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Hesitant Fuzzy Prime Ideal of Γ- ring

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Abstract

In this paper, we introduce the notions of hesitant fuzzy ideal, hesitant fuzzy prime ideal, hesitant fuzzy strongly prime ideal in gamma rings and hesitant 3-prime ideal. Also, we study the relation between the above concepts

Keywords: gamma ring, fuzzy ideal, hesitant fuzzy subring, hesitant fuzzy prime ideal.

Introduction

N. Nobusawa [11] introduces the notion of Γ- rings as more general that a ring)

W. E. Barnes[8] we a kened the conditions of definition of Γ-ring in the sence of Nobusawa Barnes, K.Yuno and Luh[1,6,9] studied the structure of Γ-ring and obtained various generalizations.

In 1965, The notions of fuzzy sets was introduced by L. A. Zadeh[15] while W. J. Liu in 1982[7] introduced the concept of fuzzy ring. Jun and Lee in [3] introduced the notions of fuzzy Γ-ring, many mathomaticians, work on this subject after W. J. Liu . In 2009, Jorra. And Narukawa Proposed a new type of fuzzy set which is called hesitant fuzzy set and some generalized concepts related to them[13].

Mohammad and others [10] in 2018 introduced the concepts hesitant fuzzy ideal, hesitant fuzzy Bi- ideal in ring and some others concepts.

In this paper, we introduce the notions of hesitant fuzzy ideal, hesitant fuzzy prime ideal, hesitant fuzzy strongly prime ideal in gamma rings and hesitant 3-prime ideal.

Also we study the relation between the above concepts.

2. Preliminaries

Definition 2.1 [11]

Let M and is Γ be two additive abelian groups.

M is called a $\Gamma-$ ring if the following conditions are satisfied fir all a, b, c ∈ M and for all $\alpha, \beta, \gamma \in \Gamma$

1) a α b ∈ M

2) (a + b) α c = a α c + b α c , a(α + β) b = a α b + a β b , a α (b + c) = a α b + a α c

3) a α (b β c) = (a α b) β c

Definition 2.2 :- [13]

Let X be a reference set a hesitant fuzzy set of X is a function $h: X \to P[0,1]$ that returns a subset of some values in [0,1] .Where $P[0,1]$ denotes the set of all subsets of [0,1] , and expressed the HFS by a mathematical symbol $A = \{< X, h_A(y) >: y \in X\}$. we will denote the set of all HFSs in X as HFS (X)

Definition 2.3 :- [4]

Let $h \in HFS(X)$. such that X be a reference set than h is called a hesitant fuzzy point (in short HFP) with the support $x \in X$ and the value t denoted by x_t if $x_t: X \to P[0,1]$ is the mapping given by : for each $y \in X$

$$x_t(y) = \{^{t \subset [0,1] \quad if\ y=x}_{\emptyset \quad other\ wise}$$

We will denote the set of all $HFPs$ in X as $HFP(X)$.

Definition 2.4 :- [4]

Let $h \in HFS(X)$ and $x_t \in HFP(X)$. Thenx_t is said to be belong to h and denoted by $x_t \in h$ if $t \subseteq h(x)$

Definition 2. 5 :- [4]

Let X be a reference set and let $h_1, h_2 \in HFS(X)$. Then the hesitant fuzzy product of h_1 and h_2 denoted by $h_1 \circ h_2$ is a $HFS(X)$. and defined by : for each $x \in X$

$$(h_1 \circ h_2)(x) = \begin{cases} \cup_{x=y\alpha z}[h_1(y) \cap h_2(z)] & \text{if } x=y \\ \emptyset & \text{if other wise} \end{cases}$$

Theorem 2.6 :- [4,2]

let $h_1, h_2 \in HFS(X)$and $x_t, y_q \in HFP(X)$ then :-

1- $x_t \circ y_q = (x\alpha y)_{t\cap q}$
2- $x_t + y_q = (x+y)_{t\cap q}$
3- $x_t + y_q = (x+y)_{t\cap q}$
4- $h_1 \circ h_2 = \cup_{x_t\in h_1, y_q\in h_2} x_t \circ y_q$

Definition 2.7 [5]

A right (left) ideal of a Γ– ring M is an additive subgroup I of M such that I Γ M ⊆ I (M Γ I ⊆ I) if I is both left and right ideal , then we say that I is an ideal or two sided of M

an ideal P of a Γ– ring M is prime if for any ideals A,B ⊂M, A ΓB ⊆P implies that A⊆P or B ⊆P

3- HESIANT FUZZY PRIME IDEAL IN Γ– RING

Definition 3.1 :-

Let M be is Γ- ring and $h \in HFS(M)$ then h is a hesitant fuzzy Γ- subring iff

1- $h(x-y) \supseteq h(x) \cap h(y)$

2- $h(x\alpha y) \supseteq h(x) \cap h(y)$
for any $x, y \in M$ and $\alpha \in \Gamma$

Definition 3.2 :-

Let M be Γ-ring and h is said to be a hesitant fuzzy ideal of M iff

1- $h(x-y) \supseteq h(x) \cap h(y)$

2- $h(x\alpha y) \supseteq h(x) \cup h(y)$
for any $x, y \in M$ and $\alpha \in \Gamma$

Definition 3.3:-

Let M be a Γ- ring and let ∅≠h∈ HFR(M) . Then h is called

1- hesitant fuzzy left ideal (in short , HFLI) of M if h(xαy)⊃h(y) , for any x,y ϵM and $\alpha \in \Gamma$
2- hesitant fuzzy right ideal (in short HFRI) of M if h(xαy)⊃h(x) , for any x,y ϵM , $\alpha \in \Gamma$
3- hesitant fuzzy ideal (in short , HFI) if it is both a HFLI and a HFRI of M
We will denote the set of all $HFLI_s$ [resp. $HFRI_s$ and HFI_s] of M as HFLI(M) [resp. HFRI(M) and HFI (M)] .

Theorem 3.4 :-

Let M be Γ – ring and let $h_1, h_2 \in HFS(M)$ and $\{h_i | i \in I\} \subset HFS(M)$, then

1- $h_1 \subset h_2$ if and only if $x_t \in h_1$ for each $x_t \in h_2$
2- $x_t \in h_1 \cap h_2$ if and only if $x_t \in h_1$ and $x_t \in h_2$
3- if $x_t \in h_1$ or $x_t \in h_2$then $x_t \in h_1 \cup h_2$

4- $x_t \in \cap_{i \in I} h_i$ if $x_t \in h_i$ for each $i \in I$

5- if $x_t \in h_i$ for some $i \in I$ then $x_t \in \cup_{i \in I} h_i$

Example 3.5 :- Let $(Z_3, +_3)$ and $(Z, +)$ are additive abelian groups, then $(Z_3, +_3)$ is $Z -$ ring , the mapping h: $Z_3 \to$ P[0,1] defined as

h(0)= [0.2,0.9] , h(1) = [0.3 , 0.6]= h(3) , h(2) = [0.2 , 0.7)

then $h \in HFI(Z_3)$

Theorem 3 .6 :- Let M be a ring and let $h \in HFR(M)$ *then* h∊ HFI(M) iff the following condition are holds

1 - For all $x_t, y_q \in h$, $x_t - y_q \in h$

2- For all $x_t \in HFP(M)$, $y_q \in h$, $x_t \alpha y_q \in h$

Proof :- suppose that $h \in HFI(M)$ and $x_t, y_q \in h$

so that $t \subseteq h(x), q \subseteq h(y)$ ((by definition 2.4)) ,

so $h(x-y) \supseteq h(x) \cap h(y) \supseteq t \cap q$,

then $x_t - y_q = (x-y)_{t \cap q} \in h$ ((by theorem 2 .6)) , thus $x_t - y_q \in h$

Now,

Let $x_t \in HFP(M), y_q \in h$

Then $h(x\alpha y) \supseteq h(x) \cup h(y) \supseteq h(x) \supseteq t \supseteq t \cap q$

and $h(x\alpha y) \supseteq h(x) \cup h(y) \supseteq h(x) \supseteq q \supseteq t \cap q$

Hence $x_t \alpha y_q = (x\alpha y)_{t \cap q} \in h$

So $x_t \alpha y_q \in$ h $\forall$ x,y $\in M$ and $\alpha \in \Gamma$

$\Leftarrow$ suppose that the conditions are satisfy $x, y \in M$

Let $k = h(x) \cup h(y)$ and $x_k, y_k \in h$ such that $x_k - y_k \in h$,

so that $(x-y)_k \in h$ it is follows $k \subseteq h(x-y)$

Thus $h(x) \cup h(y) \subseteq h(x-y)$

Also $h(x) \cap h(y) \subseteq h(x) \cup h(y) \subseteq h(x-y)$

Hence $h(x-y) \supseteq h(x) \cap h(y)$

Now ,let $x_k \in HFP(M)$,$y_k \in h$ such that $(x\alpha y)_k \in h$,

So $k \subseteq h(x\alpha y)$,this implies

$h(x) \cup h(y) \subseteq h(x\alpha y)$ for all $x, y \in M$, $\alpha \in \Gamma$

Hence $h \in HFI(M)$

Theorem 3.7 : - let M be Γ-ring and let h_1 and h_2 are two HFI of M , then $h_1 \cap h_2 \in$ HFI(M).

Proof:- Let $x_t, y_q \in h_1 \cap h_2$, implies $x_t, y_q \in h_1$ $x_t, y_q \in h_2$

hence $x_t - y_q \in h_1$, $x_t - y_q \in h_1$since h_1 , h_2 be two HFI of M. ((by theorem 2.6)) and so $x_t - y_q \in h_1 \cap h_2$.

Also $x_t \alpha y_q \in h_1$ and $x_t \alpha y_q \in h_2$ it is follows $x_t \alpha y_q \in h_1 \cap h_2$.

Thus $h_1 \cap h_2 \in$ HFI(M).

Theorem 3.8:- Let M be a Γ-ring and let $\{h_i | i \in I\}$ be a family of HFI of M, then $\cap_{i \in I} h_i \in HFI(M)$.

Proof:- Let $x_t, y_q \in \cap_{i \in I} h_i$, so that $x_t, y_q \in h_i$ for all $i \in I$

hence $x_t - y_q \in h_i$ for all $i \in I$, Hence Let $x_t - y_q \in \bigcap_{i\in I} h_i$,Also $x_t \alpha\, y_q \in h_i$ for all $i \in I$

Then $x_t\, \alpha y_q \in \bigcap_{i\in I} h_i$.

Thus $\bigcap_{i\in I} h_i \in HFI(M)$.

Definition 3.9 :- Let M be Γ-ring . An hesitant fuzzy ideal h of M is said to be a hesitant fuzzy prime ideal (in short , $HFPI$)if for any two hesitant fuzzy points $x_t , y_q \in HFP(M)$, $x_t \circ y_q \in h$, implies either $x_t \in h$ or , $y_q \in h$ will denote the set of all HFPIs $HFPI_s$ in M as $HFPI(M)$.

Theorem 3.10:- Let M be Γ-ring and let $h \in HFI(M)$ then $h \in HFPI(M)$ if and only if $h(x) \cup h(y) \supseteq h(x\alpha y)$ for all $x, y \in M$ and $\alpha \in \Gamma$

Proof :- Assume $h \in HFPI(M)$, and $h(x_o) \cup h(y_o) \subseteq h(x_o\, \alpha\, y_o)$ for some $x_0, y_0 \in M$ and $\alpha \in \Gamma$,

put $t = h(x_o\, \alpha\, y_o)$, then $h(x_o) \cup h(y_o) \subset t$ and $(x_o\, \alpha\, y_o)_t \in h$

So $h(x_o) \subset t$ this implies $(x_o)_t \notin h$ and $h(y_o) \subset t$ this implies $(y_o)_t \notin h$.

This is contradiction, therefore, for all $x,y \in M$ and $\alpha \in \Gamma$, $h(x) \cup h(y) \supseteq h(x\alpha y)$

Suppose that the condition is hold.

Now, let $x_t , y_t \in HFP(M)$, such that $x_t \circ y_t \in h$ then $(x\, \alpha\, y)_t \in h$ and let $x_t \notin h$ and $y_t \notin h$

Put $t = h(x\alpha y)$

If $x_t \notin h$, then $h(x) \subset t$, so that $h(x) \subset h(x\alpha y)$

If $y_t \notin h$, then $h(y) \subset t$, so that $h(y) \subset h(x\alpha y)$

Thus $x_t \circ y_t \in h$ implies either $x_t \in h$ and $y_t \in h$

Thus $h \in HFPI(M)$

Remark:- If $h \in HFPI(M)$ then for any x, y $\in M$ and $\alpha \in \Gamma$, $h(x\alpha y) = h(x) \cup h(y)$

Theorem 3.11:- let M be is Γ-ring then every HFPI(M) is HFI(M).

Proof :- The prove is clear

The converse of theorem 3.10 may be not true in general for example.

Example 3. 12 :- Let$(Z_4 , +_4)$, (Z , +) are additive abelian then$(Z_4 , +_4)$ is Z-ring where $Z_4=\{0,1,2,3\}$and the mapping h: $Z_4 \to P[0,1]$ and h(o) = [0.2,0.8] , h(1) = (0.3,0.7) = h(3) ,h(2)= [0.2,0.7] , then we can easily show that h ϵ HFI(M)

But $h(2 \cdot_4 1 \cdot_4 2)$ = h(o)= [0.2,0.8] $\nsubseteq$ h(2) ∪ h(2) = [0.2, 0.7] so h∉ HFPI (M)

Theorem 3.13: - If M be Γ-ring and h is hesitant fuzzy prime ideal of M then

$h_E = \{x \in M |\, h{:}\, (x) \supseteq E\}$ where E⊆ P[0,1] is a prime ideal of M .

Proof :- assume that h is hesitant fuzzy prime ideal of M and let $a, b \in M$ such that $a\alpha\, b \in h_E$

So $h(a\alpha b) \supseteq E$, $\alpha \in \Gamma$ then $(a\alpha b)_E \in h$ since h is hesitant fuzzy prime ideal of M then $a_E \in h$ or $b_E \in h$ then E ⊆ h(a) or E ⊆ h(b) , Hence $b \in h_E$, thus $a\alpha b \in h_E$ implies either $a \in h_E$ or $b \in h_E$

Hence h_E is prime ideal of M .

Definition 3.14:- Let M be Γ-ring a hesitant fuzzy ideal of M is a called a hesitant fuzzy strongly prime ideal (in short , HFSPI) if for any $x, y \in M, \alpha \in \Gamma$, $h(x\alpha y) = h(x)$ or $h(x\alpha y) = h(y)$

will denote the set of all hesitant fuzzy strongly prime ideal set $HFSPI_s$ in M as $HFSPI(M)$.

Example 3.15 :- let $(Z_2 , +_2)$,(Z , +) are additive abelian groups and then $(Z_2 , +_2)$ is Z_2 -ring and define h(0)= [0.1, 0.8] , h(1)=[0.3, 0.6] then we can easily see that h is HFI in M .

Hence h $(0._2 1._2 0)$ = h(0) , h $(0._2 1._2 1)$ = h(0) , h $(1._2 1._2 1)$ =h(1) , h $(0._2 n._2 0)$ =h(0) , $\forall$ n $\in Z_2$

Theorem :- 3.16 : - Every a hesitant strongly fuzzy prime ideal is a hesitant fuzzy prime ideal.

Proof :- suppose h is hesitant strongly fuzzy prime ideal and $x_t, y_q \in HFP(M)$ such that $x_t \circ y_q \in h$ this implies $(x\alpha y)_{t\cap q} \in h$,

Let k=t∩q , so $(x\,\alpha\, y)_k \in h$,

Then $k \subseteq h(x\alpha y)$ since $h(x\alpha y) = h(x)$ so that $k \subseteq h(x)$ it is follows $x_k \in h$ or $h(x\alpha y) = h(y)$ so that $k \subseteq h(y)$ it is follows $y_k \in h$

Thus $h \in HFPI(M)$.

Theorem:- 3.17 : Let M be Γ-ring and let $h \in HFSPI(M)$ and $h^* = \{x \in M \mid \in h(x) = h(0)\}$, then h^* is a strongly prime ideal of M .

Proof :- suppose $h \in HFSPI(M)$, since $0 \in M$ it is clear that $0 \in h^*$thus $h^* \neq \emptyset$ Let $a, b \in M$ and $a\alpha b \in h^*$

Thus $h(a\alpha b) = h(0)$ since $h \in HFSPI(M)$

Either $h(a\alpha b) = h(a) = h(0)$ then $h(a) = h(0)$ which implies that $a \in h^*$

$h(a\alpha b) = h(b) = h(0)$ then $h(b) = h(0)$ which implies that $b \in h^*$

Hence h^* strongly prime ideal of M

Theorem 3.18:- Let M be is Γ-ring . If $h \in HFPI(M)$ then

$$h(x1\ \alpha\ x2 \ldots\ldots \alpha xn) = h(x1) \cup h(x2) \ldots\ldots \cup h(xn)$$

For all x1 , x2 ...... xn ϵ M and α ϵ Γ.

Proof : - we will do the demonstration by induction on n the theorem is evident for n=2

It is follows $h(x_1\, \alpha x_2) = h(x_1) \cup h(x_2)$

Let us suppose that it is true for n = r it is follows

$$h(x_1\, \alpha x_2\, \alpha x_3\, \alpha x_4 \ldots \alpha x_r) = h(x_1) \cup h(x_2) \cup \ldots . \cup h(x_r)$$

Now, we must prove is true for n = r + 1

$$h(x_1\, \alpha x_2\, \alpha x_3\, \alpha x_4 \ldots \alpha x_r\, \alpha x_{r+1}) = h((x_1\, \alpha x_2\, \alpha x_3\, \alpha x_4 \ldots \alpha x_r)\, \alpha (x_{r+1}))$$

$$= h(x_1\, \alpha x_2\, \alpha x_3\, \alpha x_4 \ldots \alpha x_r) \cup h(x_{r+1})$$

$$= h(x_1) \cup h(x_2) \cup \ldots . \cup h(x_r) \cup h(x_{r+1})$$

This implies

$$h(x_1\, \alpha x_2\, \alpha x_3\, \alpha x_4 \ldots \alpha x_r\, \alpha x_{r+1}) = h(x_1) \cup h(x_2) \cup \ldots . \cup h(x_r) \cup h(x_{r+1})$$

Theorem 3.19:- Let M be a Γ-ring and let h ∈ HFPI(M) and K ⊂ [0,1] , then the set $h^k = \{x \in M: h(x) \subset k\}$ is prime ideal of M.

proof:--suppose that hϵHFPI(M) and K ⊂ [0,1] ,Let x, y ∈ M and α ∈ Γ and xαy ∈ h^k

Then h(xαy) ⊂ k

Since h ∈ HFPI(M) it is follows h(xαy) = h(x) ∪ h(y) ⊂ k

so h(x) ⊆ k , then x ∈ h^k and h(y) ⊆ k then y ∈ h^k

Hence h^k is prime ideal of .

Theorem:3.20:- Let M be a Γ-ring and $h_1 \in$ HFPI(M) if h_2 is hesitant fuzzy Γ- subring of M , then $h_1 \cap h_2 \in$ HFPI(M) .

Proof :- Assume $h_1 \in$ HFPI(M) and $h_2 \in$ HFR(M)

Let x, y ∈ M.

Then $(h_1 \cap h_2)(x\alpha y) = h_1(x\alpha y) \cap h_2(x\alpha y)$

$\subseteq \{h_1(x) \cup h_1(y)\} \cap \{h_2(x) \cup h_2(y)\}$

$\subseteq \{h_1(x) \cup h_2(x)\} \cap \{h_1(y) \cup h_2(y)\}$

$= (h_1 \cap h_2)(x) \cup (h_1 \cap h_2)(y)$

So $(h_1 \cap h_2)(x\alpha y) \subseteq (h_1 \cap h_2)(x) \cup (h_1 \cap h_2)(y)$

Hence $(h_1 \cap h_2) \in$ HFPI(M)

Theorem 3.21:- Let M be is Γ-ring and let $h_1, h_2 \in$ HFPI(M) .Then $h_1 \cap h_2 \in$ HFPI(M).

Proof:- suppose $h_1, h_2 \in$ HFPI(M) and $(x\alpha y)_t \in h1 \cup h2$

Then $(x\alpha y)_t \in h_1$ and $(x\alpha y)_t \in h_1$

Since $h_1 \in$ HFPI(M) and $(x\alpha y)_t \in h_1$, we have $x_t \in h_1$ or $y_t \in h_1$

A gain ,since $h_2 \in$ HFPI(M) and $(x\alpha y)_t \in h_2$, we have $x_t \in h_2$ or $y_t \in h_2$

Thus, either $x_t \in h_1 \cap h_2$ or $y_t \in h_1 \cap h_2$

So $h_1 \cap h_2 \in$ HFPI(M).

Definition 3.21

Let M be Γ-ring and a hesitant fuzzy ideal of M define :

$$h^2(x) = \{ t^2 \mid t \in h(x), x \in M\}$$

Theorem 3.22:- For every hesitant fuzzy prime ideal h of a Γ- ring M , then h^2 is hesitant fuzzy prime ideal.

Proof:- suppose h∈ HFPI(M), and x,y ∈ M , α ∈ Γ

$h^2(x\alpha y) = \{ t^2 : t \in h(x\alpha y)\} = \{ t^2 : t \in h(x) \cup h(y)\}$

$=\{ t^2 : t \in h(x) \vee t \in h(y)\}$

$=\{ t^2 : t \in h(x)\} \cup \{t^2 : t \in h(y)\} = h^2(x) \cup h^2(y)$

Hence $h^2 \in$ HFPI(M).

Theorem 3.23:- Let M be is Γ-ring and let f:M → M* be a homomorphism of Γ-rings. If f is onto and $h_M \in$ HFPI(M). Then $f(h_M) \in$ HFPI(M*).

Proof :- Let $x_t, y_q \in$ HFP(M*) such that $x_t \alpha y_q \in f(h_M)$

Since f onto homomorphism

So that there exists $a_t, b_q \in$ HFP(M) , such that

$f(a_t) = x_t$, $f(b_q) = y_q$

$f(a_t)\alpha f(b_q) \in f(h_M)$

Thus $f(a_t) f(b_q) \in f(h_M)$, this implies

$f(a_t \alpha b_q) \in f(h_M)$ which means $a_t \alpha b_q \in h_M$ implies, either $a_t \in h_M$ or $b_q \in h_M$

If $a_t \in h_M$ this implies $f(a_t) \in f(h_M)$ then $x_t \in f(h_M)$

If $b_q \in h_M$ this implies $f(b_q) \in f(h_M)$ then $y_q \in f(h_M)$

Thus $x_t \alpha y_q \in f(h_M)$ implies , either $x_t \in f(h_M)$ or $y_q \in f(h_M)$

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