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ALGEBRAIC PROOFS FERMAT'S LAST THEOREM, BEAL'S CONJECTURE

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ABSTRACT

In this paper, the following statememt of Fermat's Last Theorem is proved. If x; y; z are positive integers, _ is an odd prime and z_ = x_ + y_; then x; y; z are all even. Also, in this paper, is proved Beal's conjecture; the equation z_ = x_ + y_ has no solution in relatively prime positive integers x; y; z; with _; _; _ primes at least 3:

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x1. Fermat's Last Theorem

For other theorems named after Pierre de Fermat, see [1]. The 1670 edition of Diophantus' Arithmetica includes Fermat's commen- tary, particularly his "Last Theorem" (Observatio Domini Petri de Fermat). In number theory, Fermat's Last Theorem (sometimes called Fermat's conjecture, especially in older texts) states that no three pos-itive integers x, y, and z satisfy the equation z_ = x_ + y_ for any integer value of _ greater than two. The case _ = 2 was known to have in_nitely many solutions. This theorem was _rst conjectured by Pierre de Fermat in 1637 in the margin of a copy of Arithmetica where he claimed he had a proof that was too large to _t in the margin. The _rst proof agreed upon as successful was released in 1994 by Andrew Wiles formally published in 1995 [2],]3], after 358 years of e_ort by mathematicians. This unsolved problem stimulated the development of algebraic number theory in the 19th century and the proof of the modularity theorem in the 20th century. It is among the most notable theorems in the history of mathematics It is known that if x; y; z are relatively prime positive integers, z4 6= x4 + y4[1]: In view of this fact,
it is only necessary to prove if x; y; z; are relatively prime positive inte- gers, _ is an is odd prime, z_ = x_ +y_; then x; y; z; are each divisible by _: Before and since Wiles paper, many papers and books have been
written trying to solve this problem in an elegant algebraic way, but none have suceeded. (See [1], and go to a search engine on the com- puter and search Fermat's Last Theorem). In the remainder of this paper, _ will represent an is odd prime. The special case z4 = x4 + y4
is impossible. In view of this fact, it is only necessary to prove, if x; y; z; are positive integers, _ is an is odd prime, and z_ = x_+y_; then x; y; z are all even.

Theorem. If x, v, z are relatively prime positive integers, satisfying $z^{\pi} = x^{\pi} + y^{\pi}$, then x, y, z are all even.

Proof. Since $z^{\pi} = x^{\pi} + y^{\pi}$, x, y or z is even and the other two are odd. It will be shown x, y, z are all even.

z even x, y are odd $v > xx + y, x - y$ are even $x + y = 2^k c, x - y = 2^{k_1} c_1, k, k1$ nonenegative integers, c, c_1 odd positive integers; adding, gives $2x = 2^k c + 2^{k_1} c_1;$ if $2^k = 2^{k_1},$ this x is even; if $2^k > 2^{k_1}, 2^{k-k_1} \geq 2, 2x = 2^{k-k_1}(2^{k_1} c - c_1);$ so x is even, a contradiction. This y is even since $x + y - z$ is even. On the other hand, y is even x, z are odd, using the same reasoning as above. Hence x, y, z are all even since $x + y - z$ is even.

Fermat's Last Theorem If x, y, z are relatively prime positive integers, and π is an is odd prime, then $z^{\pi} \neq x^{\pi} + y^{\pi}$.

Proof. If $z^{\pi} = x^{\pi} + y^{\pi}$, then z, y, x are all even .

§ 2. Beals conjecture

Any solution x, y, z to the equation $z^{\xi} = x^{\mu} + y^{\nu}$ with ξ, μ, ν primes at least 3 must all be divisible bv $2.$

Proof.

$$(z^{\xi})^{\xi} = (x^{\xi})^{\mu} + (y^{\xi})^{\nu} = (x^{\mu})^{\xi} + (y^{\nu})^{\xi},$$

and by Fermats Last Theorem., $z^{\xi}, x^{\mu}, y^{\nu}$ and x, y, z are all divisible bv $2.$

Corollary. The equation $z^{\xi} = x^{\mu} + y^{\nu}$ has no solution in relatively prime positive integers $x, y, z,$ with ξ, μ, ν primes at least 3.

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