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Applications of the ideals in the measure theory and integration

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Abstract

In this paper, we will represent some applications to various problems of mass theory and integration, by using the concept of local convergences and exhaustive sequences. We will continue the idea of point-wise I -convergence, Ideal exhaustiveness that was introduced by Komisarski [3], and Kostyrko, Sal´at and Wilczy´nski [4]. The equi-integrable introduced in Bohner-type ideal integrals and a new study on the application of symmetric differences have been presented in the theory of mass and continuous functions, continuing the results of Boccuto, Das, Dimitriou, Papanastassiou [2].

Keywords: Ideal exhaustiveness, Banach spaces, Weak convergence, Bohner-type ideal integrals, Symmetric differences, Point-wise I -convergence, Weak compactness, Δ- continuity, Δ- convergence in a discrete

1. Introduction

Definition 1.1. [1], [2]

(a) Let Y be a set that is not the empty set, W≠∅. Family $I\subset\wp$(W) is called the *ideal of the set* W if and only if, that for A, B∈I it follows that, A∪B∈I, and for every A∈I and B ⊂ A we will have B ∈I.

(b) The ideal I is called *non-trivial* if and only if, I≠∅, and y∉I. A non-trivial ideal is called *acceptable* when it contains sets with only one point on them.

Let $(T,\ \Sigma,\ \mu)$ be a space with probabilistic measure μ, where T is a random set on a line, Σ-Borel's algebra, and μ is a defined measure.

Definition 1.2. The function $f:\ T\rightarrow X$, where X is a vector space is called *a simple function* according to μ, if for every family of measurable sets $\{E_i\}$ that have no common point, so $E_i\subset T$ and $E_i\cap E_j = \emptyset$, for $i\neq j$, where $T\ =\ \cup_{i=1}^{n}E_i$ and $f(t) = x_i$, for $t\in E_i$, i=1, 2, ..., n.

As we know before, the simple function is defined $f(t) = \sum_{i=1}^{n} x_i\chi_{E_i}$, where χ_{E_i} is a characteristic function of E_i.

Definition 1.3. The function $f: T\rightarrow X$ is called I*-measurable on* T if for every t∈T, $\varepsilon\ >\ 0$ and $A\subset I$ there is a sequence of simple functions $f_n: T\rightarrow X$ for which we have

$\left|f_n(t) - f(t)\right| < \varepsilon$ for $n\in N\backslash A$.

Proposition 1.4.[1] The linear combination of functions I-measurable (measurable ideals) is an I-measurable function.

Proof: Let f and g be two I*-measurable* functions. For the function f, we will find a sequence of simple functions $f_n(t)$ that I -converge to the function f.

This means that for every ε>0, also for $\frac{\varepsilon}{|\alpha|}\ >\ 0$ and a set $A_1\in I$ such that, $\left|f_n(t) - f(t)\right| < \frac{\varepsilon}{2|\alpha|}$ for $n\in N\backslash A_1$ and t∈T.

Similarly, for the I- measurable function g(x), there is the sequence of simple functions $g_n(t)$ that I -converges to g(x), i.e., for every $\varepsilon\ >\ 0$ also for $\frac{\varepsilon}{2|\beta|}\ >\ 0$ there is $A_2\in I$ so that for $n\in N\backslash A_2$ we have $\left|g_n(t) - g(t)\right| < \frac{\varepsilon}{2|\alpha|}$. Because $N\backslash A_1\ \cup\ N\backslash A_2\ \subset\ N\backslash\left(A_1\ \cup\ A_2\right)$ for an element

$n\in N\backslash\left(A_1\ \cup\ A_2\right)$ and t ∈T we will have:

$\left|\left(\alpha f_n(x) + \beta g_n(x)\right) - (\alpha f + \beta g)\right| \le |\alpha|\left|f_n(x) - f(x)\right| + |\beta|\left|g_n(x) - g(x)\right| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.$

Definition 1.5. The subsequence $\left(f_{n_k}\right)_{k\in N}$ of the sequence $\left(f_n\right)_{n\in N}$ $I\to f$ is called fundamental if, for able to include your paper in the Proceedings. When citing references in the text of the abstract, type the corresponding number in square brackets as shown at the end of this sentence [2].

$A' = \{n_1 < n_2 < \dots < n_k < \dots\}$; f_{n_k} $I\to f$ for $n\in N\backslash A'$ where $A'\subset A$.

Definition 1.6.[2] Let (I, Σ, μ) be a measurable complete space with a non-negative measure. The sequence of measured functions $\left(f_n\right)_n$ in I is I*-convergent according to the measure* μ to the function f if, for each ε> 0 and σ> 0, there is an essential subsequence $\left(f_{n_k}\right)_k$ of the sequence $\left(f_n\right)_n$ such that: $\mu\left\{t: \left|f_{n_k}(t) - f(t)\right| \ge \sigma\right\} < \varepsilon$ for $n_k\in N\backslash A'$ and t ∈I.

We denote $f_n(t)\ I - \mu\to f(t)$.

Definition 1.7. The sequence of measured functions $\left(f_n\right)_n$ with values in R is called I*-fundamental according to the measure* μ, $S\subset I$, if there is a natural number $(\sigma, S)\subset N\backslash A$ and there is a subsequence $\left(f_{n_k}\right)_k$ of $\left(f_n\right)_n$, $if\ \forall\varepsilon > 0$ and $\sigma > 0$, $\mu\left\{t: \left|f_{n_k}(t) - f(t)\right| \ge \sigma\right\} < \varepsilon$.

Proposition 1.8. If the sequence $\left(f_n\right)_n$ is I-convergent to f in I it is I-fundamental.[2]

Proof: Let $\left(f_{n_k}(t)\right)_k$ be a fundamental subsequence of the sequence $\left(f_n(t)\right)$? We choose a number $N\in N$, $N\in N\backslash A$ and consider the inequality:

$\left|f_{n_k}(t) - f_N(t)\right| \le \left|f_{n_k}(t) - f(t)\right| + \left|f(t) - f_N(t)\right|$; from here we can write:

$$\left\{t: \left|f_{n_k}(t) - f_N(t)\right| \ge \sigma\right\} \subset \left\{t: \left|f_{n_k}(t) - f(t)\right| \ge \frac{\sigma}{2}\right\} \cup \left\{t: \left|f(t) - f_N(t)\right| \ge \frac{\sigma}{2}\right\}$$

Since we get that for $n_k\in N\backslash A'$, $A'\subset A$.

$$\mu\left\{t: \left|f_{n_k}(t) - f_N(t)\right| \ge \sigma\right\} \le \mu\left\{t: \left|f_{n_k}(t) - f(t)\right| \ge \frac{\sigma}{2}\right\} + \mu\left\{t: \left|f(t) - f_N(t)\right| \ge \frac{\sigma}{2}\right\}$$

Proposition 1.9. [1] I-limit of the sequence $\left(f_n(t)\right)_n$ according to the measure μ is unique with the proximity of equivalence.

Proof: Let us assume that the statement is not true. This means that the sequence $\left(f_n\right)_n$ I-converges in two different limits $f_1(t)$ and $f_2(t)$. For every $\varepsilon > 0$ and $\sigma > 0$, there exists a fundamental substring such that,

$\mu\left\{t: \left|f_{n_k}(t) - f_1(t)\right| \ge \frac{\sigma}{2}\right\} < \frac{\varepsilon}{2}$ for $n_k\in N\backslash A_1'$ when $A_1'\subset A$ and $\mu\left\{t: \left|f_{n_k}(t) - f_2(t)\right| \ge \frac{\sigma}{2}\right\} < \frac{\varepsilon}{2}$ for $n_k\in N\backslash A_2'$ when $A_2'\subset A$ and t ∈ T, where inclusion takes place:

$$\left\{t: \left|f_1(t) - f_2(t)\right| \ge \sigma\right\} \subset \left\{t: \left|f_{n_k}(t) - f_1(t)\right| \ge \frac{\sigma}{2}\right\} \cup \left\{t: \left|f_{n_k}(t) - f_2(t)\right| \ge \frac{\sigma}{2}\right\}$$

for $n_k\in N\backslash(A_1' \cup A_2')$ when $A_1' \cup A_2'\subset A$, or taking the measures of both sides;

$$\mu\{t: |f_1(t) - f_2(t)| \geq \sigma\} \leq \mu\left\{t: \left|f_1(t) - f_{n_k}(t)\right| \geq \frac{\sigma}{2}\right\} + \mu\left\{t: \left|f_{n_k}(t) - f_2(t)\right| \geq \frac{\sigma}{2}\right\} < \varepsilon$$

The above inequality shows that $f_1(t)$ and $f_2(t)$ can be different only in a set of zero measures.

Proposition 1.10. [1] If the sequence $(f_n)_n$ is a I —fundamental sequence in the $T\subset R$, then there exists an $I - \int f_k(t)d\mu$.

Definition 1.11. [1] The function $f: T\to X$ is called *I-Bochner integrable,* if there is a fundamental sequence of simple functions $(f_k)_k$ such that,

a) $(f_k)_k$ is I- integrable to f.

b) $I - \int|f_k(t) - f_N(t)|d\mu = 0$ almost everywhere $I - B - \int f(t)d\mu$ and is called I-Bochner integral.

Proof:

a) Since $(f_n)_n$ is an I-fundamental sequence for every $\varepsilon > 0$, there exists $k\in N\backslash A$ and fixed natural N such that $|f_k(t) - f_N(t)| < \frac{\varepsilon}{\mu(T)}$ almost for all $k\in N\backslash A$.

b) We will have: $\left|\int_T f_k(t)d\mu - \int_T f_N(t)d\mu\right| \leq \int_T |f_k(t) - f_N(t)|d\mu \leq |f_k(t) - f_N(t)|\mu(T) < \varepsilon$

This shows that the sequence $\left(\int f_n(t)d\mu\right)_n$ is a fundamental sequence in R and as such is convergent.

2. Applications of symmetric difference in mass theory.

Let X be an infinite set and Σ⊂Π (X) a σ-algebra. The set of all finite sum measures with real values in Σ will be marked (Σ); and with Ca(Σ) the linear subspace of ba(Σ), consisting of all σ-sum measures in Σ.

The following are the meanings of the exhaustive ideals and the ideals of other convergences in the framework of the mass theory.[1]

Definitions 2.1.[2]

(a) For a positive mass λ ∈ ba(Σ) and A, B∈Σ (pseudo) - λ- the distance between A and B is determined by the equation: d_λ (A, B) = λ (AΔB) where Δ denotes the symmetric difference.

(b) A mass μ ∈ ba(Σ) is a constant λ- I, in the E∈Σ set, if for every ε> 0, there is δ> 0 and A ∈ I such that, for any F∈Σ that d_λ (F, E) = λ (FΔE) <δ to have | μ (F) -μ (E) | <ε for n ∈👁 \ A.

(c) It will be stated that μ is continuous λ- I in Σ if μ is continuous λ- I in any E∈Σ.

We note that, as defined in mass theory, μ is λ-absolutely continuous if μ is λ-continuous at x. By agreement, when F⊂E we will say that E is continuous λ when for any ε> 0, ∃δ> 0 such that when d_λ (F, E) =λ (E \ F) <δ we will have | μ (F) -μ (E) | = | μ (E \ F) | <ε.

Similarly, when E⊂F then d_λ (F, E) = λ (F \ E) <δ we will have for n∈👁 \ A, μ (F \ E) <ε, when E = F, d_λ(E, F) = 0.

The sequence f_n of simple functions is called the determinant sequence of functions of the function f.

That is easily proved the proposition:

If (f_n) and (g_n) are two determinant and fundamental sequences of the same function, then

$$I - f_n(t)\, d\mu = I - g_n(t)\, d\mu$$

This proves that the proposition is correct.

Note that, as defined by the measure theory, μ is λ-absolutely continuous if μ is λ- continuous ∅. By agreement, when F⊂E we will say that E is λ continuous, when for any $\varepsilon > 0, \exists\delta > 0$ such that

$d_\lambda(F, E) = \lambda(E\backslash F) < \delta$ we will have $|\mu(F) - \mu(E)| = |\mu(E\backslash F)| < \varepsilon$.

In the same way, when $E \subset F$ then $d_\lambda(F, E) = \lambda(F\backslash E) < \delta$, we will have for $n \in N\backslash A$, $\mu(F\backslash E) < \varepsilon$, when $E = F$, $d_\lambda(E, F) = 0$.

Proposition 2.2. (a) If $F_1 \subset F_2 \subset ... \subset F_n \subset$ where, $F_n \in \Sigma$, $F_n \in ba(\Sigma)$ and

$F = \cup_n F_n, \in ba(\Sigma)$, or

(b) for the sequence $E_1 \supset E_2 \supset ... \supset E_n \supset ...$ when $E_n \in ba(\Sigma)$, $E_n \in \Sigma$ and

$$E = \cap_n E_n$$

Measure μ is continuous and

$$\mu(F\Delta F_n) = \mu(E_n \Delta E) = 0$$

It follows that: $\mu(F_n) = \mu(F)$ and $\mu(E_n) = \mu(E)$

Definition 2.3.[1]

Let I-be an acceptable ideal in the set of natural numbers 👁.The sequence of measures $(\mu_n)_{n\in N}$ from $ba(\Sigma)$ is I-*exhaustive* in $E \in \Sigma$, if for each $\varepsilon > 0$, there exists $\delta > 0$ and the set $A \in I$ such that, $|\mu_n(E) - \mu_n(F)| < \varepsilon$ for each $F \in \Sigma$, that $d_\lambda(E, F) < \delta$, and for each $n \in N\backslash A$.

It says that the sequence $(\mu_n)_{n\in N}$ is I-*exhaustive in* Σ, if the sequence is I-exhaustive for each $E \in \Sigma$.

(b) It says that the sequence $(\mu_n)_{n\in N}$ is I_α-*convergent* to μ in the set $E \in \Sigma$, if for each sequence $(E_n)_n$ in Σ, for which I-limit $d_\lambda(E_n, E) = 0$ we take $I - \mu_n(E_n) = \mu(E)$.

It says that the sequence $(\mu_n)_{n\in N}$ is I_α-*convergent to* μ *in* Σ, if it is I_α-convergent to μ in every $E \in \Sigma$.

(c) It says that the sequence $(\mu_n)_n$ is I_δ-*convergent to*μ in the set $E \in \Sigma$, if for each $\varepsilon > 0$, there exists $\delta > 0$, such that, for each $F \in \Sigma$ and $d_\lambda(F, E) < \delta$, we have

$|\mu_n(F) - \mu(E)| < \varepsilon$ for $n \in N\backslash A$.

The set of all subsets $F \in \Sigma$, for which $d_\lambda(F, E) < \delta$ for a fixed δ we will denote V_δ^E and will be called the neighborhood of *E*. It seems clear that $d_\lambda(E, E) = 0$. Consequently $E \in V_\delta^E$.

Notation 2.4.[1] The sets $\{V_\delta^E\}$ are used as a substructure of a topology, where the openings are

G = {union of sets by Σ} which have a neighborhood of each element of them

It is proved that:

(1) The union of G_n is opened.

(2) The intersection of a finite number of opened sets is an opened set.

We call this, a topology of the neighborhood of the sets.

For each ε> 0, there is a δ> 0 such that, for every $F \in V_{\delta}^{E}$ we will have:

$\left|\mu_n(F) - \mu(E)\right| < \varepsilon$.

Proposition 2.5.[1] From the above definitions, it is clear that when $\left(\mu_n\right)_n$ is I_n-convergent, it is and I_{δ}-convergent.

(1) We state that if the sequence $\left(\mu_n\right)_n$ is I_{α}--convergent, it is also I_{δ_a} -convergent.

(2) Measure μ is λ- continuous.

Proof.

Let $\left(E_n\right)_n$ be a set of sets Σ, for which $I - d_{\lambda}\left(E_n, E\right) = 0$ for

$n \in N \backslash A_1, A_1 \in I$. That means they exist $n \in N \backslash A_1^{'}$ that $d_{\lambda}\left(E_n, E\right) < \delta$ where $A_1^{'} \subset A_1$.

That is, for $\in N \backslash A_1^{'}$, $E_n \in V_{\delta}^{E}$. Since there is I_{δ_a}-convergence, there exist for every $\varepsilon > 0$, the set A_2 such that for

$n \in N \backslash \left(A_2 \cup A_1^{'}\right)$

$\left|\mu_n\left(E_n\right) - \mu(E)\right| < \varepsilon$.

Proposition 2.6. If, under the above conditions, the series of measures $\left(\mu_n\right)_n$ -I_{δ}-convergent in μ, when $\mu_n \in ba(\Sigma)$ is I_{δ}-convergent and I -exhaustive in Σ, then the measure μ is continuous in Σ.

Proof: Since the series $\left(\mu_n\right)_n$ is I_{δ}-convergent to μ on a set E in Σ, for every $\varepsilon > 0$ and for $\frac{\varepsilon}{3}$, there exists $\delta_1 > 0$ such that, for every $F \in V_{\delta_1}^{E}$ and $n \in N \backslash A_1$ to have: $\left|\mu_n(F) - \mu(F)\right| < \frac{\varepsilon}{3}$ (*)

Also, since the sequence

$\left(\mu_n\right)_n$ is I -exhaustive, for each $\varepsilon > 0$, and for $\frac{\varepsilon}{3}$ will be found $\delta_2 > 0$ such that for $F \in V_{\delta_2}^{E}$ to have:

$\left|\mu_n(F) - \mu_n(E)\right| < \frac{\varepsilon}{3}$ (**) for $\in N \backslash A_2$.

Denote $\delta = min\left\{\delta_1, \delta_2\right\}$ then $V_{\delta_1}^{E} \cap V_{\delta_2}^{E} = V_{\delta}^{E}$ për $F \in V_{\delta}^{E}$ and

$n \in N \backslash \left(A_1 \cup A_2\right)$.

Consider the inequalities:

$$|\mu(E) - \mu(F)| \leq \left|\mu(E) - \mu_n(E)\right| + \left|\mu_n(E) - \mu_n(F)\right| + \left|\mu_n(F) - \mu(F)\right| < \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \varepsilon$$

having the two inequalities (*) and (**).

Proposition 2.7. Let I be an ideal in N and E is any element from Σ. If the set $\left(\mu_n\right)_n$, where $\mu_n \in ba(\Sigma)$ is I_{δ_a} -convergent to μ in E then:

(1) The sequence $\left(\mu_n\right)_n$ is exhaustive.

(2) The measure μ is λ-continuous.

Proof: (a) We start from the definition of I_{δ_a}-convergence in set E. For every $\varepsilon > 0$ and for $\frac{\varepsilon}{2}$, there exists $\delta > 0$ and $A_1 \in I$ such that, for $n \in N \setminus A_1$ and $F \in V_\delta^E$ to have:

For these n and δ>0, we consider the inequality:

$\left|\mu_n(E) - \mu_n(F)\right| \leq \left|\mu_n(E) - \mu(F)\right| + \left|\mu(F) - \mu_n(F)\right| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$

Here we used the fact that when $F \in V_\delta^E$ means that $d_\lambda(F, E) = \lambda(F \Delta E) = \lambda(E \Delta F)$.

The above inequality proves the point (a).

$$\left|\mu_n(F) - \mu(E)\right| < \frac{\varepsilon}{2}$$

(b) From point (a) we have that $\left(\mu_n\right)_n$ is an exhaustive sequence. Starting from the definition for each $\frac{\varepsilon}{3}$, $\exists \delta_2 > 0$, that for $F \in V_{\delta_2}^E$ we will have:

$\left|\mu_n(F) - \mu_n(E)\right| < \frac{\varepsilon}{3}$ for $n \in N \setminus A_1$.

For $F \in V_\delta^E \cap V_{\delta_2}^E = V_{\delta_1}^E$ and $n \in N \setminus (A \cup A_1)$.

Consider the inequality:

$|\mu(E) - \mu(F)| \leq \left|\mu(E) - \mu_n(E)\right| + \left|\mu_n(E) - \mu_n(F)\right| + \left|\mu_n(F) - \mu(F)\right| < \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \varepsilon$

Inequality that proves the statement (b).

3. Δ-Continuity

Let (X, Σ, λ) and (Y, Σ', μ) be two measured spaces and the sets ℘(X) and ℘(Y) of them.[2]

Definition 3.1. The set function f: X →Y is called *Δ-I continuous* in the subset A, if for each ε> 0 there exists δ(ε) > 0 such that, for every set G ∈℘(X)∩Σ that λ(GΔA) <δ to have μ(f(G)Δf(A)) <ε.

To denote B (A, δ)={G ∈℘(X)∩Σ: λ(GΔA) <δ} that is an *open ball* with the support of *A* and B(f(A), ε) *open ball* with the support of *f(A)* the above definition can be geometrized:

The set function f is continuous in the subset A of X if for every open ball B(f(A), ε) from ℘(Y) there exists an opened ball B (A, δ) from ℘(X) such that, for each G∈ B (A, δ) it follows that f(G) ∈ B(f(A), ε) (or f (B (A, δ) ⊂ B(f(A), ε)).

Definition 3.2.[1]

(a) It is said that the sequence (f_n) *Δ-converges in a discrete way* to an element G ∈𝒜, where 𝒜 is a collection of sets, if for each element ε> 0, there exists n_0(ε,G) ∈ℕ such that, for n > n_0(ε,G) we have μ(f_n(G)Δf(G)) <ε.

(b) It is said that the sequence (f_n) *Δ-converges in a uniform way* if, for each ε> 0 and any G ∈𝒜, there exists n_0(ε) ∈ℕ such that, for n > n_0(ε), we have μ(f_n(G)Δf(G)) <ε.

(c) It is said that the sequence (f_n) is *Δ-exhaustive* in the subset G ∈𝒜 where 𝒜 is a collection of sets if for each ε> 0, there exists a natural number n_0(ε) ∈ℕ and δ>0 such that for n > n_0(ε) and every subset H ⊂ B (G, δ) we have μ(f_n(H)Δf_n (G)) <ε.

Proposition 3.3. Let f_n, f: X →℘(Y) be a set of functions. If the sequence (f_n) is Δ-convergent in a uniformly and with terms Δ- continuous then it converges to f Δ- continuous.

Proof: The functions f_n are Δ-continuous on the subset G∈℘(X) such that, for every ε/3> 0 there exists δ_1> 0 such that, for every A∈℘(X) that λ(AΔG) < δ_1 to have that:

μ(f_n (A) Δf_n (G)) < ε/3.

According to the Δ- uniform convergence of the sequence for ε/3>0 there is an n_0(ε)∈ℕ such that, for n > n_0 and every G∈℘(X) we have μ(f_n(G)Δf(G)) < ε/3. For n > n_0 and δ_1> 0 we consider the inclusion

$$f(G)\Delta f(A) \subset f(G)\Delta f_n(G) \cup f_n(G)\Delta f_n(A) \cup f_n(A)\Delta f(A)$$

Comparing the measure μ of the two sides and the semi-integral property, we get, $\mu(f(G)\Delta f(A)) < \varepsilon$.

Proposition 3.4. If the sequence (f_n) Δ-converges in a uniform way in ⛷(X) to the function f, then it is Δ-exhaustive in each G∈⛷(X).

Proof. From the above Proposition 3.2. It turns out that the function f is Δ-continuous in the subset G∈🏛∈⛷(x), so for every ε/3> 0 there exists δ such that when for A∈🏛 that $\lambda(A\Delta G) < \delta$ we have that $\mu(f(G)\Delta f(A)) < \varepsilon/3$.

Using the Δ-uniform convergence of the sequence (f_n), for every $\varepsilon/3 > 0$ and every G∈🏛 there exists an $n_0(\varepsilon)$∈👁 such that, for $n > n_0$ to have $\mu(f_n(G)\Delta f(G)) < \varepsilon/3$. For $n > n_0$ and a given δ we consider inclusion

$$f_n(G)\Delta f_n(A) \subset f_n(G)\Delta f(G) \cup f(G)\Delta f(A) \cup f(A)\Delta f_n(A)$$

Taking the measure μ of both sides, we will have for $n > n_0$, $\mu(f_n(G)\Delta f_n(A)) < \varepsilon$.

👁 and δ >0 such that for $n > n_0(\varepsilon)$ and every set $H \subset B(G, \delta)$ to have $\mu(f(H)\Delta f_n(G)) < \varepsilon$.

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