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On a fractional differential equation with Jumarie's fractional derivative

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Abstruct:

By a counterexample, we prove that the results obtained in [1] are incorrect and there exist some theoretical mistakes in fractional complex transform.

Keyword:

Jumarie's fractional derivative; fractional complex transform; fractional differential equation

INTRUDUCTION

In [1], Zhang-Biao Li and Ji-Huan He have solved the following frctional differential equation:

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} + c\frac{\partial^{\beta} u(x,t)}{\partial x^{\beta}} = 0, \tag{1}$$

where $0 < \alpha, \beta \le 1$, c is a constant and

$$\frac{\partial^{\alpha} u}{\partial t^{\alpha}} = \frac{1}{\Gamma(1-\alpha)} \frac{\partial}{\partial t} \int_0^t (t-\xi)^{-\alpha} (u(x,\xi) - u(x,0)) d\xi, \tag{2}$$

$$\frac{\partial^{\beta} u}{\partial x^{\beta}} = \frac{1}{\Gamma(1-\beta)} \frac{\partial}{\partial x} \int_0^x (x-\xi)^{-\beta} (u(\xi,t) - u(0,t)) d\xi, \tag{3}$$

Which are called Jumarie's fractional derivative [3] .

By using the fractional complex transform [2], they obtained the general solution of Eq. (1) as

$$u_1(x,t) = f\left(\frac{t^{\alpha}}{\Gamma(1+\alpha)} - \frac{x^{\beta}}{c\Gamma(1+\beta)}\right), \tag{4}$$

or

$$u_2(x,t) = f\left(\frac{x^{\beta}}{\Gamma(1+\beta)} - \frac{ct^{\alpha}}{\Gamma(1+\alpha)}\right), \tag{5}$$

where the function $f(p)$ is an arbitrarg and first order function differentiable with respect to p.

However, after careful checking, we find that the solution (4) or (5) is not the solution of Eq. (1). In this note, we will give a counterexample to show this fact.

1. Counterexample

We first recall the dedinition of Riemann-Liouville fractional derivatives.

Dedinition: Riemann-Liouville fractional derivative of order $\alpha\,(0<\alpha<1)$ of function $f(p)$ is defined as [4]:

$$D^{\alpha} f(p)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dp}\int_0^p (p-\xi)^{-\alpha} f(\xi)d\xi. \tag{6}$$

Properties of the operators can be found in [4], we mention only the following:

$$D^{\alpha} p^{\lambda}=\frac{\Gamma(\lambda+1)}{\Gamma(\lambda+1-\alpha)} p^{\lambda-\alpha}, \tag{7}$$

where $p>0, \lambda>-1.$

From (2), (3) and (6), we can get that if $f(0)=0$, then

$$\frac{\partial^{\alpha}}{\partial p^{\alpha}} f(p)=D^{\alpha} f(p). \tag{8}$$

Next we give a counterexample to prove the (4) is not the solution of Eq. (1).

We take $\alpha=\frac{1}{2}, \beta=\frac{1}{2}, c=1, f(p)=p^2.$ Thus Eq. (1) becomes

$$\frac{\partial^{1/2} u(x,t)}{\partial t^{1/2}}+\frac{\partial^{1/2} u(x,t)}{\partial x^{1/2}}=0, \tag{9}$$

And the solution (4) becomes

$$u_1(x,t)=\frac{t}{\Gamma^2(\frac{3}{2})}+\frac{x}{\Gamma^2(\frac{3}{2})}-\frac{2}{\Gamma^2(\frac{3}{2})} t^{\frac{1}{2}} x^{\frac{1}{2}}. \tag{10}$$

Note that $u_1(x,0)=0, u_1(0,t)=0$, Thus by the formula (6)-(8), we have

$$\frac{\partial^{1/2}}{\partial t^{1/2}} u_1(x,t)=\frac{t^{\frac{1}{2}}}{\Gamma^2(\frac{3}{2})}-\frac{2x^{\frac{1}{2}}}{\Gamma(\frac{3}{2})}, \tag{11}$$

and

$$\frac{\partial^{1/2}}{\partial x^{1/2}} u_1(x,t) = \frac{x^{\frac{1}{2}}}{\Gamma^2(\frac{3}{2})} - \frac{2t^{\frac{1}{2}}}{\Gamma(\frac{3}{2})}. \tag{12}$$

From (11) and (12), we can see that :

$$\frac{\partial^{1/2} u_1(x,t)}{\partial t^{1/2}} + \frac{\partial^{1/2} u_1(x,t)}{\partial x^{1/2}} \neq 0,$$

This shows that the function (4) does not the solution of the Eq. (1). We can prove similarly that the function (5) does not the solution of the Eq. (1).

2. Conclusion

By a counterexample, we can conclude that the results obtained in [1] are incorrect and there exist some theoretical mistakes in fractional complex transform.

References

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