



The Complete Solution of some Kinds of Linear Third Order Partial Differential Equations with Three Independent Variables and variable coefficients

Rusul Hassan Naser, Wafaa Hadi Hanoon, Layla Abd Al-Jaleel Mohsin
University of Kufa, Najaf, IRAQ, Department of Computer Science Faculty of Education for Women
rusulh.nasir@uokufa.edu.iq
University of Kufa, Najaf, IRAQ, Department of Computer Science Faculty of Education for Women
wafaah.hannon@uokufa.edu.iq
University of Kufa, Najaf, IRAQ, Department of Computer Science Faculty of Education for Women
Laila.alyasiri@uokufa.edu.iq

ABSTRACT

In this paper we find the complete solution of some kinds of linear third order partial differential equations of variable coefficients with three independent variables which have the general form

$$AZ_{xxx} + BZ_{yyy} + CZ_{ttt} + DZ_{xxy} + EZ_{xxt} + FZ_{xyy} + GZ_{yyt} + HZ_{xtt} + IZ_{ytt} + JZ_{xyt} + KZ_{xx} + LZ_{yy} + MZ_{tt} + NZ_{xy} + OZ_{xt} + PZ_{yt} + QZ_x + RZ_y + SZ_t + TZ = 0,$$

Where A,B,...,T are variable coefficients . By use the some assumptions will transform the above equation to the nonlinear second order ordinary differential equations

Indexing terms/Keywords

linear Partial Differential Equations ; Third Order ; Three Independent Variables

Academic Discipline And Sub-Disciplines

Applied Mathematics, Differential Equations

SUBJECT CLASSIFICATION

E.g., Mathematics Subject Classification; Library of Congress Classification

INTRODUCTION

The differential equations are very importance in the plenty of the fields of the science as Physics, Chemistry and other sciences, therefore plenty of the scientists are studied this subject. Hani [6], and by the assumed

$Z(x, y, t) = e^{\int u(x)dx + \int v(y)dy + \int w(t)dt}$ found the solution to the linear second order partial differential equations with independent variables and of the form

$$AZ_{xx} + BZ_{xy} + CZ_{xt} + DZ_{yy} + EZ_{yt} + FZ_{tt} + GZ_x + HZ_y + IZ_t + JZ = 0$$

Where A,B,C,...,I and J are arbitrary constants .

Hanoon [7], and by the assumptions

$$Z(x,y) = e^{\int \frac{u(x)}{x} dx + \int v(y) dy}, Z(x,y) = e^{\int u(x) dx + \int \frac{v(y)}{y} dy} \text{ and } Z(x,y) = e^{\int \frac{u(x)}{x} dx + \int \frac{v(y)}{y} dy}$$

found the solution to the linear second order partial differential equations with coefficients variable and of the form

$$A(x, y)Z_{xx} + B(x, y)Z_{xy} + C(x, y)Z_{yy} + D(x, y)Z_x + E(x, y)Z_y + F(x, y)Z = 0$$

Where some of A(x,y),B(x,y),C(x,y),D(x,y),E(x,y) and F(x,y) are functions of x or y or both x and y .

Dzurina , Thandapani and Tamilvanan [5], studied the oscillation of solutions to the third-order neutral differential equations

$$a(t) ([x(t) \pm p(t)x(\delta(t))]^n)^a + q(t)x^a(\tau(t)) = 0$$

Sufficient conditions are established so that every solution is either oscillatory or converges to zero.



Ketap [8], studied the linear third order partial differential equations, with constant coefficients which have the form

$$AZ_{xxx} + BZ_{yyy} + CZ_{xxy} + DZ_{xyy} + EZ_{xx} + FZ_{yy} + GZ_{xy} + HZ_x + IZ_y + JZ = 0,$$

Where A, ... I and J are arbitrary constants, and used the assumption $Z(x, y) = e^{\int u(x) dx + \int v(y) dy}$ to find the complete solution of it.

Liu, Ume, Anderson and Kang [9], studied singular nonlinear third-order differential equation

Ademola, Ogundiran, Arawomo and Adesina [1], studied boundedness of solutions of the third nonlinear differential equation

$$\ddot{x} + f(\dot{x}) + g(\dot{y}) + h(x) = p(t, x, \dot{x}, \dot{y}), \text{ or its equivalent system of differential equations}$$

$$\dot{x} = y, \dot{y} = z, \dot{z} = p(t, x, y, z) - f(z) - g(y) - h(x)$$

where $f, g, h \in C(R, R)$,

$$p \in C(R^+ \times R \times R \times R, R), R^+ = [0, \infty)$$

and $R = (-\infty, \infty)$

Clarkson, Mansfield and Priestley [3], studies symmetry reductions of a class of nonlinear third-order partial differential equations

$$u_t - \epsilon u_{xxx} + 2ku_x = uu_{xxx} + \alpha uu_x + \beta u_x u_{xx},$$

where ϵ, k, α and β are arbitrary constants.

This paper is devoted to solve the linear third order partial differential equations with three independent variables of the general form

$$AZ_{xxx} + BZ_{yyy} + CZ_{ttt} + DZ_{xxy} + EZ_{xxt} + FZ_{xyy} + GZ_{yyt} + HZ_{xtt} + IZ_{ytt} + JZ_{xyt} +$$

$$KZ_{xx} + LZ_{yy} + MZ_{tt} + NZ_{xy} + OZ_{xt} + PZ_{yt} + QZ_x + RZ_y + SZ_t + TZ = 0,$$

where A, B, ..., T are variable coefficients.

By using the assumptions

$$Z(x, y, t) = e^{\int \frac{u(x)}{x} dx + \int \frac{v(y)}{y} dy + \int \frac{w(t)}{t} dt}, Z(x, y, t) = e^{\int \frac{u(x)}{x} dx + \int \frac{v(y)}{y} dy + \int w(t) dt}$$

$$, Z(x, y, t) = e^{\int \frac{u(x)}{x} dx + \int v(y) dy + \int \frac{w(t)}{t} dt} \text{ and } Z(x, y, t) = e^{\int u(x) dx + \int v(y) dy + \int \frac{w(t)}{t} dt}$$

The Complete Solution of linear Third Order partial Differential Equations with Three independent variables

My aim in this section is to solve the linear third order of partial differential equations, with three independent variables which have the general form

$$AZ_{xxx} + BZ_{yyy} + CZ_{ttt} + DZ_{xxy} + EZ_{xxt} + FZ_{xyy} + GZ_{yyt} + HZ_{xtt} + IZ_{ytt} + JZ_{xyt} +$$

$$KZ_{xx} + LZ_{yy} + MZ_{tt} + NZ_{xy} + OZ_{xt} + PZ_{yt} + QZ_x + RZ_y + SZ_t + TZ = 0,$$

where A, B, ..., T are variable coefficients.

So, for this purpose we will search functions $u(x)$, $v(y)$ and $w(t)$ such that the assumptions

$$Z(x, y, t) = e^{\int \frac{u(x)}{x} dx + \int \frac{v(y)}{y} dy + \int \frac{w(t)}{t} dt}, Z(x, y, t) = e^{\int \frac{u(x)}{x} dx + \int \frac{v(y)}{y} dy + \int w(t) dt},$$

$$Z(x, y, t) = e^{\int \frac{u(x)}{x} dx + \int v(y) dy + \int \frac{w(t)}{t} dt} \text{ and } Z(x, y, t) = e^{\int u(x) dx + \int v(y) dy + \int \frac{w(t)}{t} dt}$$



To give the complete solution to the above equation we need to consider many kinds :

Kind (1) :

$$A_1 t^2 y Z_{yzt} + A_2 txy Z_{xyt} + A_3 x^2 Z_{xx} + A_4 xt Z_{xt} = 0 \quad .$$

Kind (2) :

$$A_1 Z_{ttt} + A_2 x^2 Z_{xxt} + A_3 y Z_{yzt} + A_4 y^2 Z_{yyt} + A_5 x Z_x = 0 \quad .$$

Kind (3) :

$$A_1 Z_{yyy} + A_2 x Z_{xyy} + A_3 t^2 Z_{tt} + A_4 x Z_{xy} = 0$$

Kind (4) :

$$A_1 Z_{xxy} + A_2 t^2 Z_{ttx} + A_3 Z_{xy} = 0$$

Where A_1, A_2, A_3, A_4 and A_5 are real constants.

kind (1): By using the assumption

$$Z = e^{\int \frac{u(x)}{x} dx + \int \frac{v(y)}{y} dy + \int \frac{w(t)}{t} dt}, \text{ the equation } A_1 t^2 y Z_{yzt} + A_2 txy Z_{xyt} + A_3 x^2 Z_{xx} + A_4 xt Z_{xt} = 0$$

Transforms to the form

$$A_1 (tw'(t) + w^2(t) - w(t))v(y) + A_2 (u(x)v(y)w(t)) + A_3 (xu'(x) + u^2(x) - u(x)) + A_4 (u(x)w(t)) = 0 \quad \dots (1)$$

This equation can't separate the variables, so we suppose that $w(t) = \lambda_1$, where λ_1 is arbitrary constants, then

$$A_1 (\lambda_1^2 - \lambda_1)v(y) + A_2 \lambda_1 u(x)v(y) + A_3 (xu'(x) + u^2(x) - u(x)) + A_4 \lambda_1 u(x) = 0$$

$$v(y) = \frac{-A_3 (xu'(x) + u^2(x) - u(x)) - A_4 \lambda_1 u(x)}{A_1 (\lambda_1^2 - \lambda_1) + A_2 \lambda_1 u(x)} = -\lambda_2^2 \Rightarrow v(y) = -\lambda_2^2$$

$$A_3 (xu'(x) + u^2(x) - u(x)) + A_4 \lambda_1 u(x) - A_1 \lambda_2^2 (\lambda_1^2 - \lambda_1) - A_2 \lambda_2^2 \lambda_1 u(x) = 0$$

$$xu'(x) + u^2(x) + \left(\frac{A_4 \lambda_1 - A_2 \lambda_2^2 \lambda_1 - A_3}{A_3}\right)u(x) - \frac{A_1 \lambda_2^2 (\lambda_1^2 - \lambda_1)}{A_3} = 0$$

$$xu'(x) + u^2(x) + B_1 u(x) + B_2 = 0 \quad \dots (2)$$

$$; B_1 = \frac{A_4 \lambda_1 - A_2 \lambda_2^2 \lambda_1 - A_3}{A_3} \text{ and } B_2 = \frac{A_1 \lambda_2^2 (\lambda_1^2 - \lambda_1)}{A_3}$$

$$xu'(x) + (u(x) + \frac{B_1}{2})^2 + B_2 - \frac{B_1^2}{4} = 0$$

i) If $B_2 \neq \frac{B_1^2}{4}$ we get $\int \frac{du}{\left(u(x) + \frac{B_1}{2}\right)^2 + f_1^2} + \int \frac{dx}{x} = \int 0$; $f_1^2 = B_2 - \frac{B_1^2}{4}$



$$\frac{1}{f_1} \tan^{-1} \left(\frac{u(x) + \frac{B_1}{2}}{f_1} \right) = -\ln(cx) \quad ; cx > 0 \Rightarrow u(x) = -f_1 \tan(f_1 \ln(cx)) - \frac{B_1}{2}$$

then the complete solution of equation (1), is given by:

$$Z(x,y,t) = e^{\int \frac{-f_1 \tan(f_1 \ln(cx)) - \frac{B_1}{2}}{x} dx - \int \frac{\lambda_2^2}{y} dy + \int \frac{\lambda_1}{t} dt}$$

$$= e^{\ln(\cos(f_1 \ln(cx))) - \frac{B_1}{2} \ln x - \lambda_2^2 \ln y + \lambda_1 \ln t + c_1} \quad ; cx > 0, y > 0, t > 0$$

$$Z(x, y, t) = k x^{\frac{-A_4 \lambda_1 + A_2 \lambda_2^2 \lambda_1 + A_3}{2A_3}} y^{-\lambda_2^2} t^{\lambda_1} \cos(f_1 \ln(cx)); f_1 = \sqrt{\frac{4A_1 A_3 \lambda_2^2 (\lambda_1^2 - \lambda_1) - (A_4 \lambda_1 - A_2 \lambda_2^2 \lambda_1 - A_3)^2}{4A_3^2}}$$

$$k = e^{c_1}, cx > 0$$

Where K, c, λ_1 and λ_2 are arbitrary constants.

Domain : $x > 0, -\infty < y < \infty, -\infty < t < \infty$.

ii) If $B_2 = \frac{B_1^2}{4}$ we get $\Rightarrow \int \frac{du}{(u + \frac{B_1}{2})^2} + \int \frac{dx}{x} = \int 0 \Rightarrow u(x) = \frac{1}{\ln cx} - \frac{B_1}{2} \quad ; cx > 0$

then the complete solution of equation (1), is given by:

$$Z(x, y, t) = e^{\int \frac{(\frac{1}{\ln cx} - \frac{B_1}{2})}{x} dx + \int \frac{-\lambda_2^2}{y} dy + \int \frac{\lambda_1}{t} dt}$$

$$Z(x, y, t) = k y^{-\lambda_2^2} t^{\lambda_1} x^{\frac{-A_4 \lambda_1 + A_2 \lambda_2^2 \lambda_1 + A_3}{2A_3}} \ln cx ; k = e^{c_1}, cx > 0$$

Where K, c, λ_1 and λ_2 are arbitrary constants.

Domain : $x > 0, -\infty < y < \infty, -\infty < t < \infty$

kind (2) : By using the assumption

$$Z(x, y, t) = e^{\int \frac{u(x)}{x} dx + \int \frac{v(y)}{y} dy + \int w(t) dt}, \text{ the equation } A_1 Z_{ttt} + A_2 x^2 Z_{xxt} + A_3 y Z_{yft} + A_4 y^2 Z_{yft} + A_5 x Z_x = 0$$

Transforms to the form

$$A_1 (w''(t) + 3w(t) w'(t) + w^3(t)) + A_2 (xu'(x) + u^2(x) - u(x))w(t) + A_3 (w'(t) + w^2(t))v(y) + A_4 (yv'(y) + v^2(y) - v(y))w(t) + A_5 u(x) = 0 \quad \dots(3)$$

This equation can't separate the variables, so we suppose that $w(t) = \lambda_1$, where λ_1 is arbitrary constants, then



$$\Rightarrow A_1 \lambda_1^3 + A_2 \lambda_1 (xu'(x) + u^2(x) - u(x)) + A_3 \lambda_1^2 v(y) + A_4 \lambda_1 (yv'(y) + v^2(y) - v(y)) + A_5 u(x) = 0$$

This equation is variable separable equation [4].

$$\Rightarrow A_2 \lambda_1 (xu'(x) + u^2(x) - u(x)) + A_5 u(x) = -A_4 \lambda_1 (yv'(y) + v^2(y) - v(y)) - A_3 \lambda_1^2 v(y) - A_1 \lambda_1^3 = -\lambda_2$$

$$\Rightarrow xu'(x) + u^2(x) + \left(\frac{A_5}{A_2 \lambda_1} - 1\right)u(x) + \frac{\lambda_2}{A_2 \lambda_1} = 0$$

$$\text{Let } B_1 = \frac{A_5}{A_2 \lambda_1} - 1 \text{ and } B_2 = \frac{\lambda_2}{A_2 \lambda_1} \Rightarrow xu'(x) + u^2(x) + B_1 u(x) + B_2 = 0 \quad \dots(4)$$

$$yv'(y) + v^2(y) + \left(\frac{A_3 \lambda_1^2}{A_4 \lambda_1} - 1\right)v(y) + \frac{A_1 \lambda_1^3 - \lambda_2}{A_4 \lambda_1} = 0$$

$$\text{Let } B_3 = \frac{A_3 \lambda_1^2}{A_4 \lambda_1} - 1 \text{ and } B_4 = \frac{A_1 \lambda_1^3 - \lambda_2}{A_4 \lambda_1} \Rightarrow yv'(y) + v^2(y) + B_3 v(y) + B_4 = 0 \quad \dots (5),$$

The equation (4), is similar to equation (2), then the same method the solution of it is given by :

$$i) \text{ If } B_2 \neq \frac{B_1^2}{4} ; \text{ we get } \Rightarrow u(x) = -f_1 \tan(f_1 \ln(cx)) - \frac{B_1}{2} ; f_1^2 = B_2 - \frac{B_1^2}{4}$$

$$ii) \text{ If } B_2 = \frac{B_1^2}{4} \text{ we get } \Rightarrow u(x) = \frac{1}{\ln cx} - \frac{B_1}{2} ; cx > 0$$

Now the equation(5), is similar to equation (2), then the same method the solution of it is given by :

$$i) \text{ If } B_4 \neq \frac{B_3^2}{4} ; \text{ we get } \Rightarrow v(y) = -f_2 \tan(f_2 \ln(c_1 y)) - \frac{B_3}{2} ; f_2^2 = B_4 - \frac{B_3^2}{4} , c_1 y > 0$$

$$ii) \text{ If } B_4 = \frac{B_3^2}{4} \text{ we get } \Rightarrow v(y) = \frac{1}{\ln c_1 y} - \frac{B_3}{2} ; c_1 y > 0$$

Then the complete solution of the equation (3), is given by :

$$i) \text{ If } B_2 \neq \frac{B_1^2}{4} \text{ and } B_4 \neq \frac{B_3^2}{4} , \text{ we get}$$

$$\Rightarrow Z(x, y, t) = e^{\int \frac{-f_1 \tan(f_1 \ln(cx)) - \frac{B_1}{2}}{x} dx} \times e^{\int \frac{-f_2 \tan(f_2 \ln(c_1 y)) - \frac{B_3}{2}}{y} dy} + \int \lambda_1 dt$$

$$Z(x, y, t) = K x^{-\frac{B_1}{2}} y^{-\frac{B_3}{2}} e^{\lambda_1 t} \cos(f_1 \ln(cx)) \cos(f_2 \ln(c_1 y)); K = e^{c_2}, B_1 = \frac{A_5}{A_2 \lambda_1} - 1, B_3 = \frac{A_3 \lambda_1^2}{A_4 \lambda_1} - 1$$

$$, f_1 = \sqrt{\frac{\lambda_2}{A_2 \lambda_1} - \left(\frac{A_5}{2A_2 \lambda_1} - \frac{1}{2}\right)^2} , cx > 0 , c_1 y > 0 \text{ and } f_2 = \sqrt{\frac{A_1 \lambda_1^3 - \lambda_2}{A_4 \lambda_1} - \frac{(A_3 \lambda_1^2 - A_4 \lambda_1)^2}{4A_4^2 \lambda_1^2}}$$



Where K, c, c_1, λ_1 and λ_2 are arbitrary constants.

Domain : $x > 0, y > 0, -\infty < t < \infty$

ii) If $B_2 \neq \frac{B_1^2}{4}$ and $B_4 = \frac{B_3}{4}$, we get

$$Z(x, y, t) = K x^{-\frac{B_1}{2}} y^{-\frac{B_3}{2}} e^{\lambda_1 t} \cos(f_1 \ln(cx)) \ln(c_1 y); K = e^{c^2}, B_1 = \frac{A_5}{A_2 \lambda_1} - 1, B_3 = \frac{A_3 \lambda_1^2}{A_4 \lambda_1} - 1$$

$$, f_1 = \sqrt{\frac{\lambda_2}{A_2 \lambda_1} - \left(\frac{A_5}{2A_2 \lambda_1} - \frac{1}{2}\right)^2}, \quad cx > 0 \text{ and } c_1 y > 0$$

Where K, c, c_1, λ_1 and λ_2 are arbitrary constants.

Domain : $x > 0, y > 0, -\infty < t < \infty$

$$Z(x, y, t) = e^{\int \frac{1 - B_1}{x} dx + \int \frac{-f_2 \tan(f_2 \ln(c_1 y)) - B_3}{y} dy + \int \lambda_1 dt}$$

$$Z(x, y, t) = K x^{-\frac{B_1}{2}} y^{-\frac{B_3}{2}} e^{\lambda_1 t} \ln(cx) \cos(f_2 \ln(c_1 y)); K = e^{c^2}, B_1 = \frac{A_5}{A_2 \lambda_1} - 1, B_3 = \frac{A_3 \lambda_1^2}{A_4 \lambda_1} - 1$$

iii) If $B_2 = \frac{B_1^2}{4}$ and $B_4 \neq \frac{B_3}{4}$, we get

$$, f_2 = \sqrt{\frac{A_1 \lambda_1^3 + A_3 \lambda_1^2 - \lambda_2}{A_4 \lambda_1} - \frac{1}{4}}, \quad cx > 0, c_1 y > 0 \text{ Where } K, c, c_1, \lambda_1 \text{ and } \lambda_2 \text{ are arbitrary constants.}$$

Domain : $x > 0, y > 0, -\infty < t < \infty$

iv) If $B_2 = \frac{B_1^2}{4}$ and $B_4 = \frac{B_3}{4}$, we get

$$Z(x, y, t) = e^{\int \frac{1 - B_1}{x} dx + \int \frac{1 - B_3}{y} dy + \int \lambda_1 dt}$$

$$Z(x, y, t) = K x^{-\frac{B_1}{2}} y^{-\frac{B_3}{2}} e^{\lambda_1 t} \ln(cx) \ln(c_1 y); K = e^{c^2}, B_1 = \frac{A_5}{A_2 \lambda_1} - 1, B_3 = \frac{A_3 \lambda_1^2}{A_4 \lambda_1} - 1, cx > 0, c_1 y > 0$$

Where K, c, c_1 and λ_1 are arbitrary constants.

Domain : $x > 0, y > 0, -\infty < t < \infty$

kind (3) : By using the assumption $Z = e^{\int \frac{u(x)}{x} dx + \int v(y) dy + \int \frac{w(t)}{t} dt}$, the equation

$$A_1 Z_{yyy} + A_2 x Z_{xyy} + A_3 t^2 Z_{tt} + A_4 x Z_{xy} = 0$$



Transforms to the form

$$A_1 (v''(y) + 3v(y)v'(y) + v^3(y)) + A_2 u(x)(v'(y) + v^2(y)) + A_3 (tw'(t) + w^2(t) - w(t)) + A_4 u(x)v(y) = 0 \quad \dots (6)$$

This equation can't separate the variables, so we suppose that $v(y) = \lambda_1$, where λ_1 is arbitrary constants, then

$$A_1 \lambda_1^3 + A_2 \lambda_1^2 u(x) + A_3 (tw'(t) + w^2(t) - w(t)) + A_4 \lambda_1 u(x) = 0$$

This equation is variable separable equation [4].

$$A_1 \lambda_1^3 + A_2 \lambda_1^2 u(x) + A_4 \lambda_1 u(x) = -A_3 (tw'(t) + w^2(t) - w(t)) = -\lambda_2$$

$$A_1 \lambda_1^3 + A_2 \lambda_1^2 u(x) + A_4 \lambda_1 u(x) + \lambda_2 = 0 \Rightarrow u(x) = \frac{-(A_1 \lambda_1^3 + \lambda_2)}{A_2 \lambda_1^2 + A_4 \lambda_1}$$

And $tw'(t) + w^2(t) - w(t) - \frac{\lambda_2}{A_3} = 0$

Let $B = \frac{\lambda_2}{A_3} \Rightarrow tw'(t) + w^2(t) - w(t) - B = 0$

This equation is variable separable equation [4], we can solve it as follows :

$$-\frac{dw}{d^2 - \left(w(t) - \frac{1}{2}\right)^2} + \frac{dt}{t} = 0 \quad ; d^2 = B + \frac{1}{4}$$

i) if $B \neq -\frac{1}{4}$, we get $\Rightarrow -\frac{1}{d} \tanh^{-1} \left(\frac{w(t) - \frac{1}{2}}{d} \right) = -\ln(ct) \Rightarrow w(t) = d \tanh(d \ln(ct)) + \frac{1}{2} ; ct > 0$

ii) if $B = -\frac{1}{4}$, we get $\Rightarrow \frac{dw}{\left(w(t) - \frac{1}{2}\right)^2} + \frac{dt}{t} = 0 \Rightarrow w(t) = \frac{1}{\ln(ct)} + \frac{1}{2} ; ct > 0$

Then the complete solution of the equation (6), is given by :

i) if $B \neq -\frac{1}{4}$, we get

$$Z(x, y, t) = e^{\int \frac{-(A_1 \lambda_1^3 + \lambda_2)}{A_2 \lambda_1^2 + A_4 \lambda_1} dx + \int \lambda_1 dy + \int \frac{d \tanh(d \ln(ct)) + \frac{1}{2}}{t} dt}$$

$$Z(x, y, t) = K x^{\frac{-(A_1 \lambda_1^3 + \lambda_2)}{A_2 \lambda_1^2 + A_4 \lambda_1}} e^{\lambda_1 y} t^{\frac{1}{2}} \cosh \left(\left(\sqrt{\frac{\lambda_2}{A_3} + \frac{1}{4}} \right) \ln(ct) \right) ; K = e^{c_1}, ct > 0$$



Where K, c, λ_1 and λ_2 are arbitrary constants.

Domain : $-\infty < x < \infty, -\infty < y < \infty, t > 0$.

ii) if $B = -\frac{1}{4}$, we get $\Rightarrow Z(x, y, t) = e^{\int \frac{-(A_1 \lambda_1^3 + \lambda_2)}{A_2 \lambda_1^2 + A_4 \lambda_1} dx + \int \lambda_1 dy + \int \frac{1}{t} dt}$

$$Z(x, y, t) = K x^{\frac{-(A_1 \lambda_1^3 + \lambda_2)}{A_2 \lambda_1^2 + A_4 \lambda_1}} e^{\lambda_1 y} t^{\frac{1}{2}} \ln(ct) ; K = e^{c_1}, ct > 0$$

Where K, c, λ_1 and λ_2 are arbitrary constants.

Domain : $-\infty < x < \infty, -\infty < y < \infty, t > 0$.

kind (4) : By using the assumption

$$Z(x, y, t) = e^{\int u(x) dx + \int v(y) dy + \int \frac{w(t)}{t} dt}, \text{ the equation } A_1 Z_{xxy} + A_2 t^2 Z_{ttx} + A_3 Z_{xy} = 0$$

Transforms to the form

$$A_1 (u'(x) + u^2(x))v(y) + A_2 (tw'(t) + w^2(t) - w(t))u(x) + A_3 u(x)v(y) = 0 \quad \dots (7)$$

This equation can't separate the variables, so we suppose that $w(t) = \lambda_1$, where λ_1 is arbitrary constants,

then $A_1 (u'(x) + u^2(x))v(y) + A_2 (\lambda_1^2 - \lambda_1)u(x) + A_3 u(x)v(y) = 0$

This equation is variable separable equation [4].

$$\frac{v(y)}{A_2 (\lambda_1^2 - \lambda_1) + A_3 v(y)} = -\frac{u(x)}{A_1 (u'(x) + u^2(x))} = -\lambda_2 \Rightarrow v(y) = \frac{A_2 \lambda_2 (\lambda_1 - \lambda_1^2)}{\lambda_2 A_3 - 1}$$

And $u'(x) + u^2(x) - \frac{1}{A_1 \lambda_2} u(x) = 0$

This equation is similar to Bernoulli equation [2], then the solution of it is given by :

$$u(x) = \frac{e^{-\frac{1}{A_1 \lambda_2} x}}{\int e^{\frac{1}{A_1 \lambda_2} x} dx}$$

Then the complete solution of the equation (7), is given by:

$$\begin{aligned} Z(x, y, t) &= e^{\int \frac{e^{-\frac{1}{A_1 \lambda_2} x}}{e^{\frac{1}{A_1 \lambda_2} x}} dx + \int \frac{A_2 \lambda_2 (\lambda_1 - \lambda_1^2)}{\lambda_2 A_3 - 1} dy + \int \frac{\lambda_1}{t} dt} \\ &= e^{\ln \left(e^{\frac{1}{A_1 \lambda_2} x} + \frac{A_2 \lambda_2 (\lambda_1 - \lambda_1^2)}{\lambda_2 A_3 - 1} y + \lambda_1 t \right)} \\ &= K t^{\lambda_1} e^{\frac{1}{A_1 \lambda_2} x + \frac{A_2 \lambda_2 (\lambda_1 - \lambda_1^2)}{\lambda_2 A_3 - 1} y} ; K = A_1 \lambda_2 e^c \end{aligned}$$



Where K , λ_1 and λ_2 are arbitrary constant.

Domain : $-\infty < x < \infty$, $-\infty < y < \infty$, $-\infty < t < \infty$.

REFERENCES

1. Ademola T. A. , Ogundiran, M. O., Arawomo , Peter O. and Adesina , O. A.," Boundednes results for a certain third order nonlinear differential equation", Journal of Applied Mathematics and Computation 216 (2010) 3044–3049.
2. Braun, M., "Differential Equation and Their Applications", 4th ed. New. York :Spring-verlag, (1993).
3. Clarkson, P.A., Mansfield, E. L. and Priestley, T. J.
" Symmetries of a Class of Nonlinear Third -Order Partial Differential Equations", Journal of Mathl. Cornput. Modelling Vol. 25, No. 8/9, pp. 195-212, (1997) .
4. Codington, E.A. ,"An Introduction to Ordinary Differential Equations ",New York: Dover, 1989.
5. Dzurina,J. Thandapani, E. and Tamilvanan, S.
" Oscillation of solutions to third-order half-linear neutral differential equations" , Electron. J. Diff. Equ., Vol. 2012, No. 29, pp. 1-9 , (2012).
6. Hani N.N., "On Solutions of Partial Differential Equations of second order with Constant Coefficients", Msc, thesis, University of Kufa,(2008).
7. Hanoon W.H., "On Solutions of Partial Differential Equations and Their Physical Applications ", Msc, thesis, University of Kufa,, (2009).
8. Ketap, S.N., "The Complete Solution for Some Kinds of Linear Third Order Partial Differential Equations ", Msc, thesis, University of Kufa,(2011).
9. Liu , Z. ,Ume , J. S., Anderson , D. R .and Kang , S. M. ," Twin monotone positive solutions to a singular nonlinear third-order differential equation" J. Math. Anal. Appl. 334 (2007) 299–313