. Comput., 114(2/3), 115-123, (2000).
6. G. Adomian, Solving frontier problems of physics: The decomposition method, Kluwer Academic Publishers, Boston and London, 1994.
7. Rasin A.G. and Schiff J., “Backland Transformations for the Boussinesq Equation and Merging Solitons”, Journal of Physics A Mathematical and Theoretical·August (2017) 17.
8. Kundu, P. R., Fahim, M. R. A., Islam, M. E., & Akbar, M. A., “The sine-Gordon expansion method for higher-dimensional NLEEs and parametric analysis”, Heliyon, (2021) 7(3).
9. A-M. Wazwaz, Partial Differential Equations and Solitary Waves Theory , Higher Education Press, Beijing and Springer-Verlag Berlin Heidelberg, 2009.
10. N. A. Hussein, New Approach for solving (2+1)-dimensional differential equations, College of Education for Pure Sciences, University of Baghdad, 2022.
11. J. H. He and X. H. Wu, Variational iteration method: New development and applications, Computers & Mathematics with Applications, 54 (7-8): 881–894, 2007.
12. A.-M. Wazwaz, The Variational iteration method for solving linear and nonlinear systems of PDEs, Computers and Mathematics with Applications, 54: 895–902, 2007.
13. A. K. Jabber, L. N. Tawfiq, New Approach for Solving Partial Differential Equations Based on Collocation Method, Journal of Advances in Mathematics Vol 18 2020.
14. D. Zwillinger, "Handbook of Differential Equations", 3rd edition Academic Press, 1997.