Effects of Gravity Modulation and Internal Heat Generation on the onset of Rayleigh-Bénard convection in a Micropolar Fluid

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ABSTRACT

The effect of gravity modulation (time periodic body force or g-jitter) on the onset of Rayleigh-Bénard convection in a micropolar fluid with internal heat generation is investigated by making a linear stability analysis. The stability of a horizontal layer of fluid heated from below is examined by assuming time periodic body force in the presence of internal heat source. A regular perturbation method is used to arrive at an expression to compute the critical Rayleigh number for small amplitude of modulation and dimensionless internal heat source. The Venezian approach is adopted to obtain the eigen value of the problem. The results obtained during the analysis have been presented graphically.

Keywords: Rayleigh-Bénard convection; gravity modulation; internal heat generation; Venezian approach; Rayleigh number.

1. INTRODUCTION

A classical Rayleigh problem on the onset of convective instabilities in a horizontal thin layer of fluid heated from below is of fundamental importance and becomes a prototype to a more complex configuration in experiments and industrial processes. It has its origin in the experimental observations of Bénard [1] and [2]. The convective flows in a liquid layer are driven by buoyancy forces due to temperature gradients. Rayleigh's paper is the pioneering work for almost all modern theories of convection. Rayleigh [3] showed that Bénard convection, which is caused by buoyancy effects, will occur when the Rayleigh number exceeds a certain critical value.

Micropolar fluids, the fluids with microstructure, are introduced and developed by Eringen [4]. Physically, these fluids represent fluids consisting randomly oriented particles suspended in a medium, where the deformation of the fluid particles is ignored. This constitutes a substantial generalization of the Navier–Stokes model and opens a new field of potential applications including a large number of complex fluids. A detailed survey of the theory of micropolar fluid and its applications are considered in the books of Erigen[5,6], Lukasazewicz[7] and Power[8], which has become an important field of research especially in many industrially important fluids like paints, polymeric suspensions, colloidal fluids, and also in physiological fluids such as normal human blood and synovial fluids. The theory of thermomicropolar convection was studied by many authors Datta and Sastry [9], Ahmadi [10], Rama Rao [11], Bhattacharya and Jena [12], Siddheshwar and Pranesh [13,14], Pranesh and Kiran [15], Pranesh and Riya [16], Joseph et al. [17] and Pranesh [18].

A significant class of natural convection problem is anxious with the effort in evading the convection in the earth's gravitational field even when the basic temperature gradient is identical and interfacial instabilities can be overlooked. Owing to numerous inevitable sources of residual acceleration experienced by a spacecraft, the gravity field in an orbiting laboratory is not constant in a microgravity environment, but it is randomly fluctuating. This fluctuating gravity is referred to as g-jitter.

The effect of gravity modulation on a convection stable configuration can significantly influence the stability of a system by increasing or decreasing its susceptibility to convection. In general, a distribution of stratifying agency that is convectively stable under constant gravity conditions can be destabilized when a time-dependent component of the gravity field is introduced certain combinations of thermal gradients, physical properties and modulation parameters may lead to parametric resonance and hence, to the stability of the system. Gresho and Sani [19], Wheeler et al. [20], Siddheshwar and Pranesh [21,22], Malashetty and Basavaraja [23], Siddheshwar and Abraham [24], Swamy et al. [25], Bhadauria and Kiran [26] and Pranesh et al. [27] have studied the effects of gravity modulation on the onset of convection in Newtonian and non-Newtonian fluids.

The above studies on gravity modulation are made for non-internal heating systems. However, in many practically important situations the material offers its own source of heat and this leads to a setting up of different convective flow in a fluid layer through internal heating.

The mechanism of internal heating in a flowing fluid is relevant to the thermal processing of liquid foods through ohmic heating, where the internal heat generation serves for the pasteurization/sterilization of the food Ruan et al. [28]. Other important applications of flows with internal heat generation are relative to nuclear reactors, as well as to the geophysics of the earth's mantle. In both cases, the internal heating is due to the radioactive decay.The research on internal heat generation is much less extensive as compared to external heat generation. Bhattacharya and Jena [29], Takashima [30], Tasaka and Takeda [31], Bhadauria et al [32] and Pranesh and Ritu Bawa [33] have studied the effect of internal heating on the onset of Rayleigh – Bénard convection under different situations.

The main aim of the present study is to investigate the effects of gravity modulation and internal heat generation on the onset of Rayleigh-Bénard convection in micropolar fluid. This analysis based on the linear stability theory and the resulting eigenvalue problem is solved using the Venezian [34] approach by considering free-free, isothermal and no spin boundaries.

2.MATHEMATICAL FORMULATION

Consider an infinite horizontal layer of Boussinesquian, micropolar fluid of depth d, where the fluid is heated from below with the internal heat generation exists with the fluid system. Let ΔT be the temperature difference between the lower and upper surfaces with the lower boundary at a higher temperature than the upper boundary. These boundaries maintained at constant temperature. A Cartesian system is taken with origin in the lower boundary and z-axis vertically upward (see figure 1).
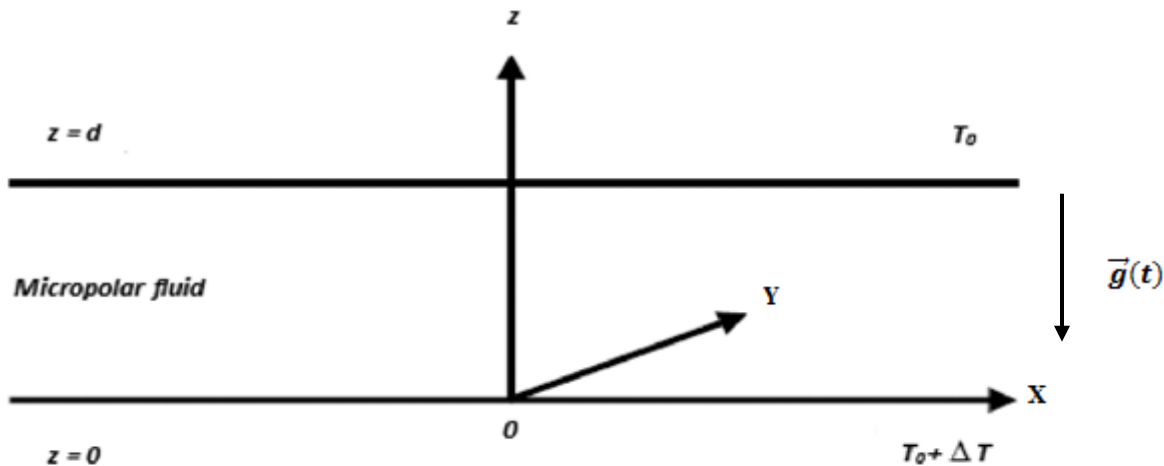

Figure 1. Schematic diagram of the Rayleigh-Bénard situation for micropolarfluid with gravity modulation.

The governing equations are

Continuity Equation:

$$\nabla . \vec{q} = 0 \ , \tag{1}$$

Conservation of Linear Momentum:

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right] = -\nabla p + \rho \vec{g}(t)\hat{k} + (2\zeta + \eta)\nabla^2 \vec{q} + \zeta(\nabla \times \vec{\omega}) \ , \tag{2}$$

$$\vec{g}(t) = -g_0(1 + \varepsilon \cos(\gamma t)) \ , \tag{3}$$

Conservation of Angular Momentum:

$$\rho_0 I \left[\frac{\partial \vec{\omega}}{\partial t} + (\vec{q}.\nabla)\vec{\omega}\right] = (\lambda' + \eta')\nabla(\nabla.\vec{\omega}) + \eta' \nabla^2 \vec{\omega} + \zeta(\nabla \times \vec{q} - 2\vec{\omega}) \ \ , \tag{4}$$

Conservation of Energy:

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \frac{\beta}{\rho_0 C_v}(\nabla \times \vec{\omega}).\nabla T + \chi\nabla^2 T + Q(T - T_0) \ , \tag{5}$$

Equation of State:

$$\rho = \rho_0[1 - \alpha(T - T_0)] \qquad , \tag{6}$$

where, $\vec{q}$ is the velocity, ρ_0 is density of the fluid at temperature T = T_0, p is the pressure, ρ is the density, $\vec{g}$ is acceleration due to gravity, g_0 is the mean gravity, ε is the small amplitude of gravity modulation, γ is the frequency, ζ is coupling viscosity coefficient or vortex viscosity, $\vec{\omega}$ is the angular velocity, I is moment of inertia, λ' and η' are bulk and shear spin viscosity coefficients, T is the temperature, χ is the thermal conductivity, β is micropolar heat conduction coefficient, α is coefficient of thermal expansion, Q is the internal heat source and t is time.

3. BASIC STATE:

The basic state of the fluid is quiescent and is described by

$$\vec{q} = \vec{q_b}(0,0,0), \ \vec{\omega} = \vec{\omega_b}(0,0,0), \ p = p_b(z), \ \rho = \rho_b(z), \ T = T_b(z) \ . \tag{7}$$

Substituting equation (7) into basic governing equations (1)-(6), we obtain the following quiescent state solutions:

$$\frac{dp_b}{dz} = -\rho_b g_0[1 + \varepsilon \cos(\gamma t)]\hat{k} \qquad , \tag{8}$$

$$\chi\frac{d^2 T_b}{dz^2} = -Q(T - T_0) \ , \tag{9}$$

$$\rho_b = \rho_0[1 - \alpha(T_b - T_0)] \ . \tag{10}$$

Solution of equation (9) subject to the conditions

$$T_b = T_0 + \Delta T \text{ at z = 0 and } T_b = T_0 \text{ at z = d,}$$

are obtained as

$$T_b(z) = T_0 + \Delta T\frac{\sin\sqrt{Ri}(1-z/d)}{\sin\sqrt{Ri}} \qquad , \tag{11}$$

where $Ri = \frac{Qd^2}{\chi}$.

4. LINEAR STABILITY ANALYSIS:

The stability of the basic state is analysed by introducing the following perturbation

$$\vec{q} = \vec{q_b} + \vec{q'}, \ \vec{\omega} = \vec{\omega_b} + \vec{\omega'}, p = p_b + p', T = T_b + T', \rho = \rho_b + \rho', \tag{12}$$

where the prime indicates that the quantities are infinitesimal perturbations.

Substituting equation (12) into the equations (1)-(6) and using the basic state solutions, we get linearized equations governing the infinitesimal perturbations in the form:

$$\nabla.\vec{q'} = 0 \ , \tag{13}$$

$$\rho_0\frac{\partial\vec{q'}}{\partial t} = -\nabla p' - \rho' g_0[1 + \varepsilon \cos(\gamma t)]\hat{k} + (2\zeta + \eta)\nabla^2\vec{q'} + \zeta(\nabla \times \vec{\omega'}), \tag{14}$$

$$\rho_0 I\left[\frac{\partial\vec{\omega'}}{\partial t} + (\vec{q'}.\nabla)\vec{\omega'}\right] = (\lambda' + \eta')\nabla(\nabla.\vec{\omega'}) + \eta'\nabla^2\vec{\omega'} + \zeta(\nabla \times \vec{q'} - 2\vec{\omega'}), \tag{15}$$

$$\frac{\partial T'}{\partial t} = \frac{\Delta T}{d}\frac{\sqrt{Ri}\cos\sqrt{Ri}(1-z/d)}{\sin\sqrt{Ri}}\left[W' - \frac{\beta}{\rho_0 C_\vartheta}\nabla \times \vec{\omega'}\right] + \chi\nabla^2 T' + QT' \ , \tag{16}$$

$$\rho' = -\alpha\rho_0 T'. \tag{17}$$

The perturbations equations (13)-(17) are non-dimensionalised using the following definitions

$$(x^*, y^*, z^*) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right), \qquad \overrightarrow{q^*} = \frac{\overrightarrow{q'}}{\frac{\chi}{d}}, \qquad T^* = \frac{T'}{\Delta T}$$

$$t^* = \frac{t}{d^2/\chi}, \qquad \Omega^* = \frac{\nabla \times \overrightarrow{\omega^*}}{\chi/d^3}, \qquad \overrightarrow{\omega^*} = \frac{\overrightarrow{\omega'}}{\chi/d^2}. \qquad (18)$$

Using equation (17) in equation (14) and operating curl twice on the resulting equation, operating curl once on equation (15) and non-dimensionalizing the two resulting equations and equation (16), using equation (18), we get

$$\frac{1}{Pr}\frac{\partial}{\partial t}(\nabla^2 W) = R[1 + \varepsilon \cos(\gamma t)]\nabla_1^2 T + (1 + N_1)\nabla^4 W + N_1 \nabla^2 \Omega, \qquad (19)$$

$$\frac{N_2}{Pr}\frac{\partial \Omega}{\partial t} = N_3 \nabla^2 \Omega - N_1 \nabla^2 W - 2N_1 \Omega, \qquad (20)$$

$$\frac{\partial T}{\partial t} = g(z)[W - N_5 \Omega] + \nabla^2 T + Ri\, T, \qquad (21)$$

where the asterisks have been dropped for simplicity. The dimensionless groups are

$Pr = \frac{\zeta + \eta}{\chi \rho_0}$,   (Prandtl Number)

$R = \frac{\rho_0 \alpha g_0 \Delta T d^3}{\chi(\zeta + \eta)}$,   (Rayleigh Number)

$N_1 = \frac{\zeta}{\zeta + \eta}$,   (Coupling Parameter)

$N_2 = \frac{I}{d^2}$,   (Inertia Parameter)

$N_3 = \frac{\lambda' + \eta'}{(\zeta + \eta)d^2}$,   (Couple Stress Parameter)

$N_5 = \frac{\beta}{\rho_0 C_\vartheta d^2}$,   (Micropolar Heat Conduction Parameter)

$Ri = \frac{Q d^2}{\chi}$,   (Internal Rayleigh Number)

and $g(z) = \frac{\sqrt{Ri}\,Cos[\sqrt{Ri}(1-z)]}{Sin[\sqrt{Ri}]}$.

Equations (19) to (21) are solved subject to the conditions, free-free, isothermal and no spin boundary conditions, given by

$$W = \frac{\partial^2 W}{\partial z^2} = \Omega = T = 0 \; at \; z = 0 \; and \; z = 1 \qquad (22)$$

Eliminating T and Ω from equations (19) to (21), we get an equation for W in the form

$$\left[\left(\frac{N_2}{Pr}\frac{\partial}{\partial t} - N_3 \nabla^2 + 2N_1\right)\left(\frac{\partial}{\partial t} - \nabla^2 - Ri\right)\left(\frac{1}{Pr}\frac{\partial}{\partial t} - (1 + N_1)\nabla^2\right)\nabla^4 + N_1^2\left(\frac{\partial}{\partial t} - \nabla^2 - Ri\right)\nabla^6\right]W =$$

$$R\nabla^2 \nabla_1^2 \left(\frac{N_2}{Pr}\varepsilon f' + (-N_3 \nabla^2 + 2N_1 + N_1 N_5 \nabla^2)(1 + \varepsilon f)\right)g(z)W \qquad (23)$$

where f = Real part of $(e^{-i\Omega t})$ and $f' = (-i\Omega)$ Real part of $(e^{-i\Omega t})$

In the dimensionless form, the velocity boundary conditions for solving equation (23) are obtainable from equations (19)-(21) and (22) in the form

$$W = \frac{\partial^2 W}{\partial z^2} = \frac{\partial^4 W}{\partial z^4} = \frac{\partial^6 W}{\partial z^6} = \frac{\partial^8 W}{\partial z^8} = 0 \quad at\; z=0,1. \qquad (24)$$

5.PERTURBATION PROCEDURE:

We now seek the eigen-function W and eigen-values R of the equation (23) in the form

$$(R, W) = (R_0, W_0) + \varepsilon(R_1, W_1) + \varepsilon^2(R_2, W_2) + \ldots\ldots\ldots\ldots\ldots \qquad (25)$$

Substituting the expression (25) into equation (23) and equating like powers of ε on both sides, we get

$$L_1 W_0 = 0, \tag{26}$$

$$L_1 W_1 = \frac{N_2}{Pr} R_0 \nabla^2 \nabla_1^2 f' g(z) W_0 + \nabla^2 \nabla_1^2 (-N_3 \nabla^2 + 2N_1 + N_1 N_5 \nabla^2) g(z)(f R_0 + R_1) W_0 \tag{27}$$

$$L_1 W_2 = \nabla^2 \nabla_1^2 \frac{N_2}{Pr} g(z) f' (R_0 W_1 + R_1 W_0) + \nabla^2 \nabla_1^2 (-N_3 \nabla^2 + 2N_1 + N_1 N_5 \nabla^2) g(z)[(R_1 + f R_0) W_1 + (R_2 + f R_1) W_0] \tag{28}$$

where

$$L_1 = \left\{ \left[\frac{N_2}{Pr} \frac{\partial}{\partial t} - N_3 \nabla^2 + 2N_1 \right] \left[\frac{\partial}{\partial t} - \nabla^2 - Ri \right] \left[\frac{1}{Pr} \frac{\partial}{\partial t} - (1 + N_1) \nabla^2 \right] \nabla^4 + N_1^2 \left[\frac{\partial}{\partial t} - \nabla^2 - Ri \right] \nabla^6 - R_0 \nabla^2 \nabla_1^2 g(z) \left[\frac{N_2}{Pr} \frac{\partial}{\partial t} - \right.$$
$$N3\nabla 2 + 2N1 + N1N5\nabla 2 \tag{29}$$

6. SOLUTION TO THE ZEROTH ORDER PROBLEM:

The zeroth order problem is equivalent to the Rayleigh-Bénard problem of micropolar fluid with internal heat generation in the absence of gravity modulation. The stability of the system in the absence of gravity modulation is investigated by introducing vertical velocity perturbation W_0 corresponding to lowest mode of convection as:

$$W_0 = Sin(\pi z) \exp[i(lx + my)] \tag{30}$$

where *l* and *m* are horizontal wave numbers in x and y direction.

Substituting equation (30) into equation (26) we obtain the expression for Rayleigh number in the form

$$R_0 = \frac{4\pi^2 - Ri}{2\pi^2} \frac{N_3(1+N_1)k^8 + N_1(2+N_1)k^6 - Ri[N_3(1+N_1)k^6 + N_1(2+N_1)k^4]}{[(N_3 - N_1 N_5)k^2 + 2N_1]a^2} \tag{31}$$

where $k^2 = \pi^2 + a^2, \quad a^2 = l^2 + m^2$.

7. SOLUTION TO THE FIRST ORDER PROBLEM:

Equation (27) for W_1 now takes the form

$$L_1 W_1 = \frac{N_2}{Pr} k^2 a^2 f' G(z) R_0 W_0 + k^2 a^2 G(z)[(N_3 - N_1 N_5)k^2 + 2N_1](f R_0 + R_1) W_0 \tag{32}$$

If the above equation is to have a solution, the right hand side must be orthogonal to the null space of the operator L_1. This implies that the time independent part of the right hand side of the equation (32) must be orthogonal to $Sin(\pi z)$. Since *f* varies sinusoidal with time, the only steady term on the right hand side of equation (32) is $k^2 a^2 G(z)[(N_3 - N_1 N_5)k^2 + 2N_1]R_1$, so that $R_1 = 0$. It follows that all the odd coefficients i.e. $R_1 = R_3 = \ldots \ldots \ldots = 0$ in equation (25).

Using equation (28), we find that

$$L_1 \big[Sin(\pi z) \exp(i(lx + my - \gamma t)) \big] = L_1(\gamma) Sin(\pi z) \exp(i(lx + my - \gamma t)),$$

$$= Y_1 + iY_2 , \tag{33}$$

where

$$Y_1 = \left[-\frac{N_2 \gamma^2}{Pr} k^2 \left[(1 + N_1)k^4 + \frac{1}{Pr} \right] \right] + \left(N_3 k^2 + \frac{2N_1}{Pr} \right) \left[(1 + N_1)k^8 - k^4 \frac{\gamma^2}{Pr} \right] + N_1^2 k^8 - R_0 k^2 a^2 G(z)[(N_3 -$$
$$N1N5k2 + 2N1 + RiN2\gamma 2Pr2k2 - N31 + N1k8 - 2N11 + N1k6 - N12k6 ,$$

$$Y_2 = \gamma \left[\frac{N_2}{Pr} \left[\frac{\gamma^2}{Pr} k^4 - (1 + N_1)k^8 + R_0 G(z) \right] - (N_3 k^2 + 2N_1) \left[(1 + N_1)k^6 + \frac{1}{Pr} k^6 \right] - N_1^2 k^6 + Ri \left[\frac{N_2}{Pr} (1 + N_1)k^6 + \right.$$
$$N3Prk6 + 2N1Prk4 \qquad .$$

$$G(z) = \int_0^1 g(z) Sin^2(\pi z) \, dz$$

The particular solution of equation (32) is

$$W_1 = \frac{R_0 k^2 a^2 G(z)}{|L_1(\gamma)|^2}\left[-\frac{N_2 \gamma}{Pr}(Y_1 Sin[\gamma t] + Y_2 Cos[\gamma t]) + A_1(Y_1 Cos[\gamma t] - Y_2 Sin[\gamma t])\right], \qquad (34)$$

where $A_1 = [(N_3 - N_1 N_5)k^2 + 2N_1]$.

The equation of W_2 is

$$L_1 W_2 = R_2 k^2 a^2 A_1 G(z) W_0 + R_0 k^2 a^2 \frac{N_2}{Pr} f' G(z) W_1 + R_0 k^2 a^2 A_1 f G(z) W_1 \qquad (35)$$

Instead of solving this equation (35), we will use this equation to determine R_2. For the existence of a solution of equation (35), it is necessary that the steady part of its right hand side is orthogonal to Sin (πz) .This gives

$$\int_0^1 \left[R_2 k^2 a^2 A_1 G(z) W_0 + R_0 k^2 a^2 \frac{N_2}{Pr} f' G(z) W_1 + R_0 k^2 a^2 A_1 f G(z) W_1\right] Sin[\pi z] dz = 0 .$$

Taking time average, we get

$$R_2 = -\frac{N_2 R_0}{A_1 Pr} G(z) \int_0^1 f' W_1 Sin[\pi z] dz - R_0 G(z) \int_0^1 f W_1 Sin[\pi z] dz, \qquad (36)$$

Finally

$$R_2 = -\frac{R_0^2 k^2 a^2 G(z)}{2|L_1(\Omega)|^2}\left[\left(\frac{N_2}{Pr}\right)^2 \frac{\Omega^2 Y_1}{A_1} + A_1 Y_1\right] . \qquad (37)$$

7.MINIMUM RAYLEIGH NUMBER FOR CONVECTION

The value of Rayleigh number R obtained by this procedure is the eigenvalue corresponding to the eigen function W. Since R is a function of the horizontal wave number a and the amplitude of modulation ε, we have

$$R(a, \varepsilon) = R_0(a, \varepsilon) + \varepsilon^2 R_2(a, \varepsilon) + \ldots\ldots\ldots\ldots\ldots\ldots \qquad (38)$$

It was shown by Venezian [34] that the critical value of thermal Rayleigh number is computed up to $O(\varepsilon^2)$, by evaluating R_0 and R_2 at $a = a_0$. It is only when one wishes to evaluate R_4 that a_2 must be taken into account where $a = a_2$ minimizes R_2. To evaluate the critical value of R_2 (denoted by R_{2c}) one has to substitute $a = a_0$ in R_2, where a_0 is the value at which R_0 given by equation (31) is minimum.

8.RESULTS AND DISCUSSIONS

We now comprehend the effect of small amplitude gravity modulation and internal heat generation on the onset of Rayleigh – Bénard convection in a horizontal layer of a micropolar fluid for a wide range of frequencies of modulation and the relevant parameters. The linear stability problem is solved based on the method proposed by Venezian. Attention is focused on the determination of the linear stability criterion.

The parameters N_1, N_2, N_3, N_5 arise due to the micropolar fluid, the parameters Pr and Ri arise due to the fluid. To study the effects of these parameters on gravity modulation, the following range of parameters are considered in this paper

$$0 \le N_1 \le 1 , \quad 0 \le N_2 \le r , \quad 0 \le N_3 \le m , \quad 0 \le N_5 \le n ,$$

where the quantities *r, m* and *n* are finite positive real numbers [see Siddheshwar and Pranesh [21]. The values of *Pr* for fluid with suspended particles are taken greater than the fluid without suspended particles because viscosity increases due to the presence of suspended particles. The values of *Ri* are considered to be moderate so that it will determinate the system.

The solutions obtained are based on the assumption that the amplitude of the gravity modulation is small. The validity of the results depends on the value of the modulating frequency γ. When γ < 1 (i.e. the period of modulation is large) the gravity modulation affects the entire volume of the fluid resulting in the growth of the disturbance. On the other hand, the effect of modulation disappears for large frequencies. This is due to the fact the buoyancy force takes a mean value leading to the equilibrium state of the unmodulated case. In view of this, we choose only moderate value of γ in our present study.

The results obtained in this paper are depicted in the figures (2)-(7).

Figure (2) is the plot of correction Rayleigh number R_{2c} versus frequency of modulation γ for different values of coupling parameter N_1. We observe that as N_1 increases, R_{2c} also increases. The increase in N_1 implies increase in the concentration of

suspended particles. These suspended particles consume the greater part of the energy in forming the gyrational velocity and as a result R_{2c} increases. Thus, increase in N_1 stabilizes the system.

Figure (3) is the plot of R_{2c} versus γ for different values of inertia parameter N_2. Increase in N_2 is representative of the increase in inertia of the fluid due to the suspended particles. Thus, as is to be expected, we find that as N_2 increases R_{2c} increases and thereby stabilizing the system.

Figure (4) is the plot of R_{2c} versus γ for different values of couple stress parameter N_3. Increase in N_3 signifies increase in couple stress of the fluid and decrease in gyrational velocities. Hence, as N_3 increases, we observe that R_{2c} decreases and destabilize the system.

Figure (5) is the plot of R_{2c} versus γ for different values of micropolar heat conduction parameter N_5. When N_5 increases, the heat induced into the fluid due to this, microelements also increase, thus reducing the heat transfer from bottom to top. We find from the figure that as N_5 increases R_{2c} increases and thus stabilizes the system.

Figure (6) is a plot of R_{2c} versus γ for different values of Prandtl number Pr. It is observed that as Pr increases R_{2c} also increases. It can be inferred from this that the effect of increasing the concentration of the suspended particle is to stabilize the system. This means that the fluids with suspended particles are more susceptible to stabilization by modulation than clean fluids.

Figure (7) is the plot of R_{2c} versus γ for different values of internal Rayleigh number Ri. We observe that the increase in the internal Rayleigh number Ri increases the heat transport in the system thereby advancing the onset of convection. Thus increase in Ri destabilizes the system.

Figure (8) is the plot of amplitude of modulation ε versus Rayleigh number R for different values of γ. From the figure, we observe that the amplitude of modulation ε increases, the Rayleigh number R also increases. Thus amplitude of modulation stabilizes the system. It can be clearly seen that as γ increases, R increases for smaller values of γ and decreases for moderate values of γ. Thus destabilizes the system.

From the figures it is observed that since R_{2c} remains always positive for all values of γ , gravity modulation leads to delay in onset of convection. The results of this study are helpful in the areas of crystal growth under microgravity conditions.

9. CONCLUSIONS

The effect of coupling parameter N_1 , inertia parameter N_2 , micropolar heat conduction parameter N_5 and Prandtl number Pr is to reduce the amount of heat transfer whereas the opposite effect is observed in the case of couple stress parameter N_3 and internal Rayleigh number Ri. It is observed that gravity modulation or g-jitter leads to delay in convection and frequency of gravity modulation also plays an important role in controlling heat transfer in the system.The effect of internal heat generation has significant influence on the Rayleigh – Bénard convection and is clearly a destabilizing factor to make the system more unstable.

10. ACKNOWLEDGMENTS

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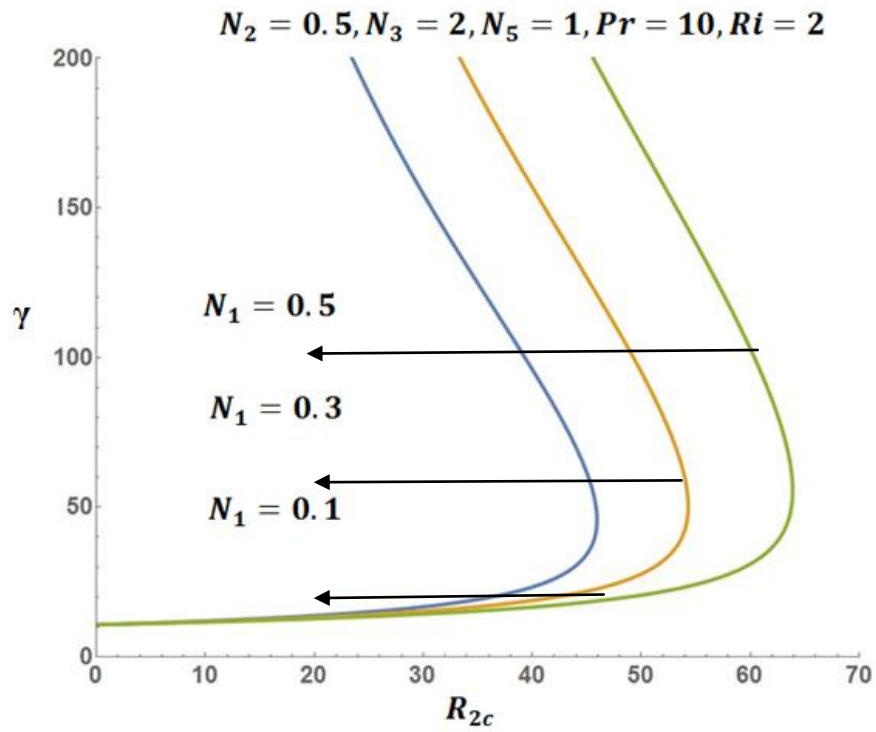
Figure 2: Plot of correction Rayleigh number R_{2c} versus frequency of gravity modulation γ for different values of coupling parameter N_1.
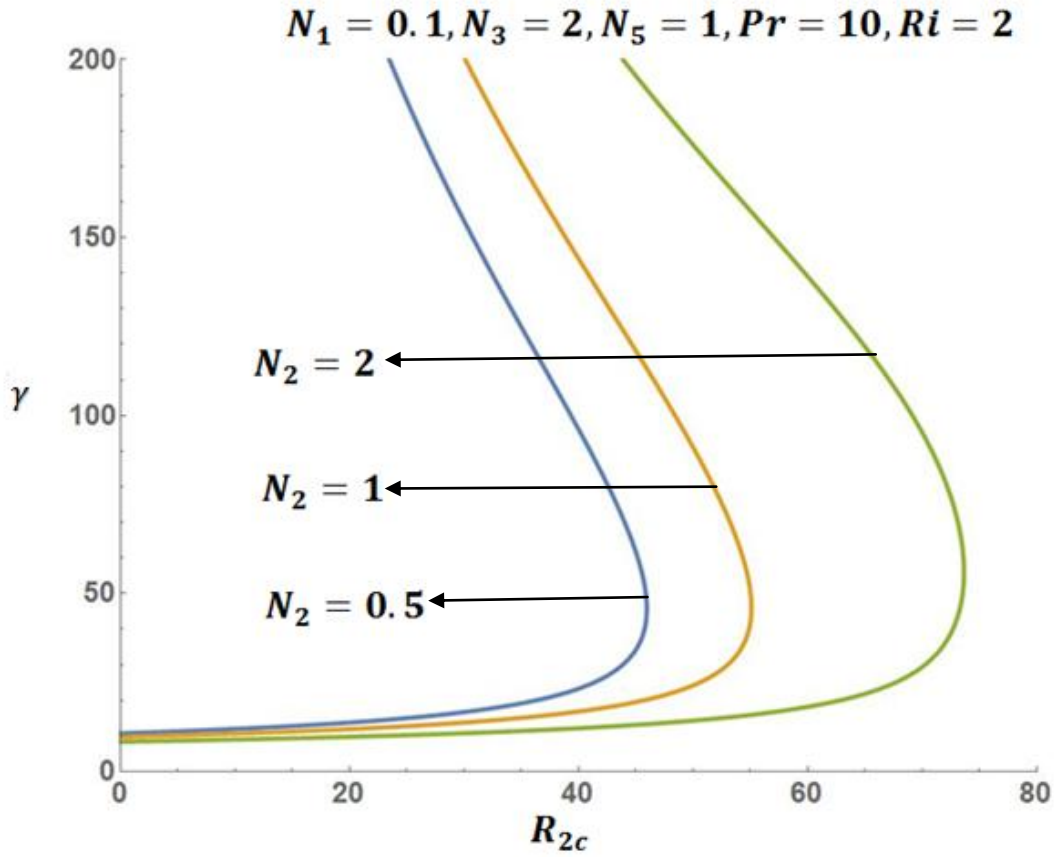
Figure 3: Plot of correction Rayleigh number R_{2c} versus frequency of gravity

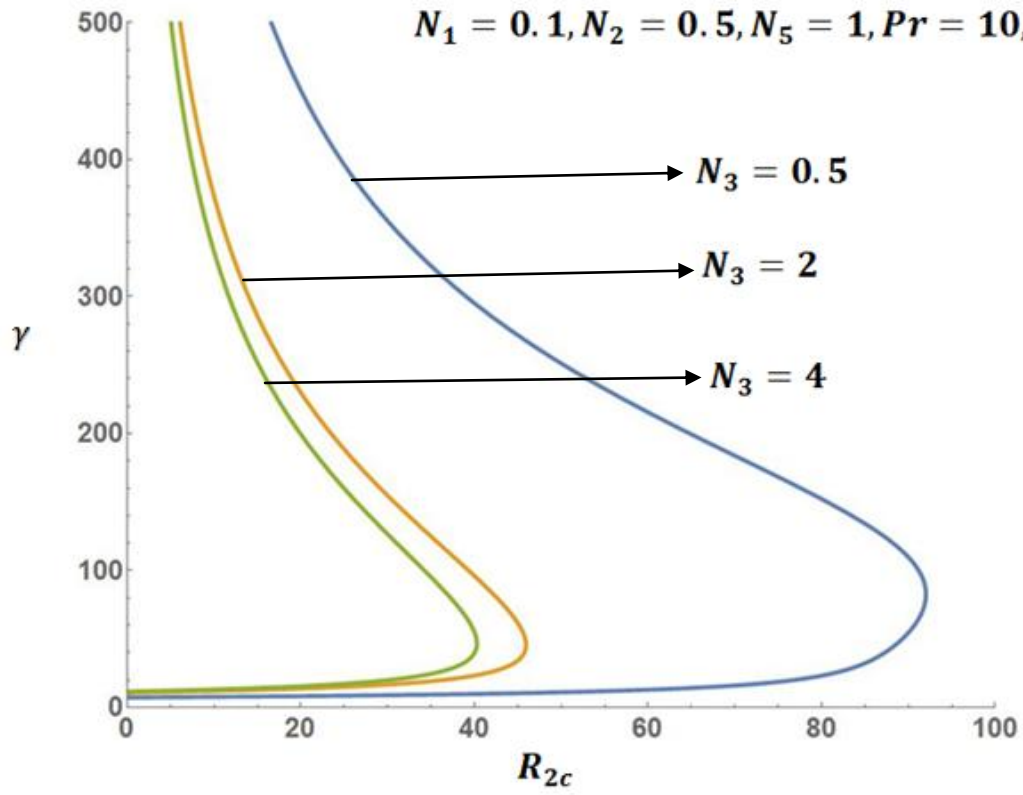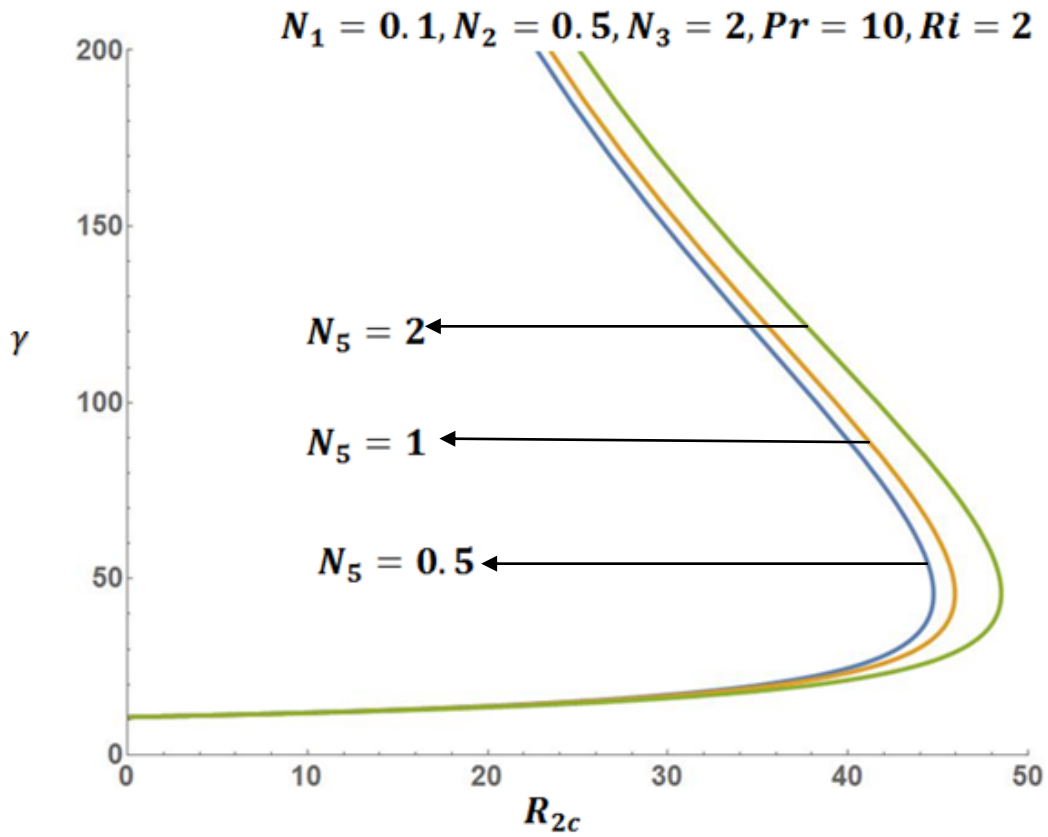modulation γ for different values of inertia parameter N_2.

Figure 4: Plot of correction Rayleigh number R_{2c} versus frequency of gravity

modulation γ for different values of Couple stress parameter N_3.

Figure 5: Plot of correction Rayleigh number R_{2c} versus frequency of gravity modulation γ for different values of micropolar heat conductionparameter N_5.
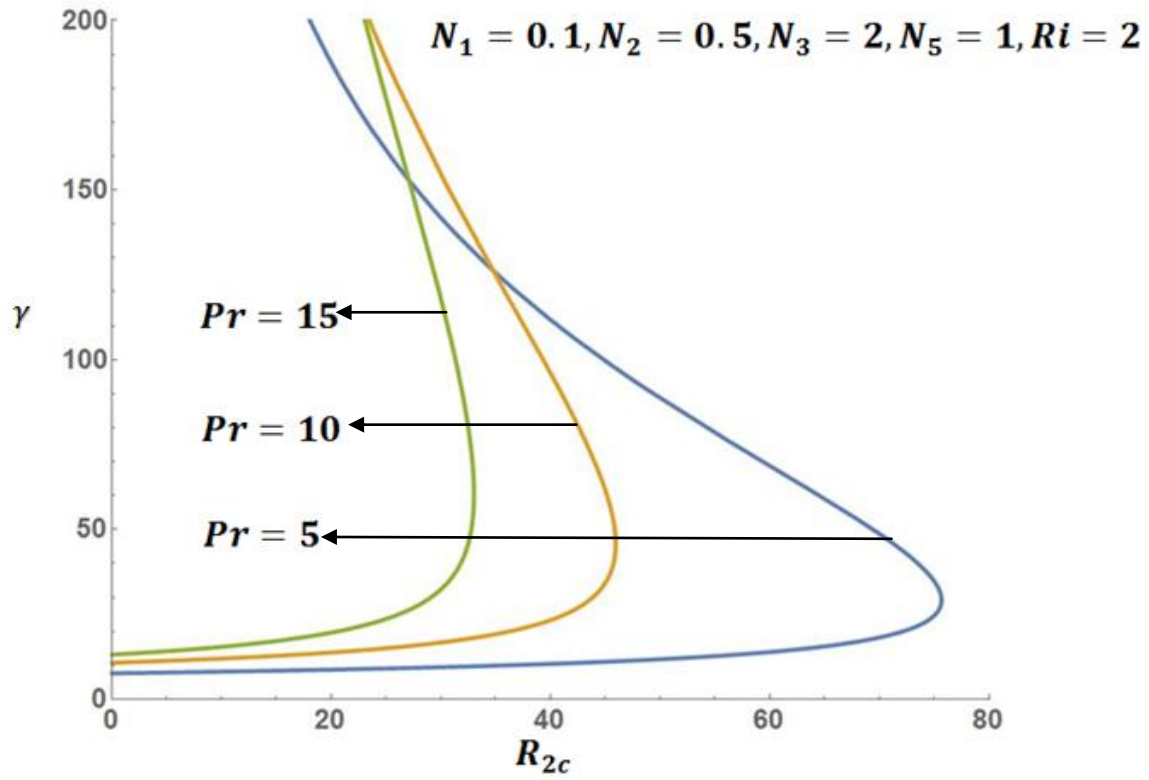
Figure 6: Plot of correction Rayleigh number R_{2c} versus frequency of gravity
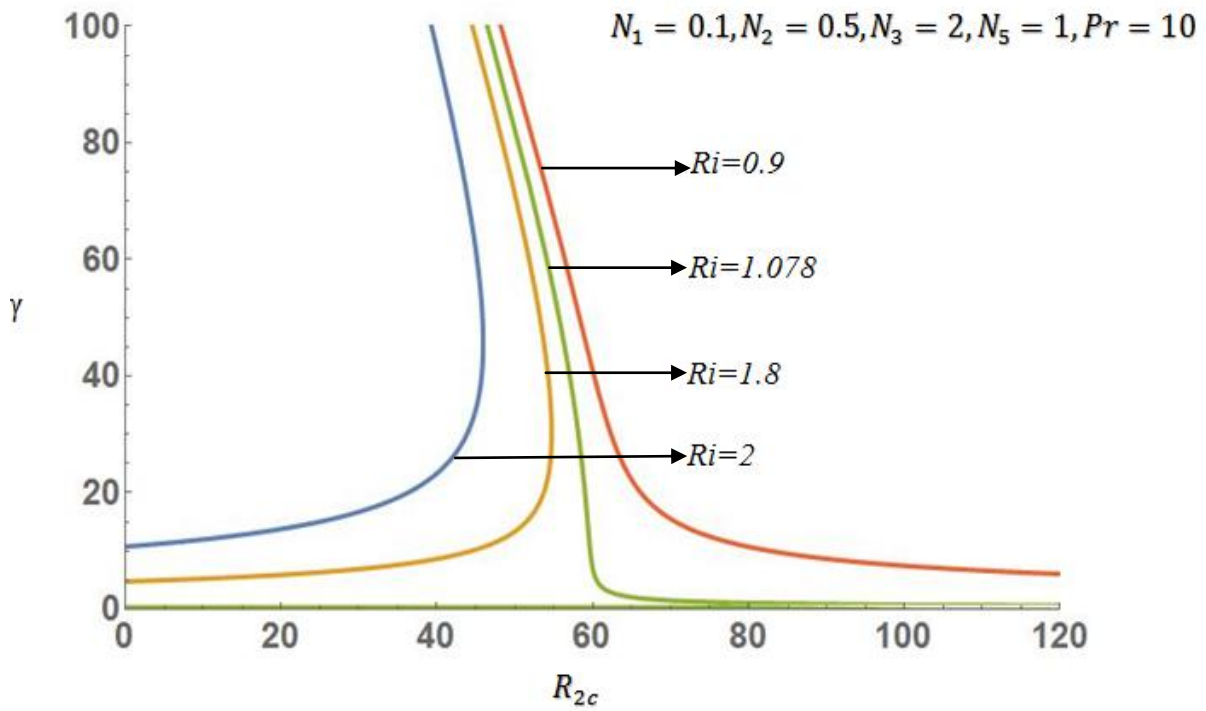modulation γ for different values of Prandtl number *Pr*.

Figure 7: Plot of correction Rayleigh number R_{2c} versus frequency of gravity

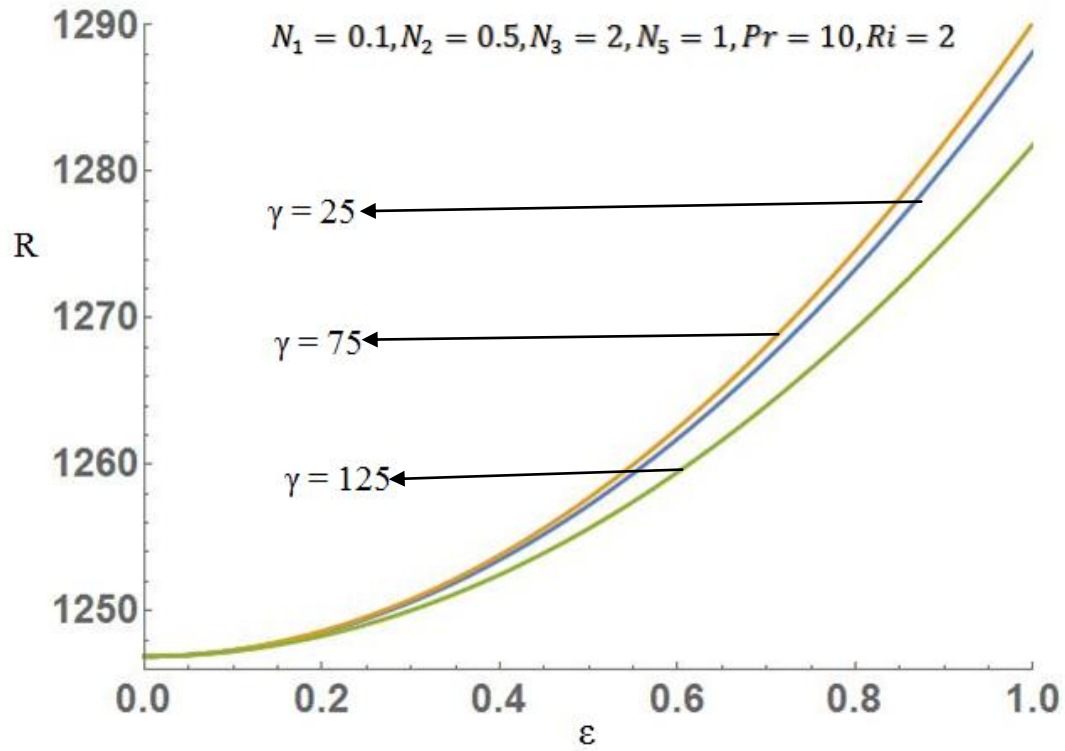modulation γ for different values of internal Rayleigh number *Ri*.

Figure 8: Plot of amplitude of modulation ε verses Rayleigh number *R* for
different values of amplitude of gravity modulation γ.