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**On hyper Z-algebras**

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**Abstract:**

This study introduces the concept of hyperZ-algebra and investigates its features. In addition, we establish and prove a number of theorems about the relation between(R̤-ḧ$Z$ , Ḉ-ḧ$Z$ , D̂-ḧ$Z$ , T̂-ḧ$Z$ , Ṿ-ḧ$Z$ ). Moreover, we explain the hyper subalgebra ,a weakhyper Z-ideal and a strong hyper V-ideal, as well as their relationship. Finally, the hyper homomorphism Z-algebra is constructed and the isomorphism theorems are examined.

**Keywords:** Z-algebra, hyper Z-algebra, hyper Z-subalgebra, weak hyper Z-ideal, strong hyper Z-ideal , hyperZ-homomorphism.

**1. Introduction**

Marty's talk about hypergroups at the Eighth Congress of Scandinavian Mathematicians in [1] 1934 was the first step in the study of hyperstructures. Since then, a lot of mathematicians and scientists have looked at the hyperstructure idea from both an applied and a theoretical point of view and expanded it to a wide range of fields. It has been applied in several fields, including Euclidian and non-Euclidian geometries, probability, information sciences, etc. The book [2] has several intriguing uses of hyper-structures that are shown throughout.
In the paper [3], Jun et al. They came up with the idea of a hyperBCK-algebra, that's also extension of a BCK-algebra, and looked at the different properties it has. Borzooei et al. [4] presented and researched hyperK-algebras. In 2006, Xin came up with the idea of hyperBCI-algebras, which are just an extension of hyperBCK-algebras. He showed that any hyperBCK-algebra is also a hyperBCI-algebra [5].
Borzooei et al. have created the concept of hyperBCC-algebra as an extension of BCC-algebras and they have specifically investigated and defined the various types of hyperBCC-ideals in [6]. J.C. Endam introduced the idea of hyperB-algebra That is an extension of B-algebra and defined the subhyperB-algebras in [7]. A.L.O.Vicedo et al,[8] Proposed and analyzed (weak, strong) hyperB-ideals. In addition, they investigated the relationships between hyperB-ideals and subhyper B-algebras, as well as a few relationships between hyperB-algebras and hypergroups. S.Niazian[9] introduced the idea of hyperstructure to BI-algebras and he defined the types of hyperBI-algebras.
Z-algebra is a unique algebraic structure based on logic that was first proposed in 2017 by Chandramouleeswaran et al. [10].
This papers is structured following: in Part 2 We present some definitions and first observations regarding Z-algebras that will be used in the next Part. Hyper Z-algebra is a concept that is introduced and some of its characteristics are looked at in Part 3.

**2.Preliminaries:**

**Denition 2.1:** [8] The map $\circledast: Z \times Z \to P(Z)$ named a hyperoperation on $Z \neq \phi$ , where $P(Z)$ is the powerset of $Z$. And , $Z$ with $\circledast$ is called a hypergroupoid. If $\phi \neq Ꝁ \subseteq Z, \phi \neq Ŧ \subseteq Z$ , then $Ꝁ \circledast Ŧ$ means for any $k \in Ꝁ$ *and* $t \in Ŧ$:

$$Ꝁ \circledast Ŧ = \cup_{k \in Ꝁ,\, t \in Ŧ} (k \circledast t),\ k \circledast Ŧ = \{k\} \circledast Ŧ = Ꝁ \circledast t = Ꝁ \circledast \{t\}$$

**Definition2.2**: [10] let $Z \neq \phi$ and * is a binary operation with constant $O$ then the algebra $(Z, *, O)$ named Z-Algebra if satisfying the following axiom:

$Z_1$: $Ʈ * O = O$

$Z_2$: $O * Ʈ = Ʈ$

$Z_3$: $Ʈ * Ʈ = Ʈ$

$Z_4$: Ꞇ * Ꝣ = Ꝣ * Ꞇ When Ꞇ≠$O$ and Ꝣ≠$O$, ∀ Ꞇ, Ꝣ∈$Z$.

**3. HyperZ-algebras:**

**Definition 3.1:** The triple $(Z, ⊛, O)$ is named a hyperZ-algebra (briefly,ḧ$Z$) where $Z$≠ϕ , ⊛ hyperoperation and $O$ is a constant if ∀ Ꞇ , Ꝣ ∈$Z$, the following conditions hold:

$(HZ_1)$ Ꞇ ≪ $O$

$(HZ_2)$ Ꞇ∈ Ꞇ⊛$O$

$(HZ_3)$ Ꞇ∈ Ꞇ⊛Ꞇ

$(HZ_4)$ Ꞇ⊛Ꝣ = Ꝣ⊛Ꞇ , ∀ Ꞇ, Ꝣ ∈$Z$, Ꞇ≠$O$, Ꝣ≠$O$

Where Ꞇ ≪ Ꝣ if $O$∈Ꞇ⊛Ꝣ.

∀ $A$, $B$⊆$Z$ , then $A$ ≪ $B$ ⇔∃ $a$∈$A$ , $b$∈$B$ such that $a$≪$b$ .We signify $A$ ≪ {Ꝣ}({Ꞇ} ≪ $B$) by $A$ ≪ Ꝣ (Ꞇ ≪ $B$)

**Example 3.2:** Let = {$O$, Ꞇ , Ꝣ, ℷ} , the following table represents the hyperoperatic ⊛ on Z.

| ⊛ | $O$ | Ꞇ | Ꝣ | ℷ |
|---|---|---|---|---|
| $O$ | {$O$} | {$O$, Ꞇ} | {Ꝣ} | {ℷ } |
| Ꞇ | {$O$ , Ꞇ} | {Ꞇ} | {$O$} | {Ꞇ, ɥ} |
| Ꝣ | {$O$} | {$O$} | {Ꝣ} | {Ꝣ , ℷ} |
| ℷ | {$O$} | {Ꞇ, ℷ} | {Ꝣ, ℷ} | {Ꞇ, Ꝣ, ℷ} |

Then $(Z, ⊛, O)$ is (ḧ$Z$)

**Example 3.3:** A Z-algebra $(Z, *, O)$ with a hyperoperation ⊛ defined on $Z$ as
Ꞇ⊛Ꝣ = {Ꞇ * Ꝣ} ∀ Ꞇ, Ꝣ ∈$Z$ then $(Z, ⊛, O)$ is (ḧ$Z$).

**Proposition3.4:** Let $(Z, ⊛, O)$ be a(ḧ$Z$), then ∀ Ꞇ, Ꝣ∈$Z$ ,∀ $(A, B, C, D)$≠ϕ, $(A, B, C, D) ⊆ Z$ the following axioms hold:

(1)(Ꞇ⊛Ꝣ)[(Ꝣ⊛Ꞇ)⊛(Ꞇ⊛Ꝣ)] = Ꝣ⊛Ꞇ , Ꞇ≠$O$, Ꝣ≠$O$

(2)$O⊛A = A$

(3)$A$ ≪ {$O$}

(4)$A⊛O$ = {$O$ }

(5)$A ⊆ B ⇒ A ≪ B$

(6)$A ⊆ B$ *and* $A≪C⇒B≪C$

(7)$A ⊆ B$ *and* $C⊆D ⇒ A⊛C⇒B⊛D$

**Definition3.5:**

1) If $O$⊛Ꞇ = {Ꞇ} , ∀ Ꞇ ∈$Z$ then $(Z, ⊛, O)$ is named Row hyperZ-algebra (briefly Ꞧ-ḧ$Z$).
2) If Ꞇ⊛$O$ = {$O$ } , ∀ Ꞇ ∈$Z$ then $(Z, ⊛, O)$ is named Column hyperZ-algebra ( briefly Ḉ-ḧ$Z$) .
3) If Ꞇ⊛Ꞇ = {Ꞇ} , ∀ Ꞇ ∈$Z$ then $(Z, ⊛, O)$ is named Diagonal hyperZ-algebra (briefly Ḓ-ḧ$Z$).
4) If $(Z, ⊛, O)$ is satisfies (1and 2) then $(Z, ⊛, O)$ is named Thin hyperZ-algebra (briefly Ṯ-ḧ$Z$).
5) If $(Z, ⊛, O)$ is satisfies (1, 2 and 3) then $(Z, ⊛, O)$ is named Verythin hyperZ-algebra . (briefly Ṿ-ḧ$Z$).

**Example3.6: ( 1)** Let $Z$ = {$O$, Ꞇ , Ꝣ, ℷ} , the following table represents the hyperoperatic ⊛ on Z.

| ⊛ | $O$ | Ꞇ | Ꝣ | ℷ |
|---|---|---|---|---|
| $O$ | {$O$} | {Ꞇ} | {Ꝣ} | {ℷ} |
| Ꞇ | {$O$, Ꞇ} | {$O$, Ꞇ} | {Ꞇ, Ꝣ} | {Ꞇ, ℷ} |
| Ꝣ | {$O$, Ꝣ} | {Ꞇ, Ꝣ} | {$O$, Ꝣ} | {Ꝣ, ℷ} |
| ℷ | {$O$, ℷ} | {Ꞇ, ℷ} | {Ꝣ, ℷ} | {$O$, ℷ} |

$(Z, ⊛, O)$is an (Ꞧ-ḧ$Z$), it is obvious that $(Z, ⊛, O)$ isn't a (Ḉ-ḧ$Z$) and (Ḓ-ḧ$Z$), since ∀ Ꞇ∈$Z$ , Ꞇ⊛$O$≠{$O$}, Ꞇ⊛Ꞇ≠{Ꞇ}.

**(2)** Let $Z$ = {$O$, Ꞇ , Ꝣ, ℷ} , the following table represents the hyperoperatic ⊛ on Z:

| ⊛ | $0$ | Ꞇ | Ꝣ | ϡ |
|---|---|---|---|---|
| $0$ | {0} | {0, Ꞇ} | {0, Ꝣ} | {0, ϡ} |
| Ꞇ | {0} | {0, Ꞇ} | {ϡ} | {Ꝣ} |
| Ꝣ | {0} | {ϡ} | {0, Ꝣ} | {Ꞇ} |
| ϡ | {0} | {Ꝣ} | {Ꞇ} | {0, ϡ} |

Then (Z, ⊛, 0) is a (Ç-ḧZ) , it is obvious that (Z, ⊛, 0) isn't a(R̥-ḧZ) and(D̥-ḧZ) ,since ∀ Ꞇ∈Z , 0⊛Ꞇ≠{Ꞇ} , Ꞇ⊛Ꞇ≠{Ꞇ}.

**(3)** Let Z = {0, Ꞇ , Ꝣ, ϡ} , the following table represents the hyperoperatic ⊛ on Z:

| ⊛ | $0$ | Ꞇ | Ꝣ | ϡ |
|---|---|---|---|---|
| $0$ | {0} | {0, Ꞇ} | {0, Ꝣ} | {0, ϡ} |
| Ꞇ | {0, Ꞇ} | {Ꞇ} | {0, ϡ} | {0, Ꝣ} |
| Ꝣ | {0, Ꝣ} | {0, ϡ} | {Ꝣ} | {0, Ꞇ} |
| ϡ | {0, ϡ} | {0, Ꝣ} | {0, Ꞇ} | {ϡ} |

Then (Z, ⊛, 0) is a (D̥-ḧZ), it is obvious that (Z, ⊛, 0) isn't an (R̥-ḧZ) and (Ç-ḧZ) ,since ∀ Ꞇ∈Z , 0⊛Ꞇ≠{Ꞇ} , Ꞇ⊛0≠{0}.

**(4)** Let Z = {0, Ꞇ , Ꝣ, ϡ} , the following table represents the hyperoperatic ⊛ on Z:

| ⊛ | $0$ | Ꞇ | Ꝣ | ϡ |
|---|---|---|---|---|
| $0$ | {0} | {Ꞇ } | {Ꝣ} | {ϡ} |
| Ꞇ | {0} | {0, Ꞇ} | {ϡ} | {Ꝣ} |
| Ꝣ | {0} | {ϡ} | {0, Ꝣ} | {Ꞇ} |
| ϡ | {0} | {Ꝣ} | {Ꞇ} | {0, ϡ} |

Then (Z, ⊛, 0) is a (T̥-ḧZ), it is obvious that (Z, ⊛, 0) isn't a (V̥-ḧZ), since it isn't a (D̥-ḧZ).

**(5)** Let Z = {0, Ꞇ , Ꝣ, ϡ} , the following table represents the hyperoperatic ⊛ on Z:

| ⊛ | $0$ | Ꞇ | Ꝣ | ϡ |
|---|---|---|---|---|
| $0$ | {0} | {Ꞇ} | {Ꝣ} | {ϡ } |
| Ꞇ | {0} | {Ꞇ} | {Ꞇ, Ꝣ} | {Ꞇ , ϡ} |
| Ꝣ | {0} | {Ꞇ, Ꝣ} | {Ꝣ} | {Ꝣ, ϡ} |
| ϡ | {0} | {Ꞇ, ϡ} | {Ꝣ, ϡ} | {ϡ} |

Then (Z, ⊛, 0) is a V-hyper Z-algebra .

**Definition3.7:** A hyper subZ-algebra of Ƶ is the set S≠ф, S⊆Ƶ, 0∈S, which (S , ⊛, 0) is a (ḧZ)w.r.t hyperoperation ⊛.

**Remark 3.8:** Let S≠ф, S⊆Ƶ , If Ƹ⊛Ϯ⊆S , ∀ Ƹ, Ϯ ∈S⇒0 ∈S.

**Theorem3.9:** Let S≠ф, S⊆Ƶ , then S is a hyper subZ-algebra of Ƶ if and only if *If* Ꞇ⊛Ꝣ⊆S , ∀ Ꞇ , Ꝣ ∈S.

**Proof:**⇒Let Sis a hypersubZ-algebra of Ƶ ⇒(S , ⊛, 0) is a (ḧZ) w.r.t hyperoperation ⊛ *then* Ꞇ⊛Ꝣ⊆S , ∀ Ꞇ, Ꝣ ∈S.
⇐Let Ꞇ⊛Ꝣ⊆S , ∀ Ꞇ, Ꝣ ∈S, then 0 ∈S.by (Remark3.8)
Then 0∈Ꞇ⊛0 ⇒ $(HZ_1)$ valid in S. So also, It is possible to show that the $(HZ_2)$ , $(HZ_3)$ & $(HZ_4)$ are valid in S ,
⇒ S is a hypersubZ-algebra of Ƶ.

**Definition 3.10:** If Ꞇ⊛S = S = S⊛Ꞇ , ∀ Ꞇ∈S where S is a hypersubZ-algebra of Ƶ, in this case S is named a stronghyper subZ-algebra of Ƶ.

**Example 3.11:**

**(1)** Let Ƶ = {0, Ꞇ, Ꝣ , ϡ} in **(**Example3.2**)** and let $S_1$ = {0} , $S_2$ = {0, Ꞇ} and $S_3$ = {0, ϡ} , then $S_1$, $S_2$ is a stronghyper subZ-algebras of Ƶ , but $S_3$ isn't a hyper subZ-algebra of Ƶ since ϡ⊛ϡ = {Ꞇ, Ꝣ, ϡ}⊈$S_3$.

**(2)** Let Ƶ in (Example 3.3) then every subZ-algebra of Ƶ is a stronghyper sub Z-algebra of Ƶ.

**(3)** Let $Z = \{0$, Ꚍ, Ꝣ, ঌ$\}$, the following table represents the hyperoperatic ⊛ on Z:

| ⊛ | 0 | Ꚍ | Ꝣ | ঌ |
|---|---|---|---|---|
| 0 | {0, Ꚍ, Ꝣ, ঌ} | {Ꚍ} | {Ꝣ} | {ঌ} |
| Ꚍ | {0} | {Ꚍ} | {Ꚍ, Ꝣ} | {Ꚍ, ঌ} |
| Ꝣ | {0} | {Ꚍ, Ꝣ} | {Ꝣ} | {Ꝣ, ঌ} |
| ঌ | {0} | {Ꚍ, ঌ} | {Ꝣ, ঌ} | {ঌ} |

Which is a Ƶ, and $S = \{0\}$ is not a stronghyper subZ-algebras of **Ƶ.**

**Remark 3.12**: $S = \{0\}$ is a stronghyper subZ-algebras of Ƶ, if Ƶ satisfied(5) in Definition3.5.

**Definition 3.13:** Let $I \neq \phi$, $I \subseteq$ Ƶ, $I$ is named a hyperZ-ideal of Ƶ if

$(HZI_1)$ $0 \in I$

$(HZI_2)$ Ꚍ⊛Ꝣ $\ll I$ & Ꝣ$\in I \Rightarrow$ Ꚍ$\in I$, ∀ Ꚍ, Ꝣ $\in Z$.

**Definition 3.14:** Let $I \neq \phi$, $I \subseteq$ Ƶ, $I$ is named a weakhyper Z-ideal of Ƶ if

$(HZI_1)$ $0 \in I$

$(HZWI_2)$ Ꚍ⊛Ꝣ $\subseteq I$ & Ꝣ$\in I \Rightarrow$ Ꚍ$\in I$, ∀ Ꚍ, Ꝣ ∈Ƶ.

**Theorem 3.15:** Every hyperZ-ideal of Ƶ is a weakhyper Z-ideal of Ƶ .

**Definition 3.16:** Let$I \neq \phi$, $I \subseteq$ Ƶ, $I$ is named a stronghyper Z-ideal of Ƶ if

$(HZI_1)$ $0 \in I$

$(HZSI_2)$ (Ꚍ⊛Ꝣ)$\cap I \neq \phi$ & Ꝣ$\in I \Rightarrow$ Ꚍ$\in I$, ∀ Ꚍ,, Ꝣ ∈Ƶ.

**Example 3.17:**

(1) Let Ƶ in (Example 3.3) then every ideal $I$ of Ƶ is a hyperZ-ideal(weakhyper Z-ideal, stronghyper Z-ideal) of **Ƶ.**

(2) If $Z = \{0$, Ꚍ, Ꝣ$\}$, the following table represents the hyperoperatic ⊛ on Z:

| ⊛ | 0 | Ꚍ | Ꝣ |
|---|---|---|---|
| 0 | {0, Ꚍ} | {0, Ꚍ} | {0, Ꝣ} |
| Ꚍ | {0} | {0, Ꚍ} | {0, Ꚍ} |
| Ꝣ | {0} | {0, Ꚍ} | {0, Ꚍ, Ꝣ} |

Which is a Ƶ, and $I_1 = \{0$, Ꚍ$\}$ is a hyperZ-ideal(weakhyper Z-ideal) of Ƶ ,and converse of (theorem3.15 ) isn't true, let $I_2 = \{0$, Ꝣ$\}$ is a weakhyper Z-ideal of Ƶ but isn't hyperZ-ideal because

Ꚍ⊛Ꝣ $= \{0$, Ꚍ$\} \ll I_2$ *and* Ꝣ$\in I_2$ *but* Ꚍ$\notin I_2$ ,and $I_3 = \{0\}$, *and* $I_4 = Z$ *are only strong hyperZ* − *ideal of* Ƶ.

**Theorem 3.18:** Let *I is a stronghyperZ − ideal of* Ƶ, then

(1) *I is a weakhyperZ − ideal of* Ƶ.

(2) *I is a hyperZ − ideal of* Ƶ.

And converse is not true.

**Proof :** from Theorem 3.15 , it is sufficient to show (2)

Let Ꚍ, Ꝣ ∈Ƶ *such that* Ꚍ⊛Ꝣ $\ll I$ *and* Ꝣ$\in I$ ,

Then ∀ ω∈Ꚍ⊛Ꝣ ∃ Կ$\in I$, *such that* ω ≪ Կ ⇒$0$∈ω⊛Կ⇒$0$∈ω⊛Կ$\cap I \neq \phi$ ,

Then by $(HZSI_2)$

ω$\in I \Rightarrow$ Ꚍ⊛Ꝣ $\subseteq I$ *and so* Ꚍ⊛Ꝣ $\cap I \neq \phi$ ,

*using* $(HZSI_2) \Rightarrow$ Ꚍ$\in I$, ⇒*I is a hyperZ − ideal of* Ƶ.

Let $I = \{0$, Ꚍ$\}$ in Example 3.17 (2) is a hyper Z-ideal (weakhyper Z-ideal) of Ƶ but isn't *a stronghyper* Z-ideal *of* Ƶ because Ꝣ⊛Ꚍ$\cap I = \{0$, Ꚍ$\} \neq \phi$ *and* Ꚍ$\in I$ *but* Ꝣ$\notin I$.

**Theorem 3.19:** Let $\{ I_\gamma : \gamma \in S \}$ and $I_\gamma \neq \phi$ be a collection of subsets of a (ḧZ),such that $0 \in I_\gamma$, $\forall \gamma \in S$.

(1) $if\ I_\gamma\ a\ hyperZ - ideal\ of\ Ƶ, \forall\gamma\in S \Rightarrow \cap_{\gamma\in S} I_\gamma\ a\ hyperZ - ideal\ of\ Ƶ.$

(2) $if\ I_\gamma\ a\ weakhyperZ - ideal\ of\ Ƶ, \forall\gamma\in S \Rightarrow \cap_{\gamma\in S} I_\gamma\ a\ weakhyperZ - ideal\ of\ Ƶ.$

(3) $if\ I_\gamma\ a\ stronghyperZ - ideal\ of\ Ƶ, \forall\gamma\in S \Rightarrow \cap_{\gamma\in S} I_\gamma\ a\ stronghyperZ - ideal\ of\ Ƶ.$

**Proof:** We only prove (3). (1) and (2) are easy to prove.

Suppose that $I_\gamma\ (\forall\gamma\in S)\ is\ a\ strong\ hyperZ - ideal\ of\ Ƶ$,

$since\ 0\in I_\gamma, \forall\gamma\in S \Rightarrow 0\in\cap_{\gamma\in S} I_\gamma, and\ so\ \cap_{\gamma\in S} I_\gamma \neq \phi.$

$suppose\ that\ \forall\ Ⴇ, Ⴘ\in Ƶ, (Ⴇ\circledast Ⴘ) \cap \cap_{\gamma\in S} I_\gamma \neq \phi\ and\ Ⴘ\in\cap_{\gamma\in S} I_\gamma$

$since\ \cap_{\gamma\in S} I_\gamma \subseteq I_\gamma, \forall\gamma\in S.\ by\ (A\cap B \neq \phi\ and\ B\subseteq C \Rightarrow A\cap C \neq \phi) \Rightarrow$

$(Ⴇ\circledast Ⴘ) \cap I_\gamma \neq \phi\ and\ Ⴘ\in I_\gamma, \forall\gamma\in S,$

$since\ I_\gamma\ (\forall\gamma\in S)\ is\ a\ strong\ hyperZ - ideal\ of\ Ƶ$

$\Rightarrow Ⴇ\in I_\gamma, \forall\gamma\in S \Rightarrow Ⴇ\in\cap_{\gamma\in S} I_\gamma \Rightarrow \cap_{\gamma\in S} I_\gamma\ is\ a\ strong\ hyperZ - ideal\ of\ Ƶ.$

**Theorem 3.20:** Let $\{I_\gamma : \gamma\in S\}$ and $I_\gamma \neq \phi$ be a collection of subsets of a $(\ddot{h}Z)$ ,such that $I_1 \subseteq I_2 \subseteq ...$

(1) $if\ I_\gamma\ a\ hyperZ - ideal\ of\ Ƶ, \forall\gamma\in S \Rightarrow \cup_{\gamma\in S} I_\gamma\ a\ hyperZ - ideal\ of\ Ƶ.$

(2) $if\ I_\gamma\ a\ weakhyperZ - ideal\ of\ Ƶ, \forall\gamma\in S \Rightarrow \cup_{\gamma\in S} I_\gamma\ a\ weakhyperZ - ideal\ of\ Ƶ.$

(3) $if\ I_\gamma\ a\ stronghyperZ - ideal\ of\ Ƶ, \forall\gamma\in S \Rightarrow \cup_{\gamma\in S} I_\gamma\ a\ stronghyperZ - ideal\ of\ Ƶ.$

**Proof:** We only prove (3) ,(1) and (2) are easy to prove.

Suppose that $I_\gamma (for\ some\ \gamma\in S)\ is\ a\ strong\ hyperZ - ideal\ of\ Z$ ,

$since\ 0\in I_\gamma\ , for\ some\ \gamma\in S \Rightarrow 0\in\cup_{\gamma\in S} I_\gamma, and\ so\ \cup_{\gamma\in S} I_\gamma \neq \phi.$

$Let\ \forall\ Ⴇ, Ⴘ\in Z, (Ⴇ\circledast Ⴘ) \cap \cup_{\gamma\in S} I_\gamma \neq \phi\ and\ Ⴘ\in\cup_{\gamma\in S} I_\gamma$

$since\ I_\gamma \subseteq \cup_{\gamma\in S} I_\gamma, \forall\gamma\in S, and\ I_1 \subseteq I_2 \subseteq ... \Rightarrow$

$(Ⴇ\circledast Ⴘ) \cap I_\gamma \neq \phi\ and\ Ⴘ\in I_\gamma, for\ some\ \gamma\in S,$

$since\ I_\gamma\ is\ a\ strong\ hyperZ - ideal\ of\ Ƶ$

$\Rightarrow Ⴇ\in I_\gamma, \forall\gamma\in S \Rightarrow Ⴇ\in\cup_{\gamma\in S} I_\gamma,$

$\Rightarrow \cup_{\gamma\in S} I_\gamma\ is\ a\ strong\ hyperZ - ideal\ of\ Ƶ.$

**Example 3.21:** This example demonstrates that a hyper subZ-algebra and a hyper Z-ideal (weak hyper Z-ideal) of Ƶ are different concepts..

(1) Let $Z$ = {O, Ƭ, ℨ},

| ⊛ | O | Ƭ | ℨ |
|---|---|---|---|
| O | {O} | {O, Ƭ} | {O, ℨ} |
| Ƭ | {O, Ƭ} | {O, Ƭ} | {O, ℨ} |
| ℨ | {O} | {O, ℨ} | {O, ℨ} |

Which is a Ƶ, $S$ = {O, ℨ} is a hyper subZ-algebra , but $S$ isn't a hyperZ-ideal(weakhyper Z-ideal) of Ƶ because Ƭ⊛ℨ = {O, ℨ}⊆$S$ *and* ℨ∈$S$ *and* Ƭ∉$S$ .

(2) Let $Z$ = {O, Ƭ, ℨ}, the following table represents the hyperoperatic ⊛ on Z:

| ⊛ | O | Ƭ | ℨ |
|---|---|---|---|
| O | {O} | {O, Ƭ} | {O, ℨ} |
| Ƭ | {O, Ƭ} | {Ƭ, ℨ} | {Ƭ, ℨ} |
| ℨ | {O, ℨ} | {Ƭ, ℨ} | {Ƭ, ℨ} |

Which is a Ƶ, and $I$ = {O, ℨ} is a hyperZ-ideal(weakhyper Z-ideal) of Ƶ,but $I$ isn't a hyper subZ-algebra because ℨ⊛ℨ = {Ƭ, ℨ}⊈ $I$ .

**Theorem 3.22:** Let $S$ a hyper subZ-algebra of Ƶ then

(1) $S$ is a weakhyper Z-ideal of Ƶ ⇔ ∀ Ƭ∈$Z$ − $S$, *and* ℨ∈$S$ *we have* Ƭ⊛ℨ⊈$S$.

(2) $S$ is a hyperZ-ideal of Ƶ ⇔ ∀ Ƭ∈$Z$ − $S$, *and* ℨ∈$S$ *we have* (Ƭ⊛ℨ)∩$S$ = ϕ.

**Proof:**

**(1)** Suppose $S$ be a weakhyper Z-ideal of Ƶ, Ƭ∈$Z$ − $S$ *and* ℨ∈$S$.

Let Ƭ⊛ℨ ⊆ $S$, by hypothesis we have Ƭ∈$S$ *which is contradictory*. ⇒Ƭ⊛ℨ⊈$S$.

Conversely, Suppose Ƭ⊛ℨ ⊆ $S$ and ℨ∈$S$ , *let* Ƭ∉$S$ then by hypothesis Ƭ⊛ℨ⊈$S$. which is contradictory.⇒Ƭ∈$S$ ⇒$S$ is a weakhyper Z-ideal of Ƶ.

**(2)** If $S$ a hyperZ-ideal of Ƶ,Ƭ∈$Z$ − $S$ & ℨ∈$S$.

Let (Ƭ⊛ℨ)∩$S$ ≠ ϕ, since ℨ∈$S$ & $S$ is a hyperZ-ideal of Ƶ , we have Ƭ∈$S$ Which is contradictory⇒ (Ƭ⊛ℨ)∩$S$ = ϕ.

Conversely, let for any Ƭ∈$Z$ − $S$ , ℨ∈$S$ , and (Ƭ⊛ℨ)∩$S$ = ϕ ,

If (Ƭ⊛ℨ)∩$S$ ≠ ϕ⇒∃ $k$∈(Ƭ⊛ℨ) *and* $k$∈$S$ since $S$ a hyper subZ-algebra of Ƶ ⇒ (Ƭ⊛ℨ)∈$S$ ⇒Ƭ∈$S$ which is contradictory ⇒ $S$ is a hyperZ-ideal of Ƶ.

**Definition 3.23:** Let Ӿ≠ϕ, Ӿ⊆Ƶ is named a downset if Ƭ ≪ ℨ and ℨ∈Ӿ ⇒ Ƭ∈Ӿ , ∀ Ƭ , ℨ ∈$Z$.

**Remark 3.24:** If Ӿ is a hyperZ-ideal of Ƶ⇒ Ӿ is a downset. Because if Ƭ ≪ ℨ and ℨ∈Ӿ.Then ∈ (Ƭ⊛ℨ)∩Ӿ . Since ℨ∈Ӿ. and Ӿ is a hyperZ-ideal of Ƶ, we getƬ∈Ӿ.

**Definition 3.25:** A mapping $f: Z_1 \to Z_2$ where ($Z_1$, ⊛, O) & ($Z_2$, ⊗, O′) be two (ḧZ), is named a "hyperZ -homomorphism" if , ∀ Ƭ , ℨ ∈$Z_1$ we have

(1)$f$(O) = O′  (2)$f$(Ƭ ⊛ℨ ) = $f$(Ƭ)⊗ $f$(ℨ), ∀ Ƭ , ℨ ∈$Z_1$

If $f$ is 1-1 and onto ⇒$f$ is named a hyper Z-isomorphism

kernel of $f$ (briefly) Ker($f$) = {Ƭ ∈$Z_1$: $f$(Ƭ) = O′} , image of $f$ (briefly) $Im$ ($f$) = {$f$(Ƭ): Ƭ ∈$Z_1$} , inverse image of $f$ *(briefly)*$Im$($f^{-1}$) = {Ƭ ∈$Z_1$: $f$(Ƭ) ∈ $Z_2$}.

**Example 3.26:** Let $Z_1$ = {O, Ƭ , ℨ} *and* $Z_2$ = {O′, ƀ, Ч} , define two the hyperoperatians "⊛, ⊗" on $Z_1$ and $Z_2$ respectively as the follows:

| ⊛ | O | Ƭ | ℨ |
|---|---|---|---|
| | | | |
| O | {O} | {Ƭ} | {ℨ} |
| Ƭ | {O, Ƭ} | {Ƭ} | {Ƭ, ℨ} |
| ℨ | {O, ℨ} | {Ƭ, ℨ} | {ℨ} |

| ⊗ | Ơ | ƀ | Կ |
|---|---|---|---|
| Ơ | {Ơ } | {ƀ } | {Կ} |
| ƀ | {Ơ, ƀ} | {ƀ } | {ƀ, Կ } |
| Կ | {Ơ, Կ} | {ƀ, Կ } | {Կ } |

Then ($Z_1$, ⊛, $O$) and ($Z_1$, ⊛, $O$) are a hyper Z-algebras.
Let $f: Z_1 \to Z_2$ define as $f(O) =$ $O'$, $f$(Ꞁ) = ƀ , $f$(Ꞅ) = Կ.
Then "$f$ is a hyper Z –homomorphism".
**Lemma3.27:** If $f: Z_1 \to Z_2$ is a hyperZ –"homomorphism"

(1) if Ꞁ ⊛Ꞅ = $O$ , ∀ Ꞁ, Ꞅ ∈$Z_1$ then $f$(Ꞁ)⊙$f$(Ꞅ) = Ơ.

(2) Ker($f$), $Im(f)$ is a hyperZ-ideal (weakhyper Z-ideal) of $Z_1$, $Z_2$ respectively.

**Theorem 3.28:** If $f: Z_1 \to Z_2$ is a hyperZ-"homomorphism", if Ꞁ ≪ Ꞅ in $Z_1$ then $f$(Ꞁ)≪$f$(Ꞅ) in $Z_2$.
**Proof:** LetꞀ, Ꞅ ∈$Z_1$, if Ꞁ ≪ Ꞅ⇒$O$∈Ꞁ⊛Ꞅ.
And $f(O) = O'$ ∈ $f$(Ꞁ⊛Ꞅ) = $f$(Ꞁ)⊗$f$(Ꞅ) ⇒ $O'$ ∈ $f$(Ꞁ)⊗$f$(Ꞅ)
⇒ $f$(Ꞁ)≪$f$(Ꞅ).
**Remark 3.29:** Let $f: Z_1 \to Z_2$ is onto hyper Z –homomorphism. If $Z_1$ is a/an ((Ç-ḧZ),(Ḓ-ḧZ), (Ț-ḧZ),(Ṿ-ḧZ) and (Ṟ-ḧZ)) therefore $Z_2$ is as well .
**Proof:** We just proof (Ṟ-ḧZ), and the explanation in all other cases is same.
Let $Z_1$ be an (Ṟ-ḧZ) and ƀ∈$Z_2$⇒∃ Ꞁ∈$Z_1$ ⇒ ƀ = $f$(Ꞁ),
$O'$ ⊗ ƀ = $f(O)$⊗$f$(Ꞁ) = $f$($O$⊛Ꞁ) = {$f$(ꞷ): ꞷ∈$O$⊛Ꞁ} = {$f$(Ꞁ)} = ƀ
Then $Z_2$ is an(Ṟ-ḧZ).
**Remark 3.30**: Let $f: Z_1 \to Z_2$ is 1-1 hyper Z-homomorphism. If $Z_2$ is a/an ((Ç-ḧZ),(Ḓ-ḧZ), (Ț-ḧZ),(Ṿ-ḧZ) and (Ṟ-ḧZ)) therefore $Z_2$ is as well.
**Proof:** We just proof (Ṟ-ḧZ), and the explanation in all other cases is same.
Let $Z_2$ be an(Ṟ-ḧZ) and Ꞁ∈$Z_1$ $and$ $f$(Ꞁ) = ƀ . Then

{$f$(ꞷ): ꞷ∈$O$⊛Ꞁ} = $f$($O$⊛Ꞁ) = $f(O)$⊗$f$(Ꞁ) = $O'$ ⊗ ƀ = {ƀ} = {$f$(Ꞁ)}

Since $f$ is 1-1, we get that $O$⊛Ꞁ = {Ꞁ} . Therefore, $Z_1$ is an (Ṟ-ḧZ).
**Theorem 3.31:** If $f: Z_1 \to Z_2$ is a hyperZ –"homomorphism",

(1) If $I$ a hyperZ-ideal of $Z_2 \Rightarrow f^{-1}(I)$ a hyperZ-ideal of $Z_1$.

(2) If $I$ a weakhyper Z-ideal of $Z_2 \Rightarrow f^{-1}(I)$ a weakhyper Z-ideal of $Z_1$.

(3) If $I$ a $stronghyperZ - ideal$ of $Z_2 \Rightarrow f^{-1}(I)$ a $stronghyperZ - ideal$ of $Z_1$.

**Proof:**
**(1)** Since $O'$ ∈ $I$ $and$ $f(O) = O'$ ⇒ $O$ ∈$f^{-1}(I)$
Let Ꞁ , Ꞅ∈$Z_1$, $and$ Ꞁ ⊛Ꞅ ≪ $f^{-1}(I)$, Ꞅ∈$f^{-1}(I)$, then $f$(Ꞅ) ∈ $I$ $and$
∀ ϒ∈Ꞁ⊛Ꞅ ∃ ϯ ∈$f^{-1}(I)$ ⇒ ϒ ≪ ϯ⇒$O$∈ϒ⊛ϯ , then
Ơ = $f(O)$∈$f$( ϒ⊛ϯ) = $f$(ϒ) ⊗ $f$(ϯ) ⊆ $f$(Ꞁ⊛Ꞅ) ⊗ $I$ = $f$(Ꞁ)⊗$f$(Ꞅ) ⊗ $I$.
So that $f$(Ꞁ)⊗$f$(Ꞅ) ≪ $I$ $since$ $I$ a hyperZ-ideal of $Z_2$ ⇒
$f$(Ꞁ)∈$I$ ⇒Ꞁ∈$f^{-1}(I)$⇒ $f^{-1}(I)$ $a\ hyperZ - ideal\ of\ Z_1$.

**(2)** Since $O' \in I$ and $f(O) = O' \Rightarrow O \in f^{-1}(I)$

Let $Ъ, Ъ \in Z_1$, and $Ъ \circledast Ъ \subseteq f^{-1}(I)$, $Ъ \in f^{-1}(I)$,

$\Rightarrow f(Ъ) \in I$ ( by Proposition 3.4 (5))

$Ъ \circledast Ъ \ll f^{-1}(I), \forall \Upsilon \in Ъ \circledast Ъ \Rightarrow \exists \dagger \in f^{-1}(I)$

Then $\Upsilon \ll \dagger \Rightarrow O \in \Upsilon \circledast \dagger$ ,then

$O' = f(O) \in f(\Upsilon \circledast \dagger) = f(\Upsilon) \otimes f(\dagger) \subseteq f(Ъ \circledast Ъ) \otimes I = f(Ъ) \otimes f(Ъ) \otimes I.$

So that $f(Ъ) \otimes f(Ъ) \subseteq I$ since $I$ a *weakhyper* Z-ideal of $Z_2 \Rightarrow$

$f(Ъ) \in I \Rightarrow Ъ \in f^{-1}(I)$, *hence forth* $f^{-1}(I)$ *a weakhyper Z − ideal* $Z_1$.

**(3)** Since $O' \in I$ and $f(O) = O' \Rightarrow O \in f^{-1}(I)$

Let $Ъ, Ъ \in Z_1$, and $(Ъ \circledast Ъ) \cap f^{-1}(I) \neq \phi$, $Ъ \in f^{-1}(I)$, then $f(Ъ) \in I$

Since $(Ъ \circledast Ъ) \cap f^{-1}(I) \neq \phi$

$\phi = f(\phi) \neq f\left[(Ъ \circledast Ъ) \cap f^{-1}(I)\right] \subseteq f(Ъ \circledast Ъ) \cap f(f^{-1}(I)) \subseteq f(Ъ) \otimes f(Ъ) \cap I$

$\Rightarrow f(Ъ) \otimes f(Ъ) \cap I \neq \phi,$

Since $Ъ \in f^{-1}(I) \Rightarrow f(Ъ) \in I$ , $I$ a *stronghyperZ − ideal of* $Z_2$,

$\Rightarrow f(Ъ) \in I \Rightarrow Ъ \in f^{-1}(I), \Rightarrow f^{-1}(I)$ *a stronghyperZ − ideal of* $Z_1$.

**Theorem 3.32:** If $f: Z_1 \to Z_2$ be an onto hyper Z –homomorphism,

(1) If $I$ a hyperZ-ideal of $Z_1$ *containing* $ker(f) \Rightarrow f(I)$ a hyperZ-ideal of $Z_2$.

(2) If $I$ a weakhyper Z-ideal of $Z_1$ *containing* $ker(f) \Rightarrow f(I)$ a weakhyper Z-ideal of $Z_2$.

(3) If $I$ a *stronghyperZ − ideal* of $Z_1$ *containing* $ker(f) \Rightarrow f(I)$ a *stronghyperZ − ideal* of $Z_2$.

**Proof:**

(1) Let $I$ a *hyperZ − ideal* of $Z_1$ *containing* $ker(f) \Rightarrow O \in ker(f) \subseteq I$

$O' = f(O) \in f(I)$ ,

*Let* $Ъ, Ъ \in Z_2$ *such that* $Ъ \otimes Ъ \ll f(I)$, *and* $Ъ \in f(I)$,

*since f is onto* $\Rightarrow \exists \dagger, \omega \in Z_1, f(\dagger) = Ъ, f(\omega) = Ъ.$ *Thus*

$f(\dagger \circledast \omega) = f(\dagger) \otimes f(\omega) = Ъ \otimes Ъ \ll f(I)$

*then* $\forall ђ \in \dagger \circledast \omega, \exists ч \in I, f(ђ) \ll f(ч) \Rightarrow$

$O' \in f(ђ) \otimes f(ч) = f(ђ \circledast ч)$, *then*

$ђ \circledast ч \subseteq ker(f) \subseteq I$, *by Proposition* 3.4(5) $\Rightarrow (ђ \otimes ч) \ll I$

*Used* $(HZI_2) \Rightarrow ђ \in I$, *then*

$\dagger \circledast \omega \subseteq I, \Rightarrow \dagger \circledast \omega \ll I$, and $\omega \in I$

*Used* $(HZI_2) \Rightarrow \dagger \in I$

*then* $f(\dagger) = Ъ \in f(I) \Rightarrow f(I)$ a hyperZ-ideal of $Z_2$.

(2) Let $I$ a weakhyper Z-ideal of $Z_1$ *containing* $ker(f)$

$\Rightarrow O \in ker(f) \subseteq I, O' = f(O) \in f(I)$ ,

Let $Ъ, Ъ \in Z_2$, *and* $Ъ \otimes Ъ \subseteq f(I)$, $Ъ \in f(I)$,

*since f is onto* $\Rightarrow \exists \omega \in I$ *and* $\dagger \in Z_1, f(\dagger) = Ъ, f(\omega) = Ъ,$

$Ъ \otimes Ъ = f(\dagger) \otimes f(\omega) = f(\dagger \circledast \omega) \subseteq f(I)$

$\Rightarrow \dagger \circledast \omega \subseteq I$ , *since* $I$ a *weakhyper* Z-ideal of $Z_1$

And $\omega \in I \Rightarrow \dagger \in I$ , $f(\dagger) \in f(I) \Rightarrow Ъ \in f(I)$

$f(I)$ is a weakhyper Z-ideal of $Z_2$.

**(3)** Let $I$ a *strong hyperZ − ideal* of $Z_1$ *containing* $ker(f)$

⇒ *O*∈ ker *ker* (*f* ) ⊆ *I* ,*Ơ* ∈ *f*(*I*),

*Let* Ƭ , Ʋ∈$Z_2$ *such that* (Ƭ ⊗ Ʋ) ∩ *f*(*I*) ≠ ϕ, *and* Ʋ∈*f*(*I*),

*since f is onto*⇒∃ɷ ∈ *I and* ϯ ∈ $Z_1$, *f*(ϯ) = Ƭ, *f*(ɷ) = Ʋ, ⇒ɷ ∈ *I*

ϕ ≠ (Ƭ ⊗ Ʋ) ∩ *f*(*I*) = (*f*(ϯ)⊗*f*(ɷ)) ∩ *f*(*I*) = *f*(ϯ⊛ɷ) ∩ *f*(*I*)⇒∃ ϡ ∈ $Z_2$

*such that* ϡ ∈ *f*(ϯ⊛ɷ) *and* ϡ ∈ *f*(*I*)⇒∃ ƅ ∈ ϯ⊛ɷ *and* ɋ ∈ *I*, *such that*

ϡ = *f*(ƅ) *and* ϡ = *f*(ɋ), since

*Ơ* ∈ *f*(ƅ) ⊗ *f*(ɋ) = *f*(ƅ⊛ɋ)⇒ (ƅ⊛ɋ) *containsan element say* ♀ *such that f*(♀) = *Ơ*

⇒♀∈*ker*(*f* ) ⊆ *I and thus* (ƅ⊛ɋ) ∩ *I* ≠ ϕ ,

Since *I* a *strong hyperZ* − *ideal* of $Z_1$, *and* ɋ ∈ *I*⇒ƅ ∈ *I* , *since* ƅ ∈ ϯ⊛ɷ

⇒(ϯ⊛ɷ) ∩ (*I*) ≠ ϕ , *since I* a *strong hyperZ* − *ideal* of $Z_1$ , and ɷ ∈ *I*

⇒ϯ ∈ *I* , *ther fore* Ƭ = *f*(ϯ) ∈ *f*(*I*) , *thenf*(*I*) *is a strong hyperZ* − *ideal* of $Z_2$.

**Conclusions**

This work introduces a new hyper algebra This is an extension of Z-algebra ,and defined hyper(subalgebras, *weakhyper Z* − *ideal* , *strong hyperZ* − *ideal*,Z-homomorphism), and particular hyperZ-algebra types were studied. isomorphism theorems were finally studied. In future publications, we will study Z-superhyperalgebra and several forms of ideals in Z-superhyperalgebra.

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