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Commutativity of Addition in Prime Near-Rings with Right (θ,θ)-3-Derivations

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Abstract

Let N be a near-ring and θ is a mapping on N . In this paper we introduce the notion of right (θ,θ)-3-derivation in near-ring N. Also, we investigate the commutativity of addition of prime near-rings satisfying certain identities involving right (θ,θ)-3-derivation.

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Introduction

Suppose that N is a near-ring and θ is a mapping on N. This paper consists of two sections. In section one, we recall some basic definitions and other concepts, which will be used later in our paper, we explain these concepts by examples and remarks. In section two, we define the concept of right (θ,θ)-3-derivation in near-ring N and we explore the commutativity of addition and ring behavior of prime near-rings satisfying certain conditions involving right (θ,θ)-3-derivations.

Basic Concepts

Definition 2.1: [1] A right near-ring (resp. a left near-ring) is a nonempty set N equipped with two binary operations + and. such that

(i) (N, +) is a group (not necessarily abelian)

(ii) (N,.) is a semigroup.

(iii) For all x, y, z $\in$N, we have

(x+y)z = xz + yz (resp. z(x+y) = zx + zy)

ISSN: 2347-1921

Volume: 14 Issue: 01

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Example 2.2: [1] Let G be a group (not necessarily abelian) then all mapping of G into itself form a right near-ring M(G) with regard to point wise addition and multiplication by composite.

Lemma 2.3: [1] Let N be left (resp. right) near-ring, then

(i) x0 = 0 (resp. 0x = 0) for all x ∈N.

(ii) – (xy) = x(-y) (resp. – (xy) = (-x)y) for all x,y ∈N.

Definition 2.4: [2] A right near-ring (resp. left near-ring) is called zero symmetric right near-ring (resp. zero symmetric left near-ring) if x0 = 0 (resp. 0x = 0), for all x ∈N.

Definition 2.5: [2] Let {N_i} be a family of near-rings (i∈I, I is an indexing set). $N = N_1 \times N_2 \times \ldots \times N_n$ with regard to component wise addition and multiplication, N is called the direct product of near-rings N_i.

Definition 2.6: [2] A near-ring N is called a prime near-ring if aNb = {0}, where a, b ∈N, implies that either a = 0 or b = 0.

Definition 2.7: [3] Let N be a near-ring. The symbol Z will denote the multiplicative center of N, that is Z = {x∈N /xy = yx for all y ∈N}.

Definition 2.8: [3] Let N be a near-ring. For any x,y ∈N the symbol (x,y) will denote the additive commutator x + y - x – y.

Definition 2.9: [3] Let N be a near-ring. For any x, y ∈N the symbol [x, y] = xy – yx stands for multiplicative commutator of x and y.

Properties 2.10: [3] Let R be a ring, then for all x, y, z ∈R, we have:

1-[x,yz] = y[x,z] + [x,y]z

2-[xy,z] = x[y,z] + [x,z]y

3-[x+y,z] = [x,z] + [y,z]

4-[x,y+z] = [x,y] + [x,z]

Definition 2.11: [4] Let N be a near-ring. An additive mapping d: N → N is said to be right derivation of N if d(xy) = d(x)y + d(y)x, for all x, y ∈N.

Definition 2.12: [4] Let N be a near-ring. An 3-additive mapping d: N x N x N → N is said to be right 3-derivation if the relations:

$d(x_1x_1^{/},x_2, x_3) = d(x_1,x_2,x_3)\, x_1^{/} + d(x_1^{/},x_2,x_3)x_1$

$d(x_1,x_2x_2^{/},x_3) = d(x_1,x_2,x_3)\, x_2^{/} + d(x_1,x_2^{/},x_3)\, x_2$

$d(x_1,x_2,x_3x_3^{/}) = d(x_1,x_2,x_3)\, x_3^{/} + d(x_1,x_2,x_3^{/})\, x_3$

hold for all $x_1 ,x_1^{/},x_2 , x_2^{/} ,x_3, x_3^{/} \in N$.

Example 2.13: [4] Let S be a 2-torsion free zero-symmetric left near-ring. Let us define:

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Journal of Advances in Mathematics

N = $\left\{\begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} : x, y, 0 \in S\right\}$

It is clear that N is a zero-symmetric near-ring with respect to matrix addition and matrix multiplication.

Define d: N x N x N → N by

$$d\left(\begin{pmatrix} 0 & x_1 & y_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & x_2 & y_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & x_3 & y_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & x_1\, x_2\, x_3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

It can be easily seen that d is a nonzero right 3-derivation of near-ring N.

Definition 2.14: [4] Let N be a near-ring. An 3-additive mapping d: N x N x N → N is said to be 3-derivation if the relations:

$d(x_1x_1^{/}, x_2, x_3) = d(x_1, x_2, x_3)\, x_1^{/} + x_1\, d(x_1^{/}, x_2, x_3)$

$d(x_1, x_2x_2^{/}, x_3) = d(x_1, x_2, x_3)\, x_2^{/} + x_2\, d(x_1, x_2^{/}, x_3)$

$d(x_1, x_2, x_3x_3^{/}) = d(x_1, x_2, x_3)\, x_3^{/} + x_3\, d(x_1, x_2, x_3^{/})$

hold for all $x_1, x_1^{/}, x_2, x_2^{/}, x_3, x_3^{/} \in N$.

Lemma 2.15: [4] Let N be a prime near-ring, d a nonzero (σ, τ)-n-derivation of N and x ∈N.

(i)If d (N , N , …., N) x = {0} , then x = 0 .

(ii)If x d (N , N , …., N) = {0} , then x = 0 .

3.Main Results

First, we introduce the basic definition in this paper

Definition 3.1: Let N be a near-ring and θ is a mapping on N. An 3-additive mapping d: N x N x N → N is said to be right (θ, θ)- 3-derivation if the relations:

$d(x_1x_1^{/}, x_2, x_3) = d(x_1, x_2, x_3)\, \theta(x_1^{/}) + d(x_1^{/}, x_2, x_3)\, \theta(x_1)$

$d(x_1, x_2x_2^{/}, x_3) = d(x_1, x_2, x_3)\, \theta(x_2^{/}) + d(x_1, x_2^{/}, x_3)\, \theta(x_2)$

$d(x_1, x_2, x_3x_3^{/}) = d(x_1, x_2, x_3)\, \theta(x_3^{/}) + d(x_1, x_2, x_3^{/})\, \theta(x_3)$

hold for $x_1, x_1^{/}, x_2, x_2^{/}, x_3, x_3^{/} \in N$.

we now explain this definition by the following example

Example 3.2: Let S be a 2-torsion free zero symmetric commutative near-ring.

Let us define

N = $\left\{\begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & 0 & z \end{pmatrix} : x, y, z, 0 \in S\right\}$.

It can be easily see that is a non-commutative 2-torsion free zero symmetric left near-ring with respect to matrix addition and matrix multiplication.

Define d: N x N x N $\rightarrow$ N by

$$d\left(\begin{pmatrix}0 & x_1 & y_1\\0 & 0 & 0\\0 & 0 & z_1\end{pmatrix},\begin{pmatrix}0 & x_2 & y_2\\0 & 0 & 0\\0 & 0 & z_2\end{pmatrix},\begin{pmatrix}0 & x_3 & y_3\\0 & 0 & 0\\0 & 0 & z_3\end{pmatrix}\right)=\begin{pmatrix}0 & x_1\,x_2\,x_3 & 0\\0 & 0 & 0\\0 & 0 & 0\end{pmatrix}$$

And θ : N $\rightarrow$ N such that

$$\theta\left(\begin{pmatrix}0 & x & y\\0 & 0 & 0\\0 & 0 & z\end{pmatrix}\right)=\begin{pmatrix}0 & x & y\\0 & 0 & 0\\0 & 0 & z\end{pmatrix}$$

It is easy to see that d is a right (θ, θ)- 3-derivation of a near-ring N.

The following lemma help us to prove the main theorems:

Lemma 3.3: Let N be a prime near-ring, d is a nonzero right (θ, θ)- 3-derivation of N and a $\in$N, where θ is an automorphism on N.

If d (N , N , N) θ(a) = {0} , then a = 0.

Proof : Given that d (N , N , N) θ(a) = {0} ,

i.e : d(x_1,x_2 ,x_3) θ(a) = 0 for all x_1 ,x_2 , x_3 $\in$N . (3.1)

Putting x_1a in place of x_1 in equation (3.1) , and using it again we get

0 = d(x_1a,x_2 ,x_3) θ(a)

=(d(x_1,x_2 ,x_3) θ(a) + d(a,x_2 ,x_3) θ(x_1)) θ(a)

= d(a,x_2,x_3) θ(x_1) θ(a) , since θ is an automorphism

So we get d(a,x_2 ,x_3) N θ(a) = {0} for all x_2 , x_3 $\in$N .

Primeness of N implies that

Either θ(a) = 0 , a = 0 or d(a,x_2 ,x_3) = 0 for all x_2 , x_3 $\in$N .

If d(a,x_2 ,x_3) = 0 for all x_2 , x_3 $\in$N . (3.2)

Since d(x(a y) ,x_2 ,x_3) = d((xa)y,x_2 ,x_3) for all x ,y ,x_2 , x_3 $\in$N .

Therefore

d(x ,x_2 ,x_3) θ(ay) + d(ay,x_2 ,x_3) θ(x) = d(xa ,x_2 ,x_3) θ(y) + d(y ,x_2 ,x_3) θ(xa) for all x ,y ,x_2 , x_3 $\in$N .

Which means that

d(x ,x_2 ,x_3) θ(ay) + (d(a ,x_2 ,x_3) θ(y) + d(y ,x_2 ,x_3) θ(a)) θ(x)

= (d(x ,x_2 ,x_3) θ(a) + d(a ,x_2 ,x_3) θ(x)) θ(y) + d(y ,x_2 ,x_3) θ(xa) for all x ,y ,x_2 , x_3 $\in$N .

Using equations (3.1) and (3.2) in previous equation , since θ is an automorphism

We get $d(y, x_2, x_3) N \theta(a) = \{0\}$ for all $y, x_2, x_3 \in N$.

Since $d \neq 0$, primeness of N implies that $a = 0$.

Now , we will prove the main results

Theorem 3.4: Let N be a prime near-ring and d_1, d_2 be any two nonzero right (θ,θ)-3- derivations, where θ is an automorphism on N. If $[d_1(N, N, N), d_2(N, N, N)] = \{0\}$, then $(N, +)$ is abelian.

Proof: Assume that $[d_1(N, N, N), d_2(N, N, N)] = \{0\}$. If both z and z + z commute element wise with $d_2(N, N, N)$, then for all $x_1, x_2, x_3 \in N$ we have

$$z\, d_2(x_1,x_2,x_3) = d_2(x_1,x_2,x_3)\, z \qquad (3.3)$$

And

$$(z + z)\, d_2(x_1,x_2,x_3) = d_2(x_1,x_2,x_3)\,(z + z) \qquad (3.4)$$

Substituting $x_1 + x_1'$ instead of x_1 in (3.4) we get

$(z + z)\, d_2(x_1 + x_1', x_2, x_3) = d_2(x_1 + x_1', x_2, x_3)\,(z + z)$ for all $x_1, x_1', x_2, x_3 \in N$.

From (3.3) and (3.4) the previous equation can be reduced to

$z\, d_2(x_1 + x_1' - x_1 - x_1', x_2, x_3) = 0$ for all $x_1, x_1', x_2, x_3 \in N$

i.e.; $z\, d_2((x_1, x_1'), x_2, x_3) = 0$ for all $x_1, x_1', x_2, x_3 \in N$ .putting $z = d_1(y_1, y_2, y_3)$, we get $d_1(y_1, y_2, y_3)\, d_2((x_1, x_1'), x_2, x_3) = 0$ for all $x_1, x_1', x_2, x_3, y_1, y_2, y_3 \in N$. By Lemma 3.3 we conclude that $d_2((x_1, x_1'), x_2, x_3) = 0$ for all $x_1, x_1', x_2, x_3 \in N$. (3.5)

Since we know that for each $w \in N$, $w(x_1, x_1') = w(x_1 + x_1' - x_1 - x_1') = w x_1 + w x_1' - w x_1 - w x_1' = (w x_1, w x_1')$ which is again an additive commutator of a near-ring N , putting $w(x_1, x_1')$ in place of additive commutator (x_1, x_1') in (3.5) we get

$d_2(w(x_1, x_1'), x_2, x_3) = 0$ for all $x_1, x_1', x_2, x_3, w \in N$.

i.e.; $d_2(w, x_2, x_3)\,\theta(x_1, x_1') + d_2((x_1, x_1'), x_2, x_3)\,\theta(w) = 0$ for all $x_1, x_1', x_2, x_3, w \in N$. Using (3.5) in previous equation yields $d_2(w, x_2, x_3)\,\theta(x_1, x_1') = 0$ for all $x_1, x_1', x_2, x_3, w \in N$, using Lemma 3.3 we conclude that $(x_1, x_1') = 0$. Hence $(N, +)$ is abelain .

Theorem 3.5: Let N be a prime near-ring and d_1, d_2 be any two-nonzero right (θ,θ)-3- derivations, where θ is an automorphism on N. If $d_1(x_1, x_2, x_3)\, d_2(y_1, y_2, y_3) + d_2(x_1, x_2, x_3)\, d_1(y_1, y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_2, y_3 \in N$, then $(N, +)$ is abelian.

Proof: By hypothesis we have,

$$d_1(x_1, x_2, x_3)\, d_2(y_1, y_2, y_3) + d_2(x_1, x_2, x_3)\, d_1(y_1, y_2, y_3) = 0 \quad \text{for all } x_1, x_2, x_3, y_1, y_2, y_3 \in N . \qquad (3.6)$$

Substituting $y_1 + y_1'$ instead of y_1 in (3.6) we get

$d_1(x_1, x_2, x_3)\, d_2(y_1 + y_1', y_2, y_3) + d_2(x_1, x_2, x_3)\, d_1(y_1 + y_1', y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_1', y_2, y_3 \in N$. So we get

$d_1(x_1, x_2, x_3)\, d_2(y_1, y_2, y_3) + d_1(x_1, x_2, x_3)\, d_2(y_1^{/}, y_2, y_3) + d_2(x_1, x_2, x_3)\, d_1(y_1, y_2, y_3) + d_2(x_1, x_2, x_3)\, d_1(y_1^{/}, y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_1^{/}, y_2, y_3 \in N$.

Using (3.6) again in last equation we get

$d_1(x_1, x_2, x_3)\, d_2(y_1, y_2, y_3) + d_1(x_1, x_2, x_3)\, d_2(y_1^{/}, y_2, y_3) + d_1(x_1, x_2, x_3)\, d_2(-y_1, y_2, y_3) + d_1(x_1, x_2, x_3)\, d_2(-y_1^{/}, y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_1^{/}, y_2, y_3 \in N$.

Thus , we get

$d_1(x_1, x_2, x_3)\, d_2((y_1, y_1^{/}), y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_1^{/}, y_2, y_3 \in N$.

Now using Lemma 3.3 we conclude that $d_2((y_1, y_1^{/}), y_2, y_3) = 0$ for all $y_1, y_1^{/}, y_2, y_3 \in N$.

Putting $w(y_1, y_1^{/})$ in place of $(y_1, y_1^{/})$, where $w \in N$, in the previous equation and using it again we get $d_2(w, y_2, y_3)\, \theta(y_1, y_1^{/}) = 0$ for all $y_1, y_1^{/}, y_2, y_3, w \in N$. using Lemma 3.3 , we conclude that $(y_1, y_1^{/}) = 0$. Hence (N ,+) is abelain .

Theorem 3.6: Let N be a prime near-ring and d_1 be a nonzero right (θ,θ)-3- derivation and d_2 be a nonzero (θ, θ)- 3-derivation, where θ is an automorphism on N. If $d_1(x_1, x_2, x_3)\, \theta\, d_2(y_1, y_2, y_3) + \theta\, d_2(x_1, x_2, x_3)\, d_1(y_1, y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_2, y_3 \in N$, then (N , +) is abelian.

Proof : By hypothesis we have ,

$d_1(x_1, x_2, x_3)\, \theta\, d_2(y_1, y_2, y_3) + \theta\, d_2(x_1, x_2, x_3)\, d_1(y_1, y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_2, y_3 \in N$. (3.7)

Substituting $y_1 + y_1^{/}$, where $y_1^{/} \in N$, for y_1 in (3.7) we get

$d_1(x_1, x_2, x_3)\, \theta\, d_2(y_1 + y_1^{/}, y_2, y_3) + \theta\, d_2(x_1, x_2, x_3)\, d_1(y_1 + y_1^{/}, y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_1^{/}, y_2, y_3 \in N$.

Thus , we get

$d_1(x_1, x_2, x_3)\, \theta\, d_2(y_1, y_2, y_3) + d_1(x_1, x_2, x_3)\, \theta\, d_2(y_1^{/}, y_2, y_3) + \theta\, d_2(x_1, x_2, x_3)\, d_1(y_1, y_2, y_3) + \theta\, d_2(x_1, x_2, x_3)\, d_1(y_1^{/}, y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_1^{/}, y_2, y_3 \in N$.

Using (3.7) in previous equation implies

$d_1(x_1, x_2, x_3)\, \theta\, d_2(y_1, y_2, y_3) + d_1(x_1, x_2, x_3)\, \theta\, d_2(y_1^{/}, y_2, y_3) + d_1(x_1, x_2, x_3)\, \theta\, d_2(-y_1, y_2, y_3) + d_1(x_1, x_2, x_3)\, \theta\, d_2(-y_1^{/}, y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_1^{/}, y_2, y_3 \in N$.

Therefore

$d_1(x_1, x_2, x_3)\, \theta\, d_2((y_1, y_1^{/}), y_2, y_3) = 0$ for all $x_1, x_2, x_3, y_1, y_1^{/}, y_2, y_3 \in N$.

Now using Lemma 3.3 , in previous equation , we conclude that

$d_2((y_1, y_1^{/}), y_2, y_3) = 0$ for all $y_1, y_1^{/}, y_2, y_3 \in N$.

Putting $w(y_1, y_1^{/})$ in place of $(y_1, y_1^{/})$, where $w \in N$, in the previous equation and using it again we get $d_2(w, y_2, y_3)\, \theta(y_1, y_1^{/}) = 0$ for all $y_1, y_1^{/}, y_2, y_3, w \in N$. Since θ is an automorphism , using Lemma 2.15, we conclude that $(y_1, y_1^{/}) = 0$. Hence (N ,+) is abelain .

Conclusion

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In present paper we define the notion of right (θ, θ)-3- derivations in near-rings. Also, we study and discuss the commutativity of addition of prime near-ring with right (θ, θ)-3- derivations.

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