

# Blind Identification of Transmission Channel with the method of Higher-OrderCummulants

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### **ABSTRACT**

The modern telecommunication systems require very high transmission rates, in this context, the problem of channels identification is a challenge major. The use of blind techniques is a great interest to have the best compromise between a suitable bit rate and quality of the information retrieved.

In this paper, we are interested to learn the algorithms for blind channel identification. We propose a hybrid method that performs a trade-off between two existing methods in order to improve the channel estimation.

### Indexing terms/Keywords

Transmission channel, telecommunication systems, blind identification, higher order cumulants.



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### INTRODUCTION

The actual progress in resolving systems became more important in the telecommunication systems, especially the blind identification channel, including the requirement of modern telecommunications systems that seek to use very high transmission rates. In this context, the application of very important in higher-order cumulants is a technique now commonly addressed by digital telecommunication systems. The use of blind identification has a great interest to have a good estimation of channel parameters, and therefore a good quality of information retrieved.

In this paper, we present tree algorithms for blind identification based on higher order cumulants [1]. The first objective is to make a comparative study of these algorithms, and then we will study our proposed algorithm, this last is a combination of two other existing algorithms [2] and [3], in order to have a better channel estimation in present of white Gaussian noise.

### PROBLEM STATEMENT

The channel is modeled by a FIR filter whose impulse responses are H (t) according to the diagram of Fig. 1.

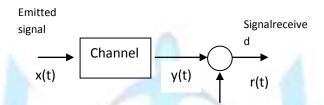


Fig. 1. Channel model

x(t): represents the sample of the input signal to the channel.

y(t): the impulse response of the channel.

The problem is to determine h (t) from a statistical analysis of y (t) (the channel response) received no information about the input signal x (t).

The cumulants for a Gaussian signal is zero, which justifies the use of statistical analysis using higher order cumulants.

Often we take samples of finished size to reduce the execution time; however, the distribution is far from Gaussian, where the higher-order cumulants are different to zero. Hybridization between algorithms reduces this error to the size of the sample (see Fig. 2.)

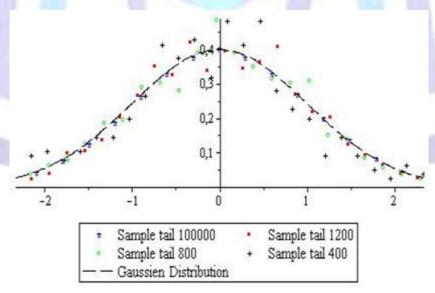


Fig. 2. Different samples of different sizes around the Gaussian distribution.

According to the Fig. 2, we note that the sample of the large size (100 000, \*) coincides perfectly with the Gaussian distribution. In contrast, other distributions are far away.

### **ALGORITHM BASED ON CUMULANTS**

In the literature there is many important algorithms based on higher order cumulant [1-4]. In this work, structure of the model is generally used single variable, discrete time, invariant in time.



### **Moment and Cumulant**

In this section we present some definitions of higher order statistics, moments and cumulants.

Let  $\{x(k), avec \ k \in \mathbb{Z}\}$ , is a real process discrete stationary, so its moment of order m is given by [4] [5] [6] [7] [2]:

$$M_{m,x}(t_1, t_2, ..., t_{m-1}) = Ex(k)x(k+t_1)x(k+t_2)...x(k+t_{m-1})$$
(1)

With E{.} represents the mathematical expectation.

The cumulant of order n of a non-Gaussian stationary process is given by:

$$C_{m,x}(t_1, t_2, ..., t_{m-1}) = M_{m,x}(t_1, t_2, ..., t_{m-1}) - M_{m,G}(t_1, t_2, ..., t_{m-1})$$
(2)

With  $M_{m,x}(t_1,t_2,...,t_{m-1})$  represents the moment of order m and  $M_{m,G}(t_1,t_2,...,t_{m-1})$  is the time of a signal equivalent Gaussian which has the same function as the autocorrelation signal x(k).

### Moment and cumulants estimation

#### Moment estimation

Let  $X = x_{i_{1}=1,\dots,k}$  a random variable representing scalar centered N samples of a stationary signal.

The simplest estimator of order k appointed conventional estimator is given by:

$$m_{k,x}(t_1, t_2, \dots, t_{k-1}) = \frac{1}{N} \sum_{i=1}^{N} x(i)x(i+t_1)x(i+t_2) \dots x(i+t_{k-1})$$
(3)

### **Cumulants estimation**

A detailed presentation of the theory of cumulants estimation can be found in [9], [4]. As cumulants are expressed in terms of moments, the estimates of cumulants are obtained as follows:

$$\hat{C}_{2,x}(t_1) = \hat{C}_2(t_1) = m_2(t_1) \tag{4}$$

$$\hat{C}_{3,x}(t_1, t_2) = m_3(t_1, t_2) \tag{5}$$

$$\hat{C}_4(t_1, t_2, t_4) = m_4(t_1, t_2, t_4) - m_2(t_1)m_2(t_2 - t_3) - m_2(t_2)m_2(t_1 - t_3) - m_2(t_3)m_2(t_1 - t_2)$$
(6)

Generally, most of the methods use different order cumulants, in this work we develop algorithms based on a single order cumulants.

### Algorithm Based on 3<sup>th</sup> Order Cumulant: Alg. 1

The m order cumulant of the system output may be expressed as a function of the coefficients of the impulse response {h (i)} [4] by:

$$C_{mv}(t_1, ..., t_{m-1}) = \gamma_{mx} \sum_{i=-\infty}^{+\infty} h(i)h(i+t_1) ... h(i+t_{m-1})$$
(7)

With  $\gamma_{mx}$  is the order cumulant m behind the input sequence.

And q is the number of channels

If m= 4, equation (7) becomes:

$$C_{4y}(t_1, t_2, t_3) = \gamma_{4x} \sum_{i=0}^{q} h(i)h(i + t_1)h(i + t_3)$$
(8)

Similarly, if m = 2, the equation (7) becomes:

$$C_{2y}(t_1) = \gamma_{2x} \sum_{i=0}^{q} h(i)h(i+t_1)$$
(9)

Applying the Fourier transform of (8) and (9) we obtain:

$$S_{2v}(\omega) = \sigma^2 H(\omega) H(-\omega)$$
 (10)

$$S_{4v}(\omega_1, \omega_2, \omega_3) = \gamma_{4x} H(\omega_1) H(\omega_2) H(\omega_3) H(-\omega_1 - \omega_2 - \omega_3)$$

$$\tag{11}$$

By applying the inverse Fourier transform [1] we have:



$$\sum_{i=0}^{q} h(i)C_{4y}(t_1 - j, t_2 - j, t_3 - j) = \epsilon \sum_{i=0}^{q} h(i)h(i + t_2 - t_1)h(t_3 - t_1)C_{2y}(t_1 - i)$$
(12)

With 
$$\epsilon = \frac{\gamma_{4e}}{\gamma_{2e}}$$

For  $t_3 = t_2$  and  $t_1 = 2q$ (with h (0) =1)

The autocorrelation function of a system to FIR vanishes for all values as: |t|>q

As the system is supposed causal (h (i) =0 for i<0 and i>q).

Equation become:

$$\sum_{i=0}^{q} h(i)C_{4y}(2q-j,t_2-j,2q-j) = \epsilon h(q)h(t_2-q)C_{2y}(q)$$
 (13)

The choice of  $t_2$  requires that  $(t_2 > 2q)$  then  $q \le t_2 \le 2q$ , and for  $t_1 = t_2 = -q$  in equation (13).

If we use the cumulants proprieties [4] [11]. We have:

$$\epsilon = \frac{C_{4y}(q,0,0)}{C_{2y}(q)}$$

So we can present the system in matrix form:

$$\begin{pmatrix} C_{4y}(2q-1,2q-1,q-1) & C_{4y}(q,q,0) \\ C_{4y}(2q-1,2q-1,q) - \epsilon' & \cdots & C_{4y}(q,q,1) \\ \vdots & \ddots & \vdots \\ 0 & \cdots & C_{4y}(q,q,0) - \epsilon' \end{pmatrix} \begin{pmatrix} h(1) \\ \vdots \\ h(q) \end{pmatrix} = \begin{pmatrix} \epsilon' - C_{4y}(2q,2q,q) \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$
(14)

With
$$\epsilon^{'} = \frac{C_{4y}(q,q,q)C_{4y}(q,0,0)}{C_{4y}(q,q,0)}$$

We can also write the system in the following form:

$$Mh_{q} = d (15)$$

The resolution of the system in the sense of least squares is given by:

$$h_{a} = (M^{T}M)^{-1}M^{T}d$$
 (16)

### Algorithm Based on 4th Order Cumulants: Alg. 2

### **Zhang algorithm**

Using equation (3), Zhang and Al. [10] have developed an equation based on the cumulants of order m given by:

$$\sum_{i=0}^{q} h(i)C_{ny}^{n-1}(i-t,q,...,0) = C_{ny}(t,0,...,0)C_{ny}^{n-3}(q,0,...,0)C_{ny}(q,q,...,0)$$
(17)

For n=4 we obtain from equation (16) the following equation:

$$\sum_{i=0}^{q} h(i)C_{4y}^{3}(i-t,q,0) = C_{4y}(t,0,0)C_{4y}(q,0,0)C_{4y}(q,q,0)$$
(18)

For t=-q, -q+1...q.

The system can be represented in the following matrix form

$$\begin{pmatrix} C_{4y}^{3}(i+q,q,0) & & C_{4y}^{3}(2q,q,0) \\ C_{4y}^{3}(i+q-1,q,0) & \cdots & C_{4y}^{3}(2q-1,q,0) \\ \vdots & \ddots & \vdots \\ C_{4y}^{3}(i-q,q,0) & \cdots & C_{4y}^{3}(0,q,0) - \epsilon' \end{pmatrix} \begin{pmatrix} h(0) \\ \vdots \\ h(q) \end{pmatrix} = \epsilon \begin{pmatrix} C_{4y}(-q,0,0) \\ C_{4y}(-q+1,0,0) \\ \vdots \\ C_{4y}(q,0,0) \end{pmatrix}$$
(19)

With  $\epsilon = C_{4v}(q, 0, 0)C_{4v}(q, q, 0)$ .

The system can be represented in a simplified form as follows:

$$Mzh_{q} = d (20)$$

To estimate the parameters  $h(i)_{i=1,\dots,q}$  we can use the method of least squares:



$$\mathbf{h}_{\mathbf{q}} = (\mathbf{M}\mathbf{z}^{\mathrm{T}}\mathbf{M}\mathbf{z})^{-1}\mathbf{M}\mathbf{z}^{\mathrm{T}}\mathbf{d} \tag{21}$$

### Proposed algorithm (Hybrid)

This algorithm is particularly clever because it performs a hybrid between the two algorithms described previously (ALG1 [4] and Zhang [10]) in order to reduce the calculation errors of cumulants for a finite sample size, this approach reduces perfectly the effect of white Gaussian noise.

Description of the proposed algorithm (hybrid)

$$\sum_{j=0}^{q} h(j) \left[ \left( C_{4y}(2q-j,t-j,2q-j) \right) + \left( C_{4y}^{3}(j-t,q,0) \right) \right] = \varepsilon^{'} h(q) h(t-q) C_{2y}(2q) \\ + C_{4y}(t,0,0) C_{4y}(q,0,0) C_{4y}(q,q,0) \quad (22)$$

With 
$$\epsilon^{'} = \frac{C_{4y}(q,q,q)C_{4y}(q,0,0)}{C_{4y}(q,q,0)}$$

For i=0..q, and t=-q..q.

The system can be represented in the following matrix form:

$$\begin{pmatrix} C_{4y}(i+q,q,0) + C_{4y}^{3}(2q-1,2q-1,q-1) & C_{4y}^{3}(2q,q,0) + C_{4y}(q,q,0) \\ C_{4y}(i+q-1,q,0) + C_{4y}^{3}(2q-1,2q-1,q) - \epsilon^{'} & C_{4y}^{3}(2q-1,q,0) + C_{4y}(q,q,1) \\ \vdots & \vdots & \vdots \\ C_{4y}^{3}(i-q,q,0) & \cdots & C_{4y}^{3}(0,q,0) - \epsilon^{'} + \left(C_{4y}(q,q,0) - \epsilon^{'}\right) + \epsilon^{''} \end{pmatrix} \begin{pmatrix} h(0) \\ \vdots \\ h(q) \end{pmatrix} = \\ \epsilon \begin{pmatrix} C_{4y}(-q,0,0) + C_{4y}(-q+1,0,0) \\ \vdots \\ C_{4y}(q,0,0) \end{pmatrix}$$
 (23)

With  $\epsilon'' = \epsilon' h(q)h(0)C_{2v}(2q)$ .

According to the properties of FIR [2],  $C_{4v}(t_1, t_2, t_3) = 0$  if  $t_1 > q$  or  $t_2 > q$  or  $t_3 > q$ .

### SIMULATION

In this section we will make a comparative study between two algorithms, one of Zhang and our proposed algorithm, this last in anhybation between the algorithm previously cited and ALG1 mentioned earlier [1], we showed that the hybrid algorithm is better than Zhang's algorithm in terms of wandering (MSE), which implies, of course, a good estimate of the channel.

### **Channel withtwoparameters**

To validate this algorithm, we applied it to different channels of a different order; remember that this algorithm is based on fourth order cumulants.

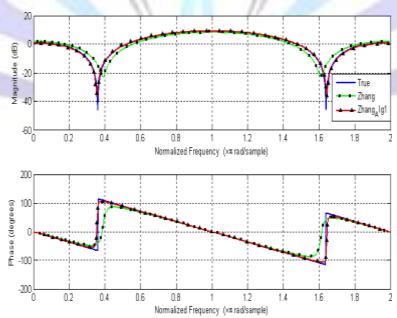


Fig. 3.Estimation of the impulse responses in amplitude and phase algorithms for N=400 samples.



According to Fig. 3, we note that the shape of the channel estimated using the proposed algorithm is very closed to those representing the true channel.

N	Algorithm	EQM	Estimetedchannel	
Truechanal			1.0000 -0.8500 1.0000	
400	Zhang	0.0288	1.0000 -0.3273 0.7535	
	Zhang+Alg1	0.0063	1.0000 -0.8243 1.0736	
800	Zhang	0.0391	1.0000 -1.0139 1.0438	
	Zhang+Alg1	0.0013	1.0000 -0.8794 1.01161	
1200	Zhang	0.0596	1.0000 -0.6533 0.9226	
	7hang+Alg1	7 2315e-04	1 0000 -0 8687 -0 9845	

Table 1. Channel estimation by the two algorithms

Table I groups the estimated values of the channel of the two algorithms, for different sample sizes, i.e. N = 400, N = 800 and N = 1200.

However, the algorithm Zhang+ALG1 (proposed) gives better performance compared to the Zhang algorithm for different sample sizes.

We note that the mean squared error decreases when the sample size increases, then we get an error of order 10-4 with the proposed algorithm.

### Channelwithtreeparameters

It has been proposed to verify the validity and performance of the proposed algorithm in the case of order channel 3; the results are shown in Fig. 4. The curve represents the magnitude of the proposed algorithm approaches the reference curve by comparing the algorithm to Zhang. Detailed results of the estimated channel for samples of different sizes are offered in Table II.

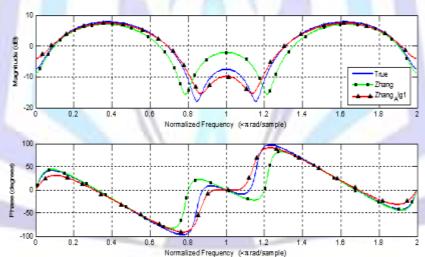


Fig. 4. Estimation of the impulse responses in amplitude and phase algorithms for N=400 samples.

Table 2.channel estimation by the two algorithms

N	Algorithm	EQM	Estimetedchannel
Thruechanal			1.0000 0.7500 -0.5800 -0.7500
400	Zhang	0.3489	1.0000 0.6127 -0.2547 -0.7699
	Zhang+Alg1	0.1815	1.0000 0.5794 -0.4417 -0.8245
800	Zhang	0.3418	1.0000 0.4164 -0.3654 -0.6873
	Zhang+Alg1	0.0926	1.0000 0.6964 -0.6665 -0.5585
1200	Zhang	0.2042	1.0000 0.8666 -0.4414 -0.4871



Zhang+Alg1	0.0262	1.0000 0.7608 -0.5802 -0.871

In the case of a noiseless channel, we note that from the MSE, and the numbers of samples (N = 400, N = 800 and N = 1200), that the algorithm of Zhang + ALG1 gives a good estimation of the system parameters FIR of order 3 (Fig. 4).

### CONCLUSION

In this paper we made a comparative study between two algorithms, the first one represent the Zhang algorithm and the second one represent our hybridization between two other algorithms, we have shown that our proposed algorithm is better compared Zhang's algorithm in terms of the mean square error, as it gives a good estimation of system parameters RIF for different channels and for different sample sizes.

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